## 68. Support of CR-hyperfunctions

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In this note we examine the possible shape of support of CR-hyperfunctions.

Let X be a complex m-dimensional complex manifold, and let N be a real k-codimensional real analytic submanifold of X, with  $0 \le k \le m$ . Throughout this note we assume that the submanifold N is a *generic* **CR**-submanifold (see [2]).

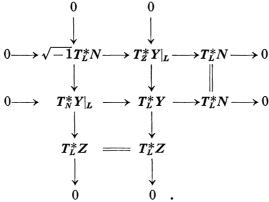
Let  $\bar{\partial}_b$  be the tangential Cauchy-Riemann system induced on N. A hyperfunction h on N which satisfies the equations  $\bar{\partial}_b h = 0$  is called a **CR**-hyperfunction. We denote by supp (h) the support of **CR**-hyperfunction h.

Remark 1. Every CR-hyperfunction defined on a Levi-flat CR-submanifold is a hyperfunction with holomorphic parameters (cf. [4]).

Let Y be a complexification of N and let  $T_N^*Y (=\sqrt{-1}T^*N)$  be the conormal bundle of N in the cotangent vector bundle  $T^*Y$  of Y. Let us denote by  $SS(\bar{\partial}_b)$  the characteristic variety of the tangential Cauchy-Riemann system  $\partial_b$ .

Note that the purely imaginary locus of the characteristic variety, denoted by  $SS(\bar{\partial}_b) \cap T_N^*Y$ , is a real 2m dimensional manifold (cf. Proposition 1.2.1 of [6]).

Let L be a real analytic submanifold of N and let Z be its complexification. Then we have the following exact sequences (cf. [5]):



Here we identify  $T_N^*Y|_L \cap T_Z^*Y|_L$  with  $\sqrt{-1}T_L^*N$ .

Definition 2. A real analytic submanifold L of N is said to be totally characteristic, if the purely imaginary conormal bundle  $\sqrt{-1}T_L^*N$  satisfies the following condition:

$$\sqrt{-1}T_L^*N\subseteq SS(\bar{\partial}_b)\cap T_N^*Y.$$

Lemma 3. If L is a totally characteristic submanifold of N, then the inequality  $\dim_{\mathbf{R}} L \ge 2m - 2k$  holds.

*Proof.* Let  $p \in L$ . If we denote by  $\sqrt{-1}T_L^*N_p$  (resp. by  $(SS(\bar{\partial}_b) \cap T_N^*Y)_p$ ) the fiber over p of the bundle  $\sqrt{-1}T_L^*N$  (resp.  $SS(\bar{\partial}_b) \cap T_N^*Y$ ), we have  $\dim_{\mathbb{R}} \sqrt{-1}T_L^*N_p = \dim_{\mathbb{R}} N - \dim_{\mathbb{R}} L$  and  $\dim_{\mathbb{R}} (SS(\bar{\partial}_b) \cap T_N^*Y)_p = k$ . Hence we have the inequality  $2m - k - \dim_{\mathbb{R}} L \le k$  which yields the conclusion. Q.E.D.

The main result of this note is the following theorem.

Theorem 4. Let h be a non-zero CR-hyperfunction on the generic CR-submanifold N. Assume that supp (h), denoted by L, is a real analytic submanifold of N. Then L is a totally characteristic submanifold of N.

Sketch of the proof. Let us denote by Z a complexification of L. An interdependence of the support and the singular spectrum of h (see [5], Proposition 3.5.2 of [3]) implies

$$\sqrt{-1}S_L^*N\subseteq SS(h)$$
,

where SS(h) denotes the singular spectrum of h. Sato's fundamental theorem implies

$$SS(h) \subseteq SS(\bar{\partial}_b) \cap S_N^*Y$$
.

Hence we have

$$\sqrt{-1}S_L^*N\subseteq SS(\bar{\partial}_b)\cap S_N^*Y.$$
 Q.E.D.

Corollary 5. Under the assumptions of Theorem 4, we have  $\dim_{\mathbb{R}} \operatorname{supp}(h) \geq 2m - 2k$ .

By the same argument as the proof of Theorem 17.1 of [1], we can conclude the following result.

Corollary 6. Let h be a CR-hyperfunction on the generic CR-submanifold N. Suppose that supp (h) is a 2m-2k dimensional real analytic submanifold of N. Then supp (h) is a complex m-k dimensional submanifold.

## References

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