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PERIODIC ACTIONS ON BRIESKORN SPHERES

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1. Introduction

In [1], Atiyah and Singer obtained an invariant for certain S^1-actions and Browder and Petrie used the invariant to distinguish certain semi-free S^1-actions [5]. In [9], we made a different approach to these problems and were able to extend the result of [5].

In the present paper, we first define invariants for some periodic actions on oriented closed manifolds (see Theorem 2.1). The idea is really a mixture of those of [9] and [10]. Then, by making use of the invariants, we distinguish periodic actions on the Brieskorn spheres (see Corollary 2.2).

2. Statements of results

Throughout this paper, we assume that p denotes an odd prime integer. We identify the group Z_p with the group $\{\exp 2\pi ai/p,\ a=0, 1, \cdots, p-1\}$. Let (M^n, φ, Z_p) be a Z_p-action on a closed oriented manifold M^n. Then the normal bundle of each component F_ν of the fixed point set has a canonical decomposition invariant under Z_p:

$$N_\nu = \sum_m N_\nu(m)$$

where the m are positive integers with $1 \leqq m \leqq (p-1)/2$ and where $N_\nu(m)$ has a unique complex structure such that $\exp(2\pi i/p)$ operates by multiplication with $\exp(2\pi mi/p)$. Therefore a fiber of the normal bundle of each component of the fixed point set has a canonical orientation. We can canonically orient F_ν so that the orientation of a fiber followed by that of F_ν yields the orientation of N_ν, where N_ν has the orientation of a tubular neighborhood of F_ν in M^n. When $N_\nu = N_\nu(m)$ for some fixed m and for all ν, we call the action a *regular* Z_p-action. Hereafter we assume that $m=1$ whenever we say *regular*. However, it will be easy to see that all the theorems in this paper still hold for any m.

Let (M^{2n-1}, φ, Z_p) be a regular Z_p-action on a closed oriented $(2n-1)$-manifold M^{2n-1}. We suppose that

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(i) The fixed point set $F(Z_p, M^{2n-1})$ is a homology sphere,

(ii) (M^{2n-1}, φ, Z_p) extends to a regular Z_p-action (W^{2n}, Φ, Z_p) ($\partial W^{2n}= M^{2n-1}$ as Z_p-manifold) such that the fixed point set $F(Z_p, W^{2n})$ is connected and the i-th Chern class of the normal complex bundle of the fixed point set is divisible by p for all $i \geqq 1$.

When $\dim F(Z_p, W^{2n}) \equiv 0 \pmod 4$, we define Pontrjagin numbers of $F(Z_p, W^{2n})$ as follows. Let $P_i(F(Z_p, W^{2n}))$ be the i-th Pontrjagin class of $F(Z_p, W^{2n})$. Since $\partial F(Z_p, W^{2n})$ $(=F(Z_p, M^{2n-1}))$ is a homology sphere, the natural homomorphism

$$j^*\colon H^i(F(Z_p, W^{2n}), \partial F(Z_p, W^{2n})) \to H^i(F(Z_p, W^{2n}))$$

is an isomorphism for $0<i \leqq \dim F(Z_p, W)-1$. Given an oriented manifold M, let $\sigma(M)$ be the orientation class.

Definition. For each nontrivial partition $\omega=\{i_1, \cdots, i_r\}$ with $d(\omega)= \dim F(Z_p, W)/4$, we define the Pontrjagin number $P_\omega[F(Z_p, W^{2n})]$ by

$$\langle j^{*-1}P_{i_1}(F(Z_p, W))\cdots j^{*-1}P_{i_r}(F(Z_p, W)), \sigma(F(Z_p, W))\rangle .$$

Then we shall obtain

Theorem 2.1. *Pontrjagin numbers $P_\omega[F(Z_p, W)]$ mod p depends only on M^{2n-1} and not on W^{2n}. If $\dim F(Z_p, M^{2n-1})<2p-3$, then the index $I(F(Z_p, W^{2n}))$ mod p depends only on M^{2n-1} and not on W^{2n}.*

As an application, we can distinguish regular Z_p-actions on the Brieskorn spheres. Recall the explicit description of homotopy spheres in bP_{4q+4r} given by Brieskorn [4] and Hirzebruch [8];

$$\Sigma^{4q+4r-1}_{3,6k-1} = \{(z_1, \cdots, z_{2q+2r+1}) \in C^{2q+2r+1} \mid z_1^3+z_2^{6k-1}+z_3^2+\cdots+z_{2q+2r+1}^2 = \varepsilon,$$
$$|z_1|^2+\cdots+|z_{2q+2r+1}|^2 = 1\}$$

where ε is a small real number. Let $\varphi\colon Z_p \to SU(2r)$ be the representation defined by

$$\varphi(e^{2\pi i/p}) = \begin{bmatrix} A(e^{2\pi i/p}) & & 0 \\ & \ddots & \\ 0 & & A(e^{2\pi i/p}) \end{bmatrix} \text{ where}$$

$$A(e^{2\pi i/p}) = \begin{bmatrix} \cos 2\pi/p & -\sin 2\pi/p \\ \sin 2\pi/p & \cos 2\pi/p \end{bmatrix}.$$

Then Z_p acts on the last $2r$ variables of $\Sigma^{4q+4r-1}_{3,6k-1}$ by means of the representation φ. Let us denote this action by $(\Sigma^{4q+4r-1}_{3,6k-1}, \varphi_{q,k}, Z_p)$. Then we shall have

Corollary 2.2. *If $4q<2p-2$ and $k\equiv k' \bmod p$, then $(\Sigma_{3,6k-1}^{4q+4r-1}, \varphi_{q,k}, Z_p)$ is not equivalent to $(\Sigma_{3,6k'-1}^{4q+4r-1}, \varphi_{q,k'}, Z_p)$.*

REMARK 2.3. Actually, if $q>r$, then $(\Sigma_{3,6k-1}^{4q+4r-1}, \varphi_{q,k}, Z_p)$ is not equivalent to $(\Sigma_{3,6k'-1}^{4q+4r-1}, \varphi_{q,k'}, Z_p)$ for $k\neq k'$ (see Ku [11]).

The proofs of Theorem 2.1 and Corollary 2.2 will be given in §3 and §4 respectively.

3. Invariants for regular Z_p-actions

Suppose a regular Z_p-action (M^{2n-1}, φ, Z_p) satisfies the hypotheses (i) and (ii) preceding Theorem 2.1 in §2. Let (W_1^{2n}, Φ_1, Z_p) and (W_2^{2n}, Φ_2, Z_p) be two extensions of the action (M^{2n-1}, φ, Z_p) satisfying (ii). Denote by ξ_1 and ξ_2 the normal complex bundles of the fixed point sets $F(Z_p, W_1)$ and $F(Z_p, W_2)$ respectively. By pasting the two Z_p-manifolds together, we obtain the action $(W, \Phi, Z_p)=(W_1 \underset{id}{\cup} (-W_2), \Phi_1\cup\Phi_2, Z_p)$ where $-W_2$ is W_2 with the opposite orientation. It follows from the uniqueness of the complex structure that the normal bundle of the fixed point set $F=F(Z_p, W_1)\cup(-(F(Z_p, W_2)))$ of the action (W, Φ, Z_p) has the complex vector bundle structure ξ whose restrictions to $F(Z_p, W_1)$ and $F(Z_p, W_2)$ are isomorphic to ξ_1 and ξ_2 respectively as complex vector bundles. Hence we have by a standard argument involving Mayer-Vietoris exact sequences that the i-th Chern class $c_i(\xi)$ is divisible by p for $0<i<\dim F/2$.

In addition to that, we shall have the following lemma.

Lemma 3.1. *$c_{i_0}(\xi)$ is divisible by p where $i_0=\dim F/2$.*

Proof. If $\dim F>\dim W/2$, $c_{i_0}(\xi)=0$ by definition. Therefore we have only to prove that $c_{i_0}(\xi)$ is divisible by p when $\dim F\leqq \dim W/2$.

We now introduce some notations. For a space X and a non-negative integer n, we denote by $\Omega_n(X)$ the bordism group in the sense of Conner and Floyd [6]. The bordism class to which $f\colon M^n\to X$ belongs is denoted by $[M^n, f]$. Following [6], we also use the notations $\Omega_n(G)(=\Omega_n(BG))$ and $\tilde{\Omega}_n(G)$ where G is a finite group. Since we can regard $\Omega_{n-2}(CP^\infty)$ (resp. $\Omega_{n-1}(Z_p)$) as the bordism group of free S^1-actions (resp. free Z_p-actions), there is a natural homomorphism

$$\mu\colon \Omega_{n-2}(CP^\infty)\to\Omega_{n-1}(Z_p)$$

defined by restricting the group S^1 to Z_p. Denote by γ_{2i+1} the element of $\Omega_{2i}(CP^\infty)$ represented by the natural free S^1-action on S^{2i+1} where $i=0, 1, 2, \cdots$. Then we can interpret the main theorem of Conner and Floyd [6] as follows.

Theorem 3.2. *There exist a sequence of manifolds $M_o^0=p$-points, M_o^4, M_o^8, $\cdots$ such that*

(i) *Ker* μ *is the submodule generated by* $\beta_1, \beta_3, \beta_5 \cdots$ *where* $\beta_{2k-1}=[M_o^0]\gamma_{2k-1}+[M_o^4]\gamma_{2k-5}+[M_o^8]\gamma_{2k-9}+\cdots$,

(ii) *the ideal of* Ω_* *generated by all the* $[M_o^{4k}]$ $(k=0, 1, 2, \cdots)$ *coincides with the ideal of all elements of* Ω_* *whose Pontrjagin numbers are all divisible by* p.

Denote by $\boldsymbol{CP}(\xi)$ the total space of the complex projective space bundle associated with ξ and by $\pi: \boldsymbol{CP}(\xi)\to F$ the projection map. Let $f: \boldsymbol{CP}(\xi)\to \boldsymbol{CP}^\infty$ be a classifying map of the canonical line bundle over $\boldsymbol{CP}(\xi)$ and $t\in H^2(\boldsymbol{CP}(\xi))$ (resp. $t_o\in H^2(\boldsymbol{CP}^\infty)$) be the first Chern class of the canonical line bundle over $\boldsymbol{CP}(\xi)$ (resp. $\boldsymbol{CP}^\infty$). Denote by f_v: $\boldsymbol{CP}^v\to \boldsymbol{CP}^\infty$, $v=0, 1, 2, \cdots$ the inclusion maps.

Denote by $D(\xi)$ (resp. $S(\xi)$) the normal disk (resp. sphere) bundle of F in W. Since the action (W, Φ, Z_p) is regular, $\mu[\boldsymbol{CP}(\xi), f]=[S(\xi), \Phi, Z_p]$, which is equal to zero by the bordism $(W-\mathrm{Int}\, D(\xi), \Phi, Z_p)$.

It follows from Theorem 3.2 that there exist $b_{2n-2k}\in\Omega_{2n-2k}$ for $k=1, 2, \cdots, n$, such that

$$[\boldsymbol{CP}(\xi), f] = \sum_{k=1}^{n} b_{2n-2k}([M_o^0]\gamma_{2k-1}+[M_o^4]\gamma_{2k-5}+\cdots)$$

in $\Omega_{2n-2}(\boldsymbol{CP}^\infty)$. Namely there exist a compact oriented manifold X^{2n-1} and a map $\bar{f}$: $X^{2n-1}\to \boldsymbol{CP}^\infty$ such that

$$\partial X = \boldsymbol{CP}(\xi)\cup -\sum_{k=1}^{n} B_{2n-2k}\times(M_o^0\times \boldsymbol{CP}^{k-1}\cup M_o^4\times \boldsymbol{CP}^{k-3}\cup\cdots)$$

and

$$\bar{f}\,|\,\boldsymbol{CP}(\xi)=f$$

and

$$\bar{f}\,|\,B_{2n-2k}\times M_o^{4i}\times \boldsymbol{CP}^{k-2i-1} = f_{k-2i-1}\circ\pi_1$$

where B_{2n-2k} is a closed oriented manifold representing the class b_{2n-2k} and π_1: $B_{2n-2k}\times M_o^{4i}\times \boldsymbol{CP}^{k-2i-1}\to \boldsymbol{CP}^{k-2i-1}$ is the projection map.

It will be convenient to introduce the following notations. Set $G=\sum_{k=1}^{n} B_{2n-2k}\times(M_o^0\times \boldsymbol{CP}^{k-1}\cup M_o^4\times \boldsymbol{CP}^{k-3}\cup\cdots)$ and set $f'=\bar{f}\,|\,G$.

Then we have

$$\begin{aligned}
&\langle f^* t_o^{n-1}, \sigma(\boldsymbol{CP}(\xi))\rangle \\
&= \langle f'^* t_o^{n-1}, \sigma(G)\rangle \\
&= \sum_{k,i} \langle \pi_1^* f_{k-2i-1}^* t_o^{n-1}, \sigma(B_{2n-2k}\times M_o^{4i}\times \boldsymbol{CP}^{k-2i-1})\rangle \\
&= \langle \pi_1^* f_{n-1}^* t_o^{n-1}, \sigma(B_0\times M_o^0\times \boldsymbol{CP}^{n-1}\rangle) \\
&= \pm p b_0\,.
\end{aligned}$$

Here we identified Ω_0 with Z. It follows that $t^{n-1}(=f^*(t_o^{n-1})\in H^{n-1}(\boldsymbol{CP}(\xi))\simeq Z)$

is divisible by p. Let x be the element of $H^{n-1}(\boldsymbol{CP}(\xi))$ such that $t^{n-1}=px$ and choose $c'_i \in H^{2i}(F)$ such that $c_i(\xi)=pc'_i$ for $0<i<\dim F/2$. According to Dold [7], $H^*(\boldsymbol{CP}(\xi))$ is a free graded $H^*(F)$-module with base $1, t, \cdots, t^{n-i_0-1}$, via the induced homomorphism π^*. It follows that there exist $y, y_1, \cdots, y_{i_0-1} \in H^{2i_0}(F)$ such that $x=\pi^*(y)t^{n-i_0-1}$ and $\pi^*(c'_i)t^{n-i-1}=\pi^*(y_i)t^{n-i_0-1}$, $i=1, \cdots, i_0-1$. On the other hand, recall the following formula (cf. Borel-Hirzebruch [2] and Bott [3]),

$$t^{n-i_0}+\sum_{i=1}^{i_0} \pi^*(c_i(\xi))\cdot t^{n-i_0-i} = 0 . \tag{3.3}$$

Hence we have

$$\begin{aligned}
&\pi^*\{p(y+\sum_{i=1}^{i_0-1} y_i)+c_{i_0}(\xi)\}\, t^{n-i_0-1} \\
&= px+p\sum_{i=1}^{i_0-1} \pi^*(c'_i)t^{n-i-1}+\pi^*(c_{i_0}(\xi))t^{n-i_0-i} \\
&= t^{n-1}+\sum_{i=1}^{i_0} \pi^*(c_i(\xi))t^{n-i-1} \\
&= t^{i_0-1}(t^{n-i_0}+\sum_{i=1}^{i_0} \pi^*(c_i(\xi))t^{n-i_0-i}) \\
&= 0 .
\end{aligned}$$

In view of the module structure of $H^*(\boldsymbol{CP}(\xi))$, we may conclude that

$$c_{i_0}(\xi) = -p(y+\sum_{i=1}^{i_0-1} y_i) .$$

This completes the proof of Lemma 3.1.

Using Lemma 3.1 and the formula (3.3), we obtain

Lemma 3.4. *t^i is divisible by p for $i \geqq n-i_0$.*

We are now ready to prove the following proposition which provides the main step in the proof of Theorem 2.1.

Proposition 3.5. *$\langle P_\omega(F), \sigma(F)\rangle \equiv \langle P_\omega(\boldsymbol{CP}(\xi))t^{n-i_0-1}, \sigma(\boldsymbol{CP}(\xi))\rangle \bmod p$ for all partitions ω with $d(\omega)=i_0/2$.*

Proof. Let η be the complex vector bundle along the fibers of the bundle $\pi: \boldsymbol{CP}(\xi) \to F$. Then the Chern class (with real coefficients) of the complex vector bundle η is generally given by the formula:

$$c(\eta) = \sum_{i=1}^{i_0} (1+t)^{n-i_0-i}\pi^*(c_i(\xi))$$

(see Borel-Hirzebruch [2]). Hereafter we say that an element $x \in H^*(X, \boldsymbol{R})$ is dividsible by p if x is in the image of $pH^*(X; Z)$ under the following natural homomorphism $H^*(X; Z) \to H^*(X; \boldsymbol{R})$ induced by the inclusion $Z \to \boldsymbol{R}$. Then we can express $c(\eta)$ as

$$c(\eta) = (1+t)^{n-i_0} + \delta_1$$

where δ_1 is an element of $H^*(\boldsymbol{CP}(\xi); \boldsymbol{R})$ divisible by p, since $c_i(\xi)$ is divisible by p for all $i \geqq 1$. The i-th Pontrjagin class $P_i(\eta)$ is in general given by $(-1)^i \sum_{j_1+j_2=2i} (-1)^{j_1} c_{j_1}(\eta) \cdot c_{j_2}(\eta)$ for a complex vector bundle η. It follows immediately that we can express $P_i(\eta)$ (with real coefficient) as

$$P_i(\eta) = (-1)^i \sum_{j_1+j_2=2i} (-1)^{j_1} \begin{bmatrix} n-i_0 \\ j_1 \end{bmatrix} \begin{bmatrix} n-i_0 \\ j_2 \end{bmatrix} t^{2i} + \delta_2$$

where δ_2 is an element of $H^*(\boldsymbol{CP}(\xi); \boldsymbol{R})$ divisible by p. Since $P(\boldsymbol{CP}(\xi)) = \pi^*(P(F))P(\eta)$ modulo 2-torsion, we have the formula (with real coefficients);

$$\begin{aligned} P_i(\boldsymbol{CP}(\xi)) &= \sum_{i_1+i_2=i} \pi^*(P_{i_1}(F)) \cdot P_{i_2}(\eta) \\ &= \sum_{i_1+i_2=i} \left\{ \pi^*(P_{i_1}(F)) \cdot (-1)^{i_2} \sum_{j_1+j_2=2i_2} (-1)^{j_1} \begin{bmatrix} n-i_0 \\ j_1 \end{bmatrix} \begin{bmatrix} n-i_0 \\ j_2 \end{bmatrix} t^{2i_2} \right\} + \delta_3 \end{aligned}$$

where δ_3 is an element of $H^*(\boldsymbol{CP}(\xi); \boldsymbol{R})$ divisible by p. Hence for each partition ω with $d(\omega) = i_0/2$, we have

$$\begin{aligned} &\langle P_\omega(\boldsymbol{CP}(\xi)) t^{n-i_0-1}, \sigma(\boldsymbol{CP}(\xi)) \rangle \\ &= \langle \pi^*(P_\omega(F)) t^{n-i_0-1} + \text{terms with higher powers of } t, \sigma(\boldsymbol{CP}(\xi)) \rangle \mod p \\ &= \langle P_\omega(F), \sigma(F) \rangle \mod p \end{aligned}$$

by Lemma 3.4. This completes the proof of Proposition 3.5.

A brief computation, using Theorem 3.2, leads to the following result.

Lemma 3.6. *The bordism Pontrjagin numbers*

$$\langle P_\omega(\boldsymbol{CP}(\xi)) f^* t_0^{n-1-2d(\omega)}, \sigma(\boldsymbol{CP}(\xi)) \rangle$$

are divisible by p for all partitions ω with $0 \leqq d(\omega) \leqq (n-1)/2$.

By combining Proposition 3.5 and Lemma 3.6, we conclude that

$$\langle P_\omega(F), \sigma(F) \rangle \equiv 0 \mod p$$

for all partitions ω with $d(\omega) = i_0/2$. We are now ready to prove our Theorem 2.1. We introduce some notatoins. Denote by a, 1 or 2. Then we set $F_a = F(Z_p, W_a)$. Let $i_a \colon F_a \to F_1 \cup (-F_2)$ be the inclusion and let $\pi \colon F_1 \cup (-F_2) \to F_1 \cup (-F_2)/\partial F_1$ be the map obtained by collapsing ∂F_1 to a point. Let $j_a \colon F_a \to F_a/\partial F_a$ be the map obtained by collapsing ∂F_a to a point and $\pi_a \colon F_1 \cup (-F_2)/\partial F_1 \to F_a/\partial F_a$ be the map obtained by collapsing F_{3-a} to a point. Since

$$j_a^*\colon H^i(F_a/\partial F_a)\to H^i(F_a)$$

is an isomorphism for $i\leqq 2i_0-1$, there exists the unique class ${}_a\hat{P}_i\in H^{4i}(F_a/\partial F_a)$ such that $j_a^*({}_a\hat{P}_i)=P_i(F_a)$ for $4i\leqq 2i_0-1$. We shall now show that $\pi^*\{\pi_1^*({}_1\hat{P}_i)+\pi_2^*({}_2\hat{P}_i)\}$ is equal to $P_i(F_1\cup(-F_2))$ for $4i\leqq 2i_0-1$. Sicne

$$i_1^*\oplus i_2^*\colon H^i(F_1\cup(-F_2))\to H^i(F_1)\oplus H^i(F_2)$$

is an isomorphism for $0<i\leqq 2i_0-1$, an element $x\in H^{4i}(F_1\cup(-F_2))$ satisfying $i_a^*x=P_i(F_a)$ $(a=1,\ 2)$ is nothing but $P_i(F_1\cup(-F_2))$. We have

$$\begin{aligned}
&i_a^*\pi^*\{\pi_1^*({}_1\hat{P}_i)+\pi_2^*({}_2\hat{P}_i)\}\\
&=i_a^*\pi^*\pi_1^*({}_1\hat{P}_i)+i_a^*\pi^*\pi_2^*({}_2\hat{P}_i)\\
&=j_a^*({}_a\hat{P}_i)\\
&=P_i(F_a)\quad\text{for }\ 0<4i\leqq 2i_0-1\,,
\end{aligned}$$

since

$$i_a^*\pi^*\pi_{a'}^*=\begin{cases} j_a^* & \text{if } a=a'\\ 0 & \text{if } a\neq a'.\end{cases}$$

Therefore we have shown that

$$\pi^*\{\pi_1^*({}_1\hat{P}_i)+\pi_2^*({}_2\hat{P}_i)\}=P_i(F_1\cup(-F_2))\,.$$

Let $\omega=(i_1,\cdots,i_r)$ be a non trivial partition of $i_0/2$, then

$$\begin{aligned}
&P_\omega(F_1\cup(-F_2))\\
&=\pi^*\{\pi_1^*({}_1\hat{P}_{i_1})+\pi_2^*({}_2\hat{P}_{i_1})\}\cdots\{\pi_1^*({}_1\hat{P}_{i_r})+\pi_2^*({}_2\hat{P}_{i_r})\}\\
&=\pi^*\{\pi_1^*({}_1\hat{P}_\omega)+\pi_2^*({}_2\hat{P}_\omega)\}\ ,
\end{aligned}$$

where ${}_a\hat{P}_\omega$ means ${}_a\hat{P}_{i_1}\cdots{}_a\hat{P}_{i_r}$, since $\pi_a^*({}_a\hat{P}_i)\cdot\pi_{a'}^*({}_{a'}\hat{P}_{i'})=0$ for $a\neq a'$. Hence we have

$$\begin{aligned}
&\langle P_\omega(F_1\cup -F_2),\ \sigma(F_1\cup(-F_2))\rangle\\
&=\langle\pi^*\{\pi_1^*({}_1\hat{P}_\omega)\}+\pi_2^*({}_2\hat{P}_\omega)\},\ \sigma(F_1\cup(-F_2))\rangle\\
&=\langle\pi_1^*({}_1\hat{P}_\omega)+\pi_2^*({}_2\hat{P}_\omega),\ \pi_*\sigma(F_1\cup(-F_2))\rangle\\
&=\langle\pi_1^*({}_1\hat{P}_\omega),\ \pi_*\sigma(F_1\cup(-F_2))\rangle+\langle\pi_2^*({}_2\hat{P}_\omega),\ \pi_*\sigma(F_1\cup(-F_2))\rangle\\
&=\langle{}_1\hat{P}_\omega,\ \pi_{1*}\pi_*\sigma(F_1\cup(-F_2))\rangle+\langle{}_2\hat{P}_\omega,\ \pi_{2*}\pi_*\sigma(F_1\cup(-F_2))\rangle\\
&=\langle{}_1\hat{P}_\omega,\ \sigma(F_1/\partial F_1)\rangle+\langle{}_2\hat{P}_\omega,\ -\sigma(F_2/\partial F_2)\rangle\\
&=P_\omega[F_1]-P_\omega(F_2)\,.
\end{aligned}$$

Thus we have that

$$P_\omega[F_1]\equiv P_\omega(F_2)\qquad \operatorname{mod} p\,.$$

If $\dim F<2p-2$, then $\langle P_\omega(F),\ \sigma(F)\rangle\equiv 0\ \operatorname{mod} p$ means that $[F]$ is divisible by p in Ω_* (see Conner-Floyd [6]). In particular, $I(F)\equiv 0 \mod p$. Since $I(F)=I(F_1)-I(F_2)$, we have that

$$I(F_1)\equiv I(F_2) \qquad \mod p\,.$$

This completes the proof of Theorem 2.1.

4. An application to Z_p-actions on Brieskorn spheres

Let $\sum_{3,6k-1}^{4q-1}\subset\sum_{3,6k-1}^{4q+4r-1}$ be the imbedding defined by

$$(z_1, \cdots, z_{2q+1}) \mapsto (z_1, \cdots, z_{2q+1}, 0, \cdots, 0)\,,$$

then the fixed point set of the action $(\sum_{3,6k-1}^{4q+4r-1}, \varphi_{q,k}, Z_p)$ is this submanifold which is a homology sphere [4], [13]. The manifold

$$\begin{aligned} W_{3,6k-1}^{4q+4r} = \{(z_1, \cdots, z_{2q+2r+1})\in C^{2q+2r+1} \,|\, z_1^3+z_2^{6k-1}+z_3^2+\cdots \\ +z_{2q+2r+1}^2=\varepsilon,\ |z_1|^2+\cdots+|z_{2q+2r+1}|^2\leqq 1\} \end{aligned}$$

admits a regular Z_p-action given in the manner of the action $(\sum_{3,6k-1}^{4q+4r-1}, \varphi_{q,k}, Z_p)$. We denote it by $(W_{3,6k-1}^{4q+4r}, \Phi_{q,k}, Z_p)$. Then the restriction $\partial(W_{3,6k-1}^{4q+4r}, \Phi_{q,k}, Z_p)$ to the boundary $\partial W_{3,6k-1}^{4p+4q}$ is nothing but $(\sum_{3,6k-1}^{4q+4r-1}, \varphi_{q,k}, Z_p)$. The fixed point set of the action $[W_{3,6k-1}^{4q+4r}, \Phi_{q,k}, Z_p]$ is $W_{3,6k-1}^{4q}$ which is connected and the normal complex bundle of the fixed point set is trivial, i.e., the hypotheses (i) and (ii) preceding the statement of Theorem 2.1 in §2 are satisfied. According to [4], $I[W_{3,6k-1}^{4q}]=(-1)^q 8k$. It follows from Theorem 2.1 that $(\sum_{3,6k-1}^{4q+4r-1}, \varphi_{q,k}, Z_p)$ is not equivalent to $(\sum_{3,6k'-1}^{4q+4r-1}, \varphi_{q,k'}, Z_p)$ if $k\not\equiv k' \mod p$. This completes the proof of Corollary 2.2.

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