On Unconditional Bases in Tensor Products of Köthe Echelon Spaces

Juan Carlos Díaz

Let us denote by e_i the sequence taking the value 1 in the ith place and 0 elsewhere, $i \in \mathbb{N}$. It is known that the tensor product basis $(e_i \otimes e_j)_{i,j\in\mathbb{N}}$ of $l_p \otimes_\pi l_q$ is not unconditional, for $p, q \in (1,\infty) \cup \{0\}$ (see [12; 13]). In the case of Fréchet spaces, the tensor product basis is unconditional in the projective tensor product $\lambda_p(A) \otimes_\pi \lambda_q(B)$ of Köthe sequence spaces if one of the spaces is nuclear. Indeed, if $\lambda_p(A)$ is nuclear then the completion of $\lambda_p(A) \otimes_\pi \lambda_q(B)$ coincides in a canonical way with the vector-valued sequence space $\lambda_p(A, \lambda_q(B))$. It follows from [5, 4.1(4)] that $(e_i \otimes e_j)_{i,j\in\mathbb{N}}$ is unconditional in $\lambda_0(A) \otimes_\pi \lambda_0(A)$ if and only if $\lambda_0(A)$ is nuclear. In this note, given Köthe matrices $A = (a^n)$ and $B = (b^n)$, we prove that the tensor product basis of $\lambda_p(A) \otimes_\pi \lambda_q(B)$ $(p, q \in (1,\infty) \cup \{0\})$ is unconditional if and only if, for each n, there exists an m such that, for every bijection $\sigma\colon \mathbb{N} \to \mathbb{N}$, the sequence $(a_i^n b_{\sigma(i)}^n / a_i^m b_{\sigma(i)}^m)$ belongs to l_1. This condition arose in a recent paper by Bonet et al. ([4], see also [9]) characterizing the coincidence of the π and ε topologies on $\lambda_p(A) \otimes \lambda_q(B)$. In fact, this article is strongly influenced by the results and techniques of [4]. We also prove that the condition just described is equivalent to the unconditionality of the tensor basis in the injective tensor product $\lambda_p(A) \otimes_\varepsilon \lambda_q(B)$ with $p, q \in [1,\infty)$.

As a further consequence we derive applications in infinite holomorphy; namely, we prove that a Montel space $\lambda_p(A)$, $p \in (1,\infty) \cup \{0\}$, is nuclear if the monomials form an unconditional basis of $(\mathcal{H}(\lambda_p(A)), \tau_0)$, the space of holomorphic functions endowed with the compact-open topology. Similar results were obtained in [6] and [5]: if E is a Montel locally convex space with basis such that the monomials form an absolute basis of $(\mathcal{H}(E), \tau_0)$ then E'_β is nuclear [6], and if E is a Fréchet Montel or a (DF) Montel space such that E'_β has an absolute basis and the monomials are an unconditional basis of $(\mathcal{H}(E), \tau_0)$ then E is nuclear [5].

We refer the reader to [1], [8], and [10] for notation and definitions not included here concerning Köthe sequence spaces and projective or injective tensor products. The space c_0 will be also denoted by l_0, and the Fréchet spaces are defined over $\mathbb{R}$ or $\mathbb{C}$. To obtain our main result we use Walsh matrices defined by

$$W_1 := \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \quad \text{and} \quad W_{k+1} := \begin{pmatrix} W_k & W_k \\ W_k & -W_k \end{pmatrix}.$$

Received October 22, 1996. Revision received February 13, 1997.
Research supported by the DGICYT project no. PB94-0441.
Michigan Math. J. 44 (1997).

An upper bound for the ε-norm of W_k as an element of $l_{p,2^k} \otimes_\varepsilon l_{q,2^k}$ was obtained in [7]. From the bounds given there and the fact that

$$2^{2k} = \langle W_k, W_k \rangle \leq \| W_k \|_\varepsilon \| W_k \|_\pi$$

we have the following estimates, where 0 stands for $1/\infty$.

1. LEMMA. *Consider W_k as an element of $l_{p,2^k} \otimes_\pi l_{q,2^k}$. Then*
(i) $\| W_k \| \geq 2^{k(1+1/q)}$ *if* $1 \leq p \leq 2 \leq q \leq \infty$,
(ii) $\| W_k \| \geq \min\{2^{k(1+1/p)}, 2^{k(1+1/q)}\}$ *if* $1 \leq p, q \leq 2$, *and*
(iii) $\| W_k \| \geq 2^{k(1/2+1/p+1/q)}$ *if* $2 \leq p, q \leq \infty$.

In Lemma 2 we shall consider W_k as an element of

$$\mathrm{sp}\{e_i;\ i = 1, \ldots, 2^k\} \otimes \mathrm{sp}\{e_i;\ i = 1, \ldots, 2^k\} \subset l_p \otimes l_q.$$

Thus, if $W_k = (\gamma_{ij}^k)_{i,j=1,\ldots,2^k}$, we define the following element in $l_p \otimes l_q$:

$$w_k := \sum_{i,j=1}^{2^k} \gamma_{ij}^k e_i \otimes e_j.$$

Before stating our main result, we recall some basic facts about symmetric tensors. We say that $z \in E \otimes E$ is *symmetric* if it has a representation

$$z = \sum_{i=1}^{n} a_i \otimes b_i + b_i \otimes a_i, \quad a_i, b_i \in E, \ \ i = 1, \ldots, n.$$

Symmetric tensors form a vector subspace of $E \otimes E$ denoted by $E \otimes^s E$. We denote by $E \otimes_\alpha^s E$ the space of symmetric tensors endowed with the topology induced by $E \otimes_\alpha E$, $\alpha = \pi$ or ε. See [11] for more details. If E has a basis $(e_i)_{i \in \mathbb{N}}$ then $(e_i \otimes e_j + e_j \otimes e_i)_{i \leq j}$ with a suitable order is a Schauder basis of $E \otimes_\alpha^s E$.

We denote by Δ the set of infinite matrices $(\delta_{ij})_{i,j \in \mathbb{N}}$ with finitely many nonvanishing coordinates and such that $|\delta_{ij}| \leq 1$, $i, j \in \mathbb{N}$; Δ_s denotes the subset of the symmetric elements of Δ. Given any $\delta \in \Delta$, we define a linear operator T_δ on the space of infinite matrices of scalars (x_{ij}) as follows: $T_\delta(x_{ij}) := (x_{ij}\delta_{ij})$.

We now present our main technical result.

2. LEMMA. *Let $A: l_p \to l_p$ and $B: l_q \to l_q$ be continuous diagonal operators defined by $A(x_i) := (x_i a_i)$ and $B(y_i) := (y_i b_i)$, where $a = (a_i)$ and $b = (b_i)$ are bounded and strictly positive sequences.*

(a) *Assume that the family of operators $\{T_\delta \circ (A \otimes B);\ \delta \in \Delta\}$ is uniformly bounded in $L(l_p \otimes_\alpha l_q, l_p \otimes_\alpha l_q)$ for $\alpha = \pi$ and $p, q \in (1, \infty) \cup \{0\}$ or for $\alpha = \varepsilon$ and $p, q \in [1, \infty)$. Then there exists r, $1 \leq r < \infty$, depending only on p and q, such that $(a_i b_{\sigma(i)}) \in l_r$ for every permutation of the integers $(\sigma(i))$.*

(b) *The same conclusion follows if $p = q$, $a = b$, and $\{T_\delta \circ (A \otimes B);\ \delta \in \Delta_s\}$ is uniformly bounded in $L(l_p \otimes_\alpha^s l_q, l_p \otimes_\alpha^s l_q)$.*

Proof. (a) We treat the case $\alpha = \pi$. Let us assume that a and b are decreasing. This hypothesis will be dropped at the end of the proof. Since a and b are decreasing it is enough to prove that $(a_i b_i) \in l_r$ for suitable r (see [4, Observation]).

Moreover in this case it suffices to show that the sequence $(a_{2^i}b_{2^i}2^{is})$ is bounded for some $s > 0$. In fact, this implies that $((a_{2^i}b_{2^i})^{2/s}2^{2i})$ is bounded, whence $((a_{2^i}b_{2^i})^{2/s}2^{i}) \in l_1$ and thus $(a_i b_i) \in l_{2/s}$.

For every $n \in \mathbb{N}$, let $c_n := \sum_{i=1}^{2^n} e_i$ and define

$$x_n := 2^{-n/p}c_n, \quad y_n := 2^{-n/q}c_n, \quad z_n := a_{2^n}b_{2^n}2^{-n(1/p+1/q)}\omega_n,$$

where $1/0$ is taken as 0. For every pair (i, j) with $1 \le i, j \le 2^n$, we have

$$\begin{aligned} |\langle z_n, e_i^* \otimes e_j^* \rangle| &= (a_{2^n}b_{2^n})2^{-n(1/p+1/q)} \le a_i b_j 2^{-n(1/p+1/q)} \\ &= \langle (A \otimes B)(x_n \otimes y_n), e_i^* \otimes e_j^* \rangle; \end{aligned}$$

for any other pair (i, j) we have $\langle z_n, e_i^* \otimes e_j^* \rangle = 0$. Thus

$$|\langle z_n, e_i^* \otimes e_j^* \rangle| \le |\langle (A \otimes B)(x_n \otimes y_n), e_i^* \otimes e_j^* \rangle| \quad \forall i, j \in \mathbb{N}.$$

This means that $z_n = T_{\delta_n} \circ (A \otimes B)(x_n \otimes y_n)$ for some $\delta_n \in \Delta$. By hypothesis there exists $M > 0$, not depending on n, with $\|z_n\| \le M\|x_n\|_{l_p}\|y_n\|_{l_q} = M$. Let us now estimate $\|z_n\|$ from Lemma 1; we consider three cases as follows.

(1) If $1 \le p \le 2 \le q < \infty$ or $q = 0$, we have

$$\|z_n\| \ge a_{2^n}b_{2^n}2^{-n(1/p+1/q)}2^{n(1+1/q)} = a_{2^n}b_{2^n}2^{n(1/p')}.$$

(*Mutatis mutandi*, the case $1 \le q \le 2 \le p < \infty$ or $p = 0$ is included here.)

(2) If $1 \le p, q \le 2$ then

$$\|z_n\| \ge a_{2^n}b_{2^n}2^{n(1/q')} \;\forall n \in \mathbb{N} \quad \text{or} \quad \|z_n\| \ge a_{2^n}b_{2^n}2^{n(1/p')} \;\forall n \in \mathbb{N}.$$

(3) If $2 \le p, q < \infty$ or $p = 0$ or $q = 0$, we obtain

$$\|z_n\| \ge a_{2^n}b_{2^n}2^{-n(1/p+1/q)}2^{(1/2+1/p+1/q)} = a_{2^n}b_{2^n}2^{n/2}.$$

Thus, in any case the sequence $(a_{2^n}b_{2^n}2^{ns})$ is bounded for some $0 < s \le 1/2$. Note that s depends only on the indices p and q. This completes the proof when a and b are decreasing.

Let us now drop the assumption that a and b are decreasing. If both a and b belong to c_0 then we consider their decreasing rearrangements and proceed as before. If one of them, say a, belongs to c_0 (thus we assume that it is decreasing) and b does not converge to zero, then there is an infinite subset $J \subset \mathbb{N}$ such that $\rho \le b_i \le K$ for some $K, \rho > 0$ and every $i \in J$. We restrict our attention to the subspace $\overline{\mathrm{sp}}\{e_i;\, i \in J\} \equiv l_q(J)$ in the right-hand side of the tensor product to conclude that

$$(\mathrm{id} \otimes (1/B)) : l_p \otimes_\pi l_q(J) \to l_p \otimes_\pi l_q(J)$$

is an isomorphism, where $1/B$ is the diagonal operator associated to $(b_i^{-1})_{i \in J}$. Moreover $A \otimes \mathrm{id}$ can be written as $(\mathrm{id} \otimes (1/B)) \circ (A \otimes B)$. Thus, the family of operators

$$\{T_\delta \circ (A \otimes \mathrm{id});\; \delta \in \Delta\}$$

is uniformly bounded. We can apply the case already considered to conclude that a belongs to l_r for some $1 < r < \infty$ and consequently ab belongs to l_r. To finish the proof we show that $a \notin c_0$ and $b \notin c_0$ yields a contradiction. In fact, proceeding as before, there are infinite subsets $I, J \subset \mathbb{N}$ such that a and b are bounded and bounded away from zero when restricted to I and J, respectively. Hence the mapping

$$((1/A) \otimes (1/B)) \colon l_p(I) \otimes_\pi l_q(J) \to l_p(I) \otimes_\pi l_q(J)$$

is an isomorphism. Therefore $\{T_\delta;\ \delta \in \Delta\}$ is uniformly bounded in

$$L\big(l_p(I) \otimes_\pi l_q(J), l_p(I) \otimes_\pi l_q(J)\big)$$

and this implies that the basis $(e_i \otimes e_j)_{i,j \in \mathbb{N}}$ is unconditional in $l_p \otimes_\pi l_q$, a contradiction.

The case $\alpha = \varepsilon$ follows by duality. If the family of operators $\{T_\delta \circ (A \otimes B);\ \delta \in \Delta\}$ is uniformly bounded for the ε-norm then we transpose and apply the preceding result. It is enough to observe that $(T_\delta \circ (A \otimes B))^t = (A \otimes B) \circ T_\delta = T_\delta \circ (A \otimes B)$. Moreover, $(l_p \otimes_\varepsilon l_q)' = l_{p'} \otimes_\pi l_{q'}$ when $p, q \in (1, \infty)$; if either p or q is unity then we use the duality $(c_0 \otimes_\pi l_q)' = l_1 \otimes_\varepsilon l_{q'}$, $q' \in [1, \infty)$. The proof can also be obtained directly, defining

$$z_n := a_{2^n} b_{2^n} c_n \otimes c_n, \quad n \in \mathbb{N}.$$

As in the previous case, there exists $\delta_n \in \Delta$ such that $z_n = T_{\delta_n} \circ (A \otimes B)(\omega_n)$ and, for some M not depending on n, we have

$$a_{2^n} b_{2^n} 2^{n(1/p+1/q)} = \|z_n\| \le M \|\omega_n\|_\varepsilon.$$

The upper bounds of $\|\omega_n\|_\varepsilon$ in [7] can be applied to conclude the proof.

The proof of part (b) follows as before if we observe that, if $p = q$ and $a = b$, all the elements defined in part (a) are symmetric. □

We can now state the announced characterization. The remarkable equivalence of (b) and (c) was obtained in [4], and condition (d) has been observed by Peris.

3. THEOREM. *For $\alpha = \pi$ and $p, q \in (1, \infty) \cup \{0\}$ or for $\alpha = \varepsilon$ and $p, q \in [1, \infty)$, the following conditions are equivalent:*

(a) *the tensor product basis of $\lambda_p(A) \otimes_\alpha \lambda_q(B)$ is unconditional;*
(b) *for each $n \in \mathbb{N}$, there exists $m > n$ such that $(a_i^n b_{\sigma(i)}^n / a_i^m b_{\sigma(i)}^m) \in l_1$ for every bijection $\sigma \colon \mathbb{N} \to \mathbb{N}$;*
(c) *$\lambda_p(A) \otimes_\pi \lambda_q(B) = \lambda_p(A) \otimes_\varepsilon \lambda_q(B)$ holds topologically;*
(d) *$\lambda_p(A) \otimes_\varepsilon \lambda_q(B)$ is barreled.*

Moreover if $p = q$ and $A = B$ then (a)–(d) *are equivalent to*

(e) *the basis $(e_i \otimes e_j + e_j \otimes e_i)_{i \le j}$ is unconditional in $\lambda_p(A) \otimes_\alpha^s \lambda_p(A)$.*

Proof. (a) $\Rightarrow$ (b) Let us denote $z_{ij} = e_i \otimes e_j$ for all i and j. By hypothesis, $(z_{ij})_{i,j}$ is an unconditional basis of $\lambda_p(A) \otimes_\alpha \lambda_q(B)$ for $\alpha = \pi$ or ε. Hence, for every $n \in \mathbb{N}$ there exist $m \ge n$ and $M > 0$ such that

$$\Big\|\sum_{i,j}\delta_{ij}a_{ij}z_{ij}\Big\|_n \le M\Big\|\sum_{i,j}a_{ij}z_{ij}\Big\|_m$$

for all $\delta = (\delta_{ij}) \in \Delta$ and every $a = \sum_{i,j} a_{ij}z_{ij} \in \lambda_p(A) \otimes_\alpha \lambda_q(B)$ [14, Thm. 1.19(2)]. Equivalently, the family $\{T_\delta;\ \delta \in \Delta_s\}$ is uniformly bounded from $l_p(a^m) \otimes_\alpha l_q(b^m)$ into $l_p(a^n) \otimes_\alpha l_q(b^n)$. We consider the following diagonal operators (if $p = 0$ or $q = 0$ then $1/p$ and respectively $1/q$ should be replaced by 1):

$$\begin{aligned}
&A_1\colon l_p \to l_p(a^m),\ (x_i) \to (x_i(a_i^m)^{-1/p}), && B_1\colon l_q \to l_q(b^m),\ (x_i) \to (x_i(b_i^m)^{-1/q}),\\
&A_2\colon l_p(a^n) \to l_p,\ (x_i) \to (x_i(a_i^n)^{1/p}), && B_2\colon l_q(b^n) \to l_q,\ (x_i) \to (x_i(b_i^n)^{1/q}),\\
&A_0\colon l_p \to l_p,\ (x_i) \to (x_i(a_i^n/a_i^m)^{1/p}), && B_0\colon l_q \to l_q,\ (x_i) \to (x_i(b_i^n/b_i^m)^{1/q}).
\end{aligned}$$

It is readily checked that $T_\delta \circ (A_0 \otimes B_0) = (A_2 \otimes B_2) \circ T_\delta \circ (A_1 \otimes B_1)$, $\delta \in \Delta$; hence $\{T_\delta \circ (A_0 \otimes B_0);\ \delta \in \Delta\}$ is uniformly bounded. From Lemma 2 we have $1 \le r < \infty$ such that $((a_i^n/a_i^m)^{1/p}(b_{\sigma(i)}^n/b_{\sigma(i)}^m)^{1/q})_i$ and therefore $(a_i^n b_{\sigma(i)}^n/a_i^m b_{\sigma(i)}^m)_i$ belongs to l_r for every permutation of the integers $(\sigma(i))$. If we repeat the process several times we obtain $(a_i^n b_{\sigma(i)}^n/a_i^m b_{\sigma(i)}^m) \in l_1$ for some $m \ge n$ and all $(\sigma(i))$.

(c) $\Rightarrow$ (a) In the vector-valued space $\lambda_p(A, \lambda_q(B))$, denote by $h(i, j)$ the element taking the value e_j in the ith place and 0 elsewhere. Then $(h(i, j))_{i,j}$ is an unconditional basis. Our hypothesis implies that the completion of the space $\lambda_p(A) \otimes_\pi \lambda_q(B)$ coincides with $\lambda_p(A, \lambda_q(B))$ [4, Prop. 8]; moreover, $e_i \otimes e_j$ can be identified with $h(i, j)$ for $i, j \in \mathbb{N}$. The assertion is proved.

The equivalence of (c) and (d) follows from [8, 15.6.6] and [10, 11.5.8]; see also [3].

(a) $\Rightarrow$ (e) The basis $(e_i \otimes e_j + e_j \otimes e_i)_{i\le j}$ is a block basic sequence of $(e_i \otimes e_j)_{i,j\in\mathbb{N}}$, and every block basic sequence of an unconditional basis is unconditional (see [14, 1.22]). Finally (e) $\Rightarrow$ (b) is proved in the same way as (a) $\Rightarrow$ (b). □

We complete this note with a consequence in infinite holomorphy. Let E be a complex Fréchet space with a basis $(e_n)_{n\in\mathbb{N}}$, and let $(e_n^*)_{n\in\mathbb{N}} \subset E'$ denote the associated sequence of biorthogonal functionals. For every finitely nonzero sequence $m = (m_k)$ of nonnegative integers, the function

$$e^m\colon z \in E \to \prod_k (e_k^*(z))^{m_k} \in \mathbb{C}$$

is said to be a monomial on E with respect to $(e_n^*)_{n\in\mathbb{N}}$. From results in [2], if E is nuclear then the monomials form an absolute basis of $(\mathcal{H}(E), \tau_0)$. Certain converses of this theorem were obtained in [6] and [5]. Let us denote by $\mathcal{P}(^2E)$ the space of 2-homogeneous continuous polynomials P on E. A 2-homogeneous polynomial P is the restriction of a symmetric 2-homogeneous linear form on $E \times E$ to the diagonal. If $i, j \in \mathbb{N}$ then we define the element $e_i^* \odot e_j^* \in \mathcal{P}(^2E)$ by $e_i^* \odot e_j^*(z) := e_i^*(z)e_j^*(z)$, $z \in E$. If E is a Montel Köthe echelon space $\lambda_p(A)$ then $(e_i^* \odot e_j^*)_{i\le j}$ is a Schauder basis of $(\mathcal{P}(^2\lambda_p(A)), \tau_0)$. In our next result we show that this basis is not unconditional unless $\lambda_p(A)$ is nuclear.

4. THEOREM. *Let $\lambda_p(A)$ be Montel and let $p \in (1, \infty) \cup \{0\}$. Then the following are equivalent.*

(a) *The monomials form an unconditional basis of $(\mathcal{H}(\lambda_p), \tau_0)$.*
(b) *The family of 2-homogeneous polynomials $(e_i^* \odot e_j^*)_{i \leq j}$ forms an unconditional basis of the space $(\mathcal{P}(^2\lambda_p(A)), \tau_0)$.*
(c) *$\lambda_p(A)$ is nuclear.*

Proof. The proof of (a) $\Rightarrow$ (b) can be found in [5]; (c) $\Rightarrow$ (a) is a particular case of [2]. To show (b) $\Rightarrow$ (c), observe that $(\mathcal{P}(^2\lambda_p(A)), \tau_0)$ is the strong dual of $\lambda_p(A) \otimes_\pi^s \lambda_p(A)$, with the duality given by

$$(e_n^* \odot e_m^*)(x \otimes y + y \otimes x) = e_n^*(x+y)e_m^*(x+y) - e_n^*(x)e_m^*(x) - e_n^*(y)e_m^*(y).$$

Moreover, $(e_i^* \odot e_j^*)_{i \leq j}$ is biorthogonal to $(e_i \otimes e_j + e_j \otimes e_i)_{i \leq j}$. Thus the hypothesis (b) implies part (e) of Theorem 3 with $\alpha = \pi$. According to Theorem 3, for every n there exists m such that $(a_i^n / a_i^m) \in l_2$. This readily implies that $\lambda_p(A)$ is nuclear. □

References

[1] K. D. Biersdtedt, R. Meise, and W. Summers, *Köthe sets and Köthe sequence spaces,* Functional analysis, holomorphy and approximation theory, North-Holland Math. Stud., 71, pp. 27–91, North-Holland, Amsterdam, 1982.
[2] P. J. Boland and S. Dineen, *Holomorphic functions on fully nuclear spaces,* Bull. Soc. Math. France 106 (1978), 311–336.
[3] J. Bonet, *Una nota sobre la coincidencia de topologías en productos tensoriales,* Rev. Real Acad. Cienc. Exact. Fís. Natur. Madrid 81 (1987), 87–89.
[4] J. Bonet, A. Defant, A. Peris, and M. S. Ramanujan, *Coincidence of topologies on tensor products of Köthe echelon spaces,* Studia Math. 111 (1994), 263–281.
[5] A. Defant and L. A. Moraes, *On unconditional bases in spaces of holomorphic functions in infinite dimension,* Arch. Math. (Basel) 56 (1991), 163–173.
[6] S. Dineen and R. M. Timoney, *Absolute bases, tensor products and a theorem of Bohr,* Studia Math. 94 (1989), 227–234.
[7] V. I. Gurarii, M. E. Kadec, and V. I. Macaev, *Distance between finite-dimensional analogs of the L_p-spaces,* Mat. Sb. (N.S.) 70 (1966), 481–489.
[8] H. Jarchow, *Locally convex spaces,* Teubner, Stuttgart, 1981.
[9] K. John, *Counterexample to a conjecture of Grothendieck,* Math. Ann. 265 (1983), 169–179.
[10] P. Pérez Carreras and J. Bonet, *Barrelled locally convex spaces,* North-Holland Math. Stud., 131, North-Holland, Amsterdam, 1987.
[11] R. Ryan, *Applications of topological tensor products to infinite dimensional holomorphy,* Ph.D. thesis, Trinity College, Dublin, 1980.
[12] C. Schütt, *Unconditionality in tensor products,* Israel J. Math. 31 (1978), 209–216.

[13] I. Singer, *Bases in Banach spaces I,* Springer, Berlin, 1970.
[14] L. J. Weill, *Unconditional and shrinking bases in locally convex spaces,* Pacific J. Math. 29 (1969), 467–483.

Matemáticas
E.T.S.I.A.M.
Universidad de Córdoba
14004 Córdoba
Spain

ma1dialj@lucano.uco.es