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On the homotopy type of FDiff $(S^3, \mathscr{F}_R)$

By

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§1. Introduction.

In this article, we study some topological subgroups of the group of diffeomorphisms of a manifold.

Our main result is that the topological group of foliation preserving diffeomorphisms of S^3 with respect to the Reeb foliation has the homotopy type of a torus $S^1 \times S^1$.

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§2. Spaces of foliation preserving diffeomorphisms and leaf preserving diffeomorphisms.

Let M be a smooth manifold of dimension n with a smooth foliation of codimension q.

Definition. A diffeomorphism $f: M \to M$ is called a *foliation preserving diffeomorphism* (*resp. a leaf preserving diffeomorphism*) if for each point x of M, the leaf through x is mapped into the leaf through $f(x)$ (*resp.* x), that is, $f(L_x) = L_{f(x)}$ (*resp.* $f(L_x) = L_x$), where L_x is the leaf that contains x. Equivalently, if there is a homeomorphism $\bar{f}$ (*resp. id*) of the leaf space $M/\mathscr{F}$, such that the diagram commutes, where vertical arrows are canonical projections (*see Reeb* [7]).

$$\begin{array}{ccc} M & \xrightarrow{f} & M \\ \downarrow & & \downarrow \\ M/\mathscr{F} & \xrightarrow{\bar{f}} & M/\mathscr{F} \end{array} \qquad \left(resp. \quad \begin{array}{ccc} M & \xrightarrow{f} & M \\ \downarrow & & \downarrow \\ M/\mathscr{F} & \xrightarrow{id} & M/\mathscr{F} \end{array} \right)$$

Let FDiff$(M, \mathscr{F})$ (*resp.* LDiff$(M, \mathscr{F})$) denote the space of all foliation (*resp. leaf*) preserving diffeomorphisms of M with respect to $\mathscr{F}$.

It is clear that LDiff$(M, \mathscr{F}) \subset$ FDiff$(M, \mathscr{F}) \subset$ Diff(M). Topologies of the spaces are induced by a relevant C^∞-topology of Diff(M). Clearly there is an exact sequence of topological groups:

$$0 \longrightarrow \mathrm{LDiff}(M, \mathscr{F}) \longrightarrow \mathrm{FDiff}(M, \mathscr{F}) \xrightarrow{\pi} \mathrm{Homeo}(M/\mathscr{F}),$$

where the second arrow is the inclusion map and the map π is defined as in the definition of the foliation preserving diffeomorphisms, $\pi : f \longrightarrow \bar{f}$.

Remark. In many important cases as foliations constructed by geometric manners, the homotopy types of the spaces of foliation preserving diffeomorphisms are not trivial. For example, π_1(FDiff$(S^5, \mathscr{F}_L)$) has non trivial element, where $\mathscr{F}_L$ is the codimension 1 foliation constructed by Lawson [2].

§3. Reeb foliation $\mathscr{F}_R$.

Let α be a C^∞-function $\alpha : [0, 1) \longrightarrow R$, such that $\alpha(0) = 0$, $\alpha'(t) > 0$ for all $t \in (0, 1)$, $\alpha^{(k)}(0) = 0$, $\lim_{t \to 1} \alpha^{(k)}(t) = \infty$ for all k. α defines a homeomorphism of $[0, 1)$ onto $[0, \infty)$, and α restricted to $(0, 1)$ is a diffeomorphism $(0, 1) \cong (0, \infty)$. Let $\beta = \alpha^{-1} : [0, \infty) \to [0, 1)$.

Three dimensional sphere can be decomposed into two solid tori, $S^3 = D^2 \times S^1 \cup_h S^1 \times D^2$, where h is a diffeomorphism $h : \partial D^2 \times S^1 \to S^1 \times \partial D^2$ defined by $h(x, y) = (x, y)$. Let $X = D^2 \times S^1 \subset S^3$ be one of the components. X can be considered as the quotient space $I \times S^1$

$\times S^1/\sim$, where $S^1=\mathbf{R}/\mathbf{Z}$, $I=[0, 1]$, the equivalence relation is defined by $(0, x, y) \sim (0, x', y)$. Express a point p of X as $p=(t, x, y)$, $(t, x)\in D^2$, $y\in S^1$, t is the radius and x is the polar angle mod 1.

Define a foliation in X as follows: for two points $p_1=(t_1, x_1, y_1)$, $p_2=(t_2, x_2, y_2)$ of X, $L_{p_1}=L_{p_2}$ if and only if $t_1=t_2=1$ or $\alpha(t_1)-y_1\equiv\alpha(t_2)-y_2 \pmod 1$. Introduce the same foliation in the other component, then these together define a foliation in S^3. This foliation is called the *Reeb foliation*. Let us denote $(S^3, \mathscr{F}_R)$ or simply $\mathscr{F}_R$.

Let μ: int$X \longrightarrow S^1$ be a map defined by $\mu(t, x, y)=\alpha(t)-y \pmod 1$. Then μ induces a diffeomorphism $\bar{\mu}$: int$X/(\mathscr{F}_R|_{\mathrm{int}X}) \longrightarrow S^1$, so that the following diagram commutes,

$$\begin{tikzcd}
\mathrm{int}X \arrow[r, "\mu"] \arrow[d] & S^1 \\
\mathrm{int}X/(\mathscr{F}_R|_{\mathrm{int}X}). \arrow[ur, "\bar{\mu}"'] &
\end{tikzcd}$$

§4. Lemmas.

Lemma 1. *There is a splitting exact sequence*:

$$0 \longrightarrow \mathrm{LDiff}(S^3, \mathscr{F}_R) \longrightarrow \mathrm{FDiff}_0(S^3, \mathscr{F}_R) \underset{\longleftarrow}{\overset{\pi}{\longrightarrow}} S^1\times S^1 \longrightarrow 0,$$

where $\mathrm{FDiff}_0(S^3, \mathscr{F}_R)$ *is the identity component of* $\mathrm{FDiff}(S^3, \mathscr{F}_R)$, $\mathrm{LDiff}(S^3, \mathscr{F}_R)$ *is the space of leaf preserving diffeomorphisms in* FDiff $(S^3, \mathscr{F}_R)$, $S^1\times S^1$ *is the subgroup of standard rotations* $SO(2)\times SO(2)$ *in* $\mathrm{Homeo}(S^3/\mathscr{F}_R)$, *and the global section*$\leftarrow$*is the natural lifting of standard rotations.*

Proof. It is clear that the image of π contains the standard rotations $S^1\times S^1$. Let us prove that the image of π is contained in $S^1\times S^1$. Suppose for some f in $\mathrm{FDiff}_0(\mathscr{F}_R)$, $\pi f \notin SO(2)\times SO(2)$. We shall deduce a contradiction from this supposition.

We may assume that the foliation preserving diffeomorphism restricted to the component X induces a homeomorphism $\bar{f}: S^1\longrightarrow$

S^1 by

$$\begin{array}{ccc} \mathrm{int}X & \xrightarrow{f|X} & \mathrm{int}X \\ \mu\downarrow & & \downarrow\mu \\ S^1 & \xrightarrow{\bar{f}} & S^1 \end{array}$$

such that $\bar{f} \notin SO(2)$.

We may assume $\bar{f}(0)=0$ and $\bar{f}\neq id$, eventually by composing some standard rotation.

Let us regard the unit interval $I=[0, 1]$ as embedded in X by identifying $I\ni t\longleftrightarrow(t, 0, 0,)\in X$. Then $f(I)$ and ∂X are transversal at $f(1)$, $1\in I\subset X$, because I and ∂X are transversal at $1=(1, 0, 0)$, and f is a diffeomorphism.

We can assume $f(1)=1$, eventually by composing a leaf preserving diffeomorphism induced by a vector field on ∂X (extended near ∂X). Moreover, eventually by composing a leaf preserving diffeomorphism which is the identity outside a sufficiently small neighbourhood of 1, induced by a smooth vector field tangent to the leaves, we assume that for some small ε, $\delta>0$, f maps $(1-\varepsilon, 1]$ diffeomorphically onto $(1-\delta, 1]$. Let $\varphi=t\circ f|I : I\longrightarrow I$, where t is the radius function $t: X\longrightarrow I$. This continuous map restricted to some neighbourhood of 1 is a diffeomorphism.

Let $\sum=\{0, \beta(1), \beta(2),\ldots, \beta(k),\ldots\}\subset I$, and $\sum_n=\{\beta(n), \beta(n+1),\ldots\}\subset\sum$. Note that $L_0\cap I=\sum$. Then φ induces a map $\bar{\varphi}=\varphi|\sum_{n_1}: \sum_{n_1}\longrightarrow\sum_{n_2}$ for sufficiently large numbers n_1, n_2. $\bar{\varphi}$ can be expressed as $\bar{\varphi}(\beta(k))=\beta(k+\rho)$ for some integer ρ.

Again by composing a leaf preserving diffeomorphism which is of the form $(t, x, y)\longrightarrow(\eta(t), x, y)$ in some small tubular neighbourhood of ∂X and is the identity in the complement of some tubular neighbourhood of ∂X, induced by a vector field of ∂X rotating $\partial X-\rho$ times, mapping a neighbourhood of 1 in I into a neighbourhood of 1 in I, we may assume $\bar{\varphi}=id\sum_{n_1}$. Now recall the following diagram (μ is not defined on ∂X),

$$\begin{tikzcd}
I \arrow[r] \arrow[d, "\varphi"'] & X \arrow[r, "\mu"] \arrow[d, "f"'] & S^1 \arrow[d, "\bar{f}"'] \\
I \arrow[r, "t"] & X \arrow[r, "\mu"] & S^1.
\end{tikzcd}$$

Consider a universal covering:

$$\begin{tikzcd}
\mathrm{int}D^2 \times \mathbf{R}^1 \arrow[r, "\tilde{\mu}"] \arrow[d] & \mathbf{R}^1 \arrow[d] \\
\mathrm{int}X = \mathrm{int}D^2 \times \mathbf{R}^1 / \mathbf{Z} \arrow[r, "\mu"] & S^1 = \mathbf{R}^1 / \mathbf{Z},
\end{tikzcd}$$

where horizontal arrows are canonical profections $\mathbf{R}^1 \longrightarrow S^2 = \mathbf{R}/\mathbf{Z}$, and $\tilde{\mu}$ is defined by $\tilde{\mu}(t, x, \tilde{y}) = \alpha(t) - \tilde{y}$, $(t, x) \in D^2$, $\tilde{y} \in \mathbf{R}^1$, t is the radius. Let $\tilde{f}$ be the lifting of $\bar{f}$:

$$\begin{tikzcd}
\mathbf{R}^1 \arrow[r, "\tilde{f}"] \arrow[d] & \mathbf{R}^1 \arrow[d] \\
S^1 \arrow[r, "\bar{f}"] & S^1.
\end{tikzcd}$$

Getting together the diagrams above, and defining the maps properly,

$$\begin{tikzcd}
 & I-\{1\} & & \mathrm{int}D^2 \times \mathbf{R}^1 \arrow[ll, "t"'] \arrow[rr, "\tilde{\mu}"] \arrow[dd] & & \mathbf{R}^1 \\
I-\{1\} \arrow[ur] \arrow[rr] \arrow[ddrr] & & \mathrm{int}D^2 \times \mathbf{R}^1 \arrow[ul] \arrow[ur] \arrow[rr, "\tilde{\mu}"] \arrow[dr] \arrow[dd] & & \mathbf{R}^1 \arrow[ur, "f"'] \arrow[dd] & \\
 & & & \mathrm{int}X \arrow[rr, "\mu"] & & S^1, \arrow[uu] \\
 & & \mathrm{int}X \arrow[ur, "f"] \arrow[rr, "\mu"] & & S^1 \arrow[ur, "f"'] &
\end{tikzcd}$$

it is easy to check that $\varphi(t) = \beta \circ \tilde{f} \circ \beta^{-1}(t)$ for $t \in (1-\varepsilon, 1)$, and $\varphi(1) = 1$. Consider the derived function $\varphi'(t) = \frac{\mathrm{d}\varphi}{\mathrm{d}t}(t)$. $1 \in I$ is the cluster point of $\sum$ and φ is the identity on $\sum_{n_1}$, hence $\varphi'(1)$ must be equal to 1.

On the other hand, in $(1-\varepsilon, 1)$, $\varphi'(t)=(\beta\circ\tilde{f}\circ\beta^{-1})'(t)=\beta'(\tilde{f}(y))\circ\tilde{f}'(y)/\beta'(y)$, where $y=\beta^{-1}(t)$. We shall now deduce that $\varphi'(t)$ is not continuous at 1. This contradiction will complete our proof of lemma 1.

Note that $\tilde{f}(0)=0$. Let $f_I=\tilde{f}\,|\,[0, 1] : [0, 1]\longrightarrow[0, 1]$.
Then $f_I(0)=0$, $f_I(1)=1$, and f_I is a diffeomorphism of the unit interval. From the assumption, $f_I\neq id$, hence there is $y_0\in I$ that satisfies following 1) or 2) :

1) $y_0\leqq f_I(y_0)$ and $f'_I(y_0)<1$,

2) $y_0\geqq f_I(y_0)$ and $f'_I(y_0)>1$.

By the way, $\lim_{t\to 1}\alpha''(t)=+\infty$, hence in some neighbourhood of 1 in I, $\alpha''(t)>0$. Differentiating the formula $\alpha\circ\beta=id$ twice, we get $\alpha''(\beta)\cdot(\beta')^2+\alpha'(\beta)\cdot\beta''=0$. Therefore there is a sufficiently large number N_0 such that for any $y\geqq N_0$, $\beta''(y)<0$.
Let $y_n=y_0+n$. When y_0 satisfies the condition 1), then for any sufficiently large number n, $\varphi'(\beta(y_n))=\tilde{f}'(y_n)\cdot\beta'(\tilde{f}(y_n))/\beta'(y_n)\leqq\tilde{f}'(y_0)<1$ (for, $\beta''(y)<0$, hence $\beta'(\tilde{f}(y_n))/\beta'(y_n)\leqq 1$).
Hence $\varphi'(\beta(y_n))$ cannot converge to 1. But $\beta(y_n)$ converge to 1. These lead to a contradiction. Analogously, when y_0 satisfies the condition 2), $\varphi'(\beta(y_n))\geqq\tilde{f}'(y_0)>1$. Q. E. D.

Let $\mathrm{LDiff}_T(\mathscr{F}_R)$ denote the space of leaf preserving diffeomorphisms of S^3 with respect to $\mathscr{F}_R$ such that in some tubular neighbourhood N of $T=\partial X$, the following diagram commutes:

$$\begin{tikzcd} N \arrow[r, "f|N"] \arrow[d] & S^3-\{t=0\} \arrow[d] \\ T \arrow[r, "f|T"] & T, \end{tikzcd}$$

where $\{t=0\}$ is the axes of two solid tori diffeomorphic to two copies of the circle S^1, and the vertical arrows are projections which are expressed in each component as $(t, x, y)\longrightarrow(x, y)$.

Lemma 2. *The inclusion map*

$$\mathrm{LDiff}_T(\mathscr{F}_R) \longrightarrow \mathrm{LDiff}(S^3, \mathscr{F}_R)$$

is a weak homotopy equivalence.

Proof Let K be any compact set and let $\Psi : K \longrightarrow \mathrm{LDiff}(S^3, \mathscr{F}_R)$ be a continuous map. We will make a homotopy Ψ_τ such that $\Psi_0 = \Psi$, $\Psi_1(k) \subset \mathrm{LDiff}_T(\mathscr{F}_R)$. Let $\chi : [0, 1] \longrightarrow \mathbf{R}$ be a smooth function such that $\chi(x) = 0$ for $x \in [1, 1 - \frac{2}{3}\nu]$, $\chi(x) = 1$ for $x \in [1 - \frac{1}{3}\nu, 1]$, and $\chi'(x) > 0$ for $x \in (1 - \frac{2}{3}\nu, 1 - \frac{1}{3}\nu)$, where ν is a properly chosen positive number.
Let $\lambda_\tau : [0, 1] \longrightarrow [0, 1]$ be a family of smooth maps defined by $\lambda_\tau(t) = t + \tau \cdot \chi(t) \cdot (1-t)$ for τ, $t \in [0, 1]$.

Let $f \in \mathrm{Im}(\Psi) \subset \mathrm{LDiff}(S^3, \mathscr{F}_R)$. Define a homotopy $f_\tau(t, x, y) = (\beta(f_y(\lambda_\tau(t), x, y) - f_y(t, x, y) + \alpha(f_t(t, x, y))), f_x(\lambda_\tau(t), x, y), f_y(\lambda_\tau(t), x, y))$, where f_t, f_x, f_y are components of f, *i. e.*,

$f(t, x, y) = (f_t(t, x, y), f_x(t, x, y), f_y(t, x, y))$.

Choosing ν small enough, they define a homotopy Ψ_τ. Q. E. D.

Remark. $\mathrm{LDiff}_T(\mathscr{F}_R)$ and $\mathrm{LDiff}(\mathscr{F}_R)$ are Fréchet manifolds, hence they have homotopy types of CW-complexes ([3], [4], [5], [6]). Therefore the inclusion map is also a homotopy equivalence.

$\mathrm{LDiff}_T(\mathscr{F}_R)$ is included in $\mathrm{FDiff}_0(S^3, \mathscr{F}_R)$, hence the restrictions to T belong to the identity component $\mathrm{Diff}_0(\mathrm{T})$ *of* $\mathrm{Diff}(T)$. Let $r : \mathrm{LDiff}_T(\mathscr{F}_R) \longrightarrow \mathrm{Diff}_0(T)$ be the restriction map, *i. e.*, $r(f) = f|T$.

Lemma 3. *There is an exact sequence*:

$$0 \longrightarrow \mathscr{I} \longrightarrow \mathrm{LDiff}_T(\mathscr{F}_R) \xrightarrow{r} \mathrm{Diff}_0(T) \longrightarrow 0,$$

where $\mathscr{I}$ is the kernel of r, and r is a locally trivial fibration.

Proof First we show r is surjective. For any φ in $\mathrm{Diff}_0(T)$, take a smooth path φ_s from identity to φ in $\mathrm{Diff}_0(T)$, $\varphi_0 = id_T$, $\varphi_1 = \varphi$. Consider a vector field defined by $\frac{\partial \varphi_s}{\partial s}$ on $T \times I$ in $S^3 \times I$.
Take small tubular neighbourhoods N_1, N_2 of $T \times I$ in $S^3 \times I$, $N_1 \supset$

$\bar{N}_2$, extend the vector field on $S^3\times I$ so that it vanishes outside N_1, tangent to the leaves, and that in N_2 it commutes with the differential map of the projection to T, next, modify the s-component of the vector field to be the unit vector field $\frac{\partial}{\partial s}$. Then integrating the vector field, we obtain an element of $\mathrm{LDiff}_T(\mathscr{F}_R)$ which is mapped to φ by the restriction r.

Next, define a local section as follows. Let $\theta\in\mathrm{LDiff}_T(\mathscr{F}_R)$, $\bar{\theta}=r(\theta)$. Note that a neighbourhood of θ in $\mathrm{LDiff}\,(S^3, \mathscr{F}_R)$ is homeomorphic to a neighbourhood of the zero section of the space of smooth sections of $\tau\mathscr{F}_R$, the sub-bundle of the tangent bundle consisting of tangent planes of leaves, and that the neighbourhood of $\bar{\theta}$ in $\mathrm{Diff}_0(T)$ is homeomorphic to that of $\tau(T)$, tangent bundle of the torus, which is canonically embedded in $\tau\mathscr{F}_R$. Extend the sections of $\tau(T)$ to sections of $\tau\mathscr{F}_R$ so that it commutes with the projection to T in some tubular neighbourhood of T.
We can give such extensions simultaneously, hence it gives a continuous local section of r. Q. E. D.

Let $\widetilde{\mathrm{Diff}}(S^2, D^2_+)$ denote the space of diffeomorphisms of the two dimensional sphere S^2, which fix some neighbourhood of the north hemisphere D^2_+. Let $L^d(\widetilde{\mathrm{Diff}}(S^2, D^2_+))$ denote the space of differentiable loops in $\widetilde{\mathrm{Diff}}(S^2, D^2_+)$.

Lemma 4. *There is a splitting exact sequence*:

$$0\longrightarrow L^d(\widetilde{\mathrm{Diff}}(S^2, D^2_+))\oplus L^d(\widetilde{\mathrm{Diff}}(S^2, D^2_+))\longrightarrow \mathscr{I}\overset{\rho\oplus\rho}{\rightleftarrows}\mathbf{Z}\oplus\mathbf{Z}\longrightarrow 0,$$

where ρ *is defined as in the proof of lemma* 1 *in each component*, $\mathbf{Z}$ *is the group of integers.*

Proof The global section is given for $(\rho_1, \rho_2)\in\mathbf{Z}\oplus\mathbf{Z}$, by integrating a vector field on $S^3\times I$ associated to the standard rotation of (ρ_1, ρ_2) times, as in the proof of the surjectivity of r in lemma 3.

The kernel of this map $\rho\oplus\rho$ is the space of leaf preserving diffeomorphisms which fix some neighbourhood of T. The restriction on each leaf may be regarded as an element of $\widetilde{\mathrm{Diff}}(S^2, D^2_+)$. Hence the kernel is homeomorphic to $L^d(\widetilde{\mathrm{Diff}}(S^2, D^2_+))\oplus L^d(\widetilde{\mathrm{Diff}}(S^2, D^2_+))$. Q. E. D.

Lemma 5. *The inclusion map:*

$$\widetilde{\mathrm{Diff}}(S^2, D^2_+) \longrightarrow \mathrm{Diff}(S^2, D^2_+)$$

is a homotopy equivalence, where $\mathrm{Diff}(S^2, D^2_+)$ *denotes the space of diffeomorphisms of* S^2 *which fix* D^2_+.

Proof Let q_t be a smooth path in the space of diffeomorphisms of S^2, such that $q_0 = id$, and q_1 shifts the equator towards the north pole. Define a homotopy $q^*_t : \mathrm{Diff}(S^2, D^2_+) \longrightarrow \mathrm{Diff}(S^2, D^2_+)$ by $q^*_t(f) = q_t^{-1} \circ f \circ q_t$. Then $q^*_1 : \mathrm{Diff}(S^2, \mathrm{D}^2_+) \longrightarrow \widetilde{\mathrm{Diff}}(S^2, D^2_+)$ defines a homotopy inverse of the inclusion map. Q. E. D.

§5. Theorem.

Theorem. *The space* $\mathrm{FDiff}_0(S^3, \mathscr{F}_R)$ *has the homotopy type of a torus* $S^1 \times S^1$.

Proof By a result of S. Smale [8], $\mathrm{Diff}(S^2, D^2_+)$ is contractible. By lemma 5, $\widetilde{\mathrm{Diff}}(S^2, D^2_+)$ is also contractible. Therefore $L^d(\widetilde{\mathrm{Diff}}(S^2, D^2_+))\oplus L^d(\widetilde{\mathrm{Diff}}(S^2, D^2_+))$ is contractible. By lemma 4, $\mathscr{I}$ is homotopy equivalent to $\mathbf{Z}\oplus\mathbf{Z}$. Consider the homotopy exact sequence of the fibration r in Lemma 3,

$$\cdots \longrightarrow \pi_2(\mathrm{Diff}_0(T)) \longrightarrow \pi_1(\mathscr{I}) \longrightarrow \pi_1(\mathrm{LDiff}_T(\mathscr{F}_R))$$
$$\longrightarrow \pi_1(\mathrm{Diff}_0(T)) \longrightarrow \pi_0(\mathscr{I}) \longrightarrow \pi_0(\mathrm{LDiff}_T(\mathscr{F}_R)) \longrightarrow 0.$$

Taking in considerations that $\mathrm{Diff}(T)$ has the homotopy type of a torus [9], and that $\mathrm{LDiff}_T(\mathscr{F}_R)$ is connected, we can deduce that all the homotopy groups of $\mathrm{LDiff}_T(\mathscr{F}_R)$ vanish. By lemma 2, all the homotopy groups of $\mathrm{LDiff}(S^3, \mathscr{F}_R)$ vanish (*i. e.*, contractible). Hence by lemma 1, $\mathrm{FDiff}_0(S^3, \mathscr{F}_R)$ is homotopy equivalent to $S^1 \times S^1$.

Q. E. D.

Corollary. *Let* $(S^3, \mathscr{F}_{R'})$ *be the codimension one foliation defined* *as follows. Decompose* $S^3=S^1\times D^2\cup S^1\times S^1\times I\cup D^2\times S^1$, *and introduce the Reeb foliation in two solid tori, and the bundle foliation induced by the submersion onto I defined by the projection in* $S^1\times S^1\times I$. *Then* $\mathrm{FDiff}_0(S^3, \mathscr{F}_{R'})$ *has the homotopy type of a torus* $S^1\times S^1$.

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