J. Math. Kyoto Univ. (JMKYAZ)
37-2 (1997) 241-249

Hecke algebras and quantum general linear groups

Dedicated to Professor Takeshi Hirai on his sixtieth birthday

By

Shin-ichi Kato

Hecke algebras of the Weyl groups are q-analogues of the group algebras of them. There is another kind of q-analogues, quantum groups, due to Drinfeld and Jimbo. Although there is a direct relationship between these two objects in the classical case, such simple relation seems not to be known in the q-analogue case.

In this paper, we shall show that Hecke algebras of the symmetric groups arise naturally from quantum general linear groups (quantum $\mathbf{GL}_n$), or from quantum matrix spaces, in a way analogous to the classical case (see Sect. 1). This construction of the Hecke algebras fits representation theory of quantum $\mathbf{GL}_n$ well, and enables us to adapt the work of Green [G] on polynomial representations for $\mathbf{GL}_n$ to quantum $\mathbf{GL}_n$ in a straightforward manner. Namely, in Section 2, we obtain certain representations of the Hecke algebras, which are essentially identical to the Specht modules defined in [DJ], from polynomial representations of these quantum groups, and study some of their properties. In this way, we could look again the relations between Hecke algebras and quantum $\mathbf{GL}_n$. Further study on this subject will be given in [HKU].

The application in Sect. 2 was inspired by conversation with T. Umeda, to whom the author would like to express his deep gratitude. Thanks are also due to R. Howe for helpful discussion and H. Naruse for critical comments on this work, especially on the Specht modules.

1. Construction

Let $R=\mathbf{Z}[q, q^{-1}]$ be the Laurent polynomial ring of the indeterminate q over $\mathbf{Z}$. We define $A=A(n)=R[M_q(n)]$ a q-analogue of the coordinate ring of the $n\times n$ matrix space over R as follows. The noncommutative R-algebra A is generated by x_{ij} $(1\leq i, j\leq n)$ with respect to the following conditions:

Received May 2, 1996

$$
\begin{array}{llll}
(1) & x_{ij}x_{il} & = q^{-1}x_{il}x_{ij} & (j<l), \\
(2) & x_{ij}x_{kj} & = q^{-1}x_{kj}x_{ij} & (i<k), \\
(3) & x_{ij}x_{kl} & = x_{kl}x_{ij} & (i<k,\ j>l), \\
(4) & [x_{ij}, x_{kl}] & = (q-q^{-1})x_{il}x_{kj} & (i>k,\ j>l).
\end{array}
$$

This algebra is a bialgebra under the comultiplication $\Delta : A \to A \otimes A$ and the counit $\varepsilon : A \to R$ given by $\Delta(x_{ij}) = \sum_{k=1}^{n} x_{ik} \otimes x_{kj}$ and $\varepsilon(x_{ij}) = \delta_{ij}$, respectively (see [D], [RTF]).

We can define a Hopf algebra, the quantum general linear group $R[GL_q(n)] = A[\det_q^{-1}]$ as the localization of A by certain central group-like element, the quantum determinant $\det_q = \det_q(1, \dots, n; 1, \dots, n) \in A$. (We refer to Sect. 2 for the definition of the quantum determinant; we will not use this in the present section.) But we do not go into details about $R[GL_q(n)]$ since we will work on A in this paper. For example, we deal with only *polynomial* representations of the quantum general linear group, namely A-comodules, in Section 2 (cf. [G]).

Let I be the two-sided ideal of A generated by $x_{ij}x_{il}$ and $x_{ij}x_{kj}$ for $1 \leq i, j, k, l \leq n$ with $j \neq l$, $i \neq k$. We see easily that this I is also a coideal of A, i. e., we have $\Delta(I) \subset I \otimes A + A \otimes I$ and $\varepsilon(I) = 0$. Set $F = A/I$. Then F becomes a bialgebra in a natural way. We denote the comultiplication and the counit of F by the same letters Δ and ε. Note that F is a q-analogue of the coordinate ring of monomial matrices.

Let J be the ideal of A generated by x_{ij} $(i \neq j)$. Then J is also a coideal and we have a bialgebra $D = A/J$. We shall identify D with the polynomial ring of n-variables (the coordinate ring of diagonal matrices in both classical and quantum senses), $R[y_1, \dots, y_n]$ by the correspondence x_{ii} (mod J) $\longleftrightarrow y_i$. The R-module A has a left and right D-comodule structure under the comodule structure maps $\Delta_D = \Delta$ mod $A \otimes J : A \to A \otimes D$ and ${}_D\Delta = \Delta$ mod $J \otimes A : A \to D \otimes A$ given by $\Delta_D(x_{ij}) = x_{ij} \otimes y_j$ and ${}_D\Delta(x_{ij}) = y_i \otimes x_{ij}$. Similarly, I and F have D-bicomodule structures.

Now we take the "invariants under a maximal torus" in F. Set $\delta = (1, \dots, 1) \in \mathbf{Z}^n$ (the determinant weight). We put $y^\delta = y_1 \cdots y_n \in D$. We define by

$$F^\delta = \{f \in F \mid \Delta_D(f) = f \otimes y^\delta\},$$

the right δ-eigenspace of F. If $m \in A$ is a right δ-eigenvector, then m can be written as an R-linear combination of monomials of the form $x_{i_1 1} \cdots x_{i_n n}$ $(1 \leq i_1, \dots, i_n \leq n)$. But if $i_l = i_k$ for $l \neq k$ in the above, the corresponding monomial is in I. This implies that F^δ coincides with the left δ-eigenspace of F, i. e.,

$$F^\delta = \{f \in F \mid {}_D\Delta(f) = y^\delta \otimes f\}.$$

Hence F^δ is the (δ, δ)-eigenspace ($=$left δ- and right δ-eigenspace) for D of

F. Let $\mathbf{S}_n$ be the symmetric group of the n-th order. We see that F^δ is a free R-module with basis

$$[x_{w(1)1}x_{w(2)2}\cdots x_{w(n)n}]=x_{w(1)1}x_{w(2)2}\cdots x_{w(n)n} \bmod I \qquad (w\in\mathbf{S}_n),$$

since there is no(δ,δ)-eigenvector in I so that these monomials are linearly independent. Note that

$$\Delta([x_{w(1)1}\cdots x_{w(n)n}])=\sum_{i_1,\dots,i_n \text{ distinct}}[x_{w(1)i_1}\cdots x_{w(n)i_n}]\otimes[x_{i_11}\cdots x_{i_nn}].$$

This shows that F^δ is a subcoalgebra of F. Set $H=\mathrm{Hom}_R(F^\delta, R)$. Then H has a free R-basis T_w $(w\in\mathbf{S}_n)$ with

$$T_w([x_{v(1)1}\cdots x_{v(n)n}])=\delta_{wv} \qquad (v\in\mathbf{S}_n).$$

The coalgebra structure on F^δ defines an algebra structure on H.

Theorem 1. *The basis elements T_w $(w\in\mathbf{S}_n)$ satisfy the following multiplication rule*

$$T_w\cdot T_s=\begin{cases} T_{ws} & \text{if } w(i)<w(i+1) \\ (q-q^{-1})T_w+T_{ws} & \text{if } w(i)>w(i+1)\end{cases},$$

where s is the transposition $(i\ i+1)$ $(1\le i\le n-1)$. Namely the algebra H is isomorphic to the Hecke algebra of $\mathbf{S}_n$.

Proof. For $v\in\mathbf{S}_n$, we have, by using commutation relations (3) and (4),

$$\begin{aligned}
&(T_w\cdot T_s)([x_{v(1)1}\cdots x_{v(n)n}])\\
&=\sum_{k_1,\dots,k_n}T_w([x_{v(1)k_1}\cdots x_{v(n)k_n}])\cdot T_s([x_{k_11}\cdots x_{k_nn}])\\
&=T_w([x_{v(1)1}\cdots x_{v(i)i+1}x_{v(i+1)i}\cdots x_{v(n)n}])\\
&=\begin{cases} T_w([x_{v(1)1}\cdots x_{v(i+1)i}x_{v(i)i+1}\cdots x_{v(n)n}]) & \text{if } v(i)<v(i+1)\\[2ex] T_w([x_{v(1)1}\cdots x_{v(i+1)i}x_{v(i)i+1}\cdots x_{v(n)n}]) & \\ \quad +(q-q^{-1})T_w([x_{v(1)1}\cdots x_{v(i)i}x_{v(i+1)i+1}\cdots x_{v(n)n}]) & \text{if } v(i)>v(i+1)\end{cases}\\
&=\begin{cases}\delta_{w,vs} & \text{if } v(i)<v(i+1)\\ \delta_{w,vs}+(q-q^{-1})\delta_{w,v} & \text{if } v(i)>v(i+1)\end{cases}.
\end{aligned}$$

Remark. We have seen that our Hecke algebra relation is a simple consequence of the defining relations of A, (1)-(4). Note that these relations are derived from certain R-matrix (see [RTF]). But, as was shown by Jimbo [J], this R-matrix is written as (transposition)$\times$(Hecke algebra action). This factorization explains why the Hecke algebra of the symmetric group arises from the quantum matrix algebra, or the quantum general linear group in our way. Hence our construction should work for other quantum groups provided that the corresponding R-matrix admits a factorization as above.

2. Representations

We construct representations of H, the quantum Specht modules, on the determinant weight spaces of certain polynomial represntations for quantum general linear groups (i. e. certain A-comodules) as in the case of $\mathbf{S}_n$ (cf. [G]).

First, we give a general remark about how to obtain H-modules from A-comodules. Let L be a right A-comodule and $\Delta_L: L \to L\otimes A$ the comodule structure map of L. Naturally L becomes a D-comodule. Let L^δ be the δ-eigenspace of L:

$$L^\delta = \{l \in L \mid \Delta_L(l) \bmod L\otimes J = l\otimes y^\delta\}.$$

It is easily seen that this L^δ becomes a right F^δ-comodule, hence a left H-module.

Let $\lambda=(\lambda_1, \dots, \lambda_n)$ be a partition of $n: \lambda_1 \geq \cdots \geq \lambda_n \geq 0$ and $\sum_{i=1}^n \lambda_i = n$. We set $y^\lambda = y_1^{\lambda_1}\cdots y_n^{\lambda_n} \in D$ (cf. y^δ in Sect. 1). Let ${}^\lambda A$ be the left λ-eigenspace of A,

$${}^\lambda A = \{m \in A \mid \Delta(m) \bmod J\otimes A = y^\lambda \otimes m\}.$$

Then ${}^\lambda A$ is a right A-comodule. Let $A^{\lambda,\delta} = ({}^\lambda A)^\delta$ be the (λ, δ)-eigenspace of A,

$$A^{\lambda,\delta} = \left\{ m \in A \;\middle|\; \begin{matrix} \Delta(m) \bmod A\otimes J = m\otimes y^\delta \\ \Delta(m) \bmod J\otimes A = y^\lambda\otimes m \end{matrix} \right\}.$$

This $A^{\lambda,\delta}$ is an H-module; see the remark given above. As an R-free basis of $A^{\lambda,\delta}$, we may choose the set of elements $x_{i_1 1}x_{i_2 2}\cdots x_{i_n n}$ where the letter k appears λ_k times for each $1 \leq k \leq n$ in the first indices $i_1, \dots, i_n$. Note that the H-action on $A^{\lambda,\delta}$ is given by

$$T_w(x_{i_1 1}\cdots x_{i_n n}) = x_{i_1 w(1)}\cdots x_{i_n w(n)}.$$

In order to define the quantum Specht module, certain H-submodule of $A^{\lambda,\delta}$, we need quantum determinants. Let $I=\{i_1, \dots, i_r\}$ and $J=\{j_1, \dots, j_r\}$ be two ordered subsets of $\{1, \dots, n\}$ with r-elements. We assume that $i_1 < \cdots < i_r$. We set

$$\begin{aligned} \det_q(I;J) &= \det_q(i_1, \dots, i_r; j_1, \dots, j_r) \\ &= \sum_{w\in \mathbf{S}_r} (-q)^{-\ell(w)} x_{i_{w(1)} j_1}\cdots x_{i_{w(r)} j_r}. \end{aligned}$$

Here $\ell(w)$ is the length of w, given by the cardinality of $\{(i, j) \mid i<j,\ w(i)$

$>w(j)\}$.

Hereafter we fix a partition λ of n unless otherwise stated. We let $\mathbf{T}(\lambda)$ be the set of Young tableaux of shape λ : elements of $\mathbf{T}(\lambda)$ are of the form

$$t=\begin{array}{llllll} t(1,1) & t(1,2) & \cdots & \cdots & \cdots & t(1,\lambda_1) \\ t(2,1) & t(2,2) & \cdots & t(2,\lambda_2) & & \\ \vdots & \vdots & \cdots & & & \\ \vdots & t(\mu_2,2) & & & & \\ \vdots & & & & & \\ t(\mu_1,1) & & & & & \end{array}$$

where $t(i,j)$ runs over 1 to n exactly once. Here $\mu=(\mu_1,\cdots,\mu_l)\,(l=\lambda_1)$ is the dual partition of λ. The group $\mathbf{S}_n$ acts on $\mathbf{T}(\lambda)$ in the way $(wt)(i,j)=w(t(i,j))$ for $w\in\mathbf{S}_n$, $t\in\mathbf{T}(\lambda)$. Similarly, we define $\mathbf{T}(\lambda,n)$ the set of tableaux of shape λ with entries from $\{1,\dots,n\}$, by replacing the condition for $t(i,j)$ above with the one, $1\le t(i,j)\le n$ for any i,j. We call $t\in\mathbf{T}(\lambda)$ *standard* (resp. $t\in\mathbf{T}(\lambda,n)$ *semistandard*) if $t(i,j)<t(i+1,j)$ and $t(i,j)<t(i,j+1)$ for any i,j, (resp. $t(i,j)<t(i+1,j)$ and $t(i,j)\le t(i,j+1)$ for any i,j). The set of standard (resp. semistandard) tableaux of shape λ (with entries from $\{1,\dots,n\}$) is denoted by $\mathbf{ST}(\lambda)$ (resp. $\mathbf{SST}(\lambda,n)$).

For $t,t'\in\mathbf{SST}(\lambda,n)$, we define the quantum bideterminant $d(t;t')$ by

$$d(t;t')=\det_q(C_1(t);C_1(t'))\det_q(C_2(t);C_2(t'))\cdots\det_q(C_l(t);C_l(t'))$$

where $l=\lambda_1$. Here $C_i(t)=(t(1,i),\dots,t(\mu_i,i))\in\{1,\dots,n\}^{\mu_i}$, the transpose of the i-th column of t. Note that from the definition of $\det_q(\,\cdot\,)$, we may allow t' above to be an arbitary tableau in $\mathbf{T}(\lambda,n)$. Let $t_l\in\mathbf{SST}(\lambda,n)$ be the special semistandard tableau uniquely determined by the condition $t(i,j)=i$:

$$t_l=\begin{array}{llllll} 1 & 1 & \cdots & \cdots & \cdots & 1 \\ 2 & 2 & \cdots & 2 & & \\ \vdots & \vdots & \cdots & & & \\ \vdots & \mu_2 & & & & \\ \mu_1 & & & & & \end{array}$$

It is easily seen that $d(t_l;t)\in A^{\lambda,\delta}$ for any $t\in\mathbf{T}(\lambda)$. We put $d_t=d(t_l;t)$ for simplicity.

Now we define the *quantum Specht module* of type λ, S_λ by

$$S_\lambda=\sum_{t\in\mathbf{T}(\lambda)}R\cdot d_t.$$

Let

$$t_0=\begin{array}{llll} 1 & \mu_1+1 & \cdots & \cdots \\ 2 & \mu_1+2 & \cdots & \cdots \\ \vdots & \cdots & \cdots & \\ \vdots & \mu_1+\mu_2 & & \\ \mu_1 & & & \end{array} \in \mathbf{ST}(\lambda)$$

be the leading standard tableau of shape λ. Since $T_w \cdot d_{t_0}=d_{wt_0}$ for $w\in\mathbf{S}_n$, we see $S_\lambda=H \cdot d_{t_0}$. Hence S_λ is the cyclic H-submodule of $A^{\lambda,\delta}$ generated by d_{t_0}. (Compare our definition with the "classical" definition of Specht modules for $\mathbf{S}_n$: If we put $q=1$, this construction coincides with the realization of the Specht modules for $\mathbf{S}_n$ given in [G].)

We shall show that our quantum Specht module S_λ is essentially identical to the Specht modules of H defined by Dipper and James in [DJ]. For that purpose, we recall the construction of Specht modules in [DJ] in our formulation. But before the review, we note the following: First our Hecke algebra H is the Hecke algebra in [DJ] with the parameter q^2, and that our $q^{\ell(w)}T_w$ corresponds to their T_w for $w\in\mathbf{S}_n$. Secondly all H-modules in [DJ] are *right* H-modules. Hence we define for any right H-module M, the left action of H on M by $T_w \cdot m=m \cdot T_{w^{-1}}$ so that we regard all H-modules appearing in [DJ] as left H-modules.

We put

$$m_\lambda=x_{11}\cdots x_{1\lambda_1}x_{2\lambda_1+1}\cdots x_{2\lambda_1+\lambda_2}\cdots x_{\mu_l n}.$$

Then we have $m_\lambda\in A^{\lambda,\delta}$. (Our m_λ and $A^{\lambda,\delta}$ correspond to x^λ and M^λ in [DJ] respectively.) Since H acts on $A^{\lambda,\delta}$ in the way described before, we have

$$\begin{aligned} T_{w_\lambda} \cdot m_\lambda &= x_{11}x_{1\mu_1+1}\cdots x_{22}x_{2\mu_1+2}\cdots \\ &= x_{11}x_{22}\cdots x_{\mu_1\mu_1}x_{1\mu_1+1}x_{2\mu_1+1}\cdots \end{aligned}$$

for the permutation

$$w_\lambda=\begin{pmatrix} 1 & 2 & \cdots & \lambda_1 & \lambda_1+1 & \lambda_1+2 & \cdots \\ 1 & \mu_1+1 & \cdots & \mu_1+\cdots+\mu_{l-1}+1 & 2 & \mu_1+2 & \cdots \end{pmatrix}$$

which transforms

$$\begin{array}{lllll} 1 & 2 & \cdots & \cdots & \lambda_1 \\ \lambda_1+1 & \lambda_1+2 & \cdots & \lambda_1+\lambda_2 & \\ \vdots & \vdots & \cdots & & \\ \vdots & \vdots & & & \\ \vdots & & & & \end{array} \in \mathbf{ST}(\lambda)$$

to t_0, the leading standard tableau. Here we used the relation (3) to have the second equality. Denote by $\mathbf{S}_\mu$ the column stabilizer of the tableau t_0, i. e., the stabilizer of the subsets $\{1, \ldots, \mu_1\}, \{\mu_1+1, \ldots, \mu_1+\mu_2\}, \ldots, \{(\sum_{k=1}^{l}\mu_k)$

$+1, \ldots, n\}$ in $\mathbf{S}_n$. Then we have

$$\sum_{w \in \mathbf{S}_\mu} (-q)^{-l(w)} T_w T_{w_\lambda} m_\lambda = d_{t_0}.$$

But the left hand side above is nothing but the element in $A^{\lambda,\delta}$ corresponing to Dipper-James' z^λ which generates their Specht module S^λ [DJ; 4.1] by definition (under certain shifts of the parameter and basis of H as explained).

Thus we have proved the following theorem.

Theorem 2. *For a partition λ of n, the quantum Specht module S_λ is isomorphic to the Specht module S^λ defined by Dipper and James.*

The properties of quantum Specht modules can be decuced from the work of Dipper-James. But we may obtain some of them together with the relation to polynomial representations of quantum $\mathbf{GL}_n$ (i. e. A-comodules) also in our formulation.

As an example, we handle the Garnir relations in the following way. Let t be a tableau in $\mathbf{T}(\lambda)$. There exists a unique element w_t in $\mathbf{S}_n$ with $w_t(t_0)=t$. We define the right $\mathbf{S}_n$-action on $\mathbf{T}(\lambda)$ by $t * y = w_t y(t_0)$ for $y \in \mathbf{S}_n$. The l-th and $l+1$-th colums of $t*y$ are given in the following form:

$$\begin{array}{ll} w_t y(a+1) & w_t y(a+p+1) \\ w_t y(a+2) & w_t y(a+p+2) \\ \vdots & \vdots \\ \vdots & w_t y(a+p+q) \\ w_t y(a+p) & \end{array}$$

where $a=\sum_{k=1}^{l-1} \mu_k$, $p=\mu_l$ and $q=\mu_{l+1}$. Set

$$\begin{array}{l} X=\{a+b, \ldots, a+p\} \\ Y=\{a+p+1, \ldots, a+p+c\} \end{array} \quad (1 \leq b \leq c \leq q)$$

and let $G_{X,Y}$ be the set of the representative elements for $\mathbf{S}_{X\cup Y}/\mathbf{S}_X \times \mathbf{S}_Y$ with minimal length. (Here, for any subset Z of $\{1, \ldots, n\}$, we denote by $\mathbf{S}_Z$ the subgroup of $\mathbf{S}_n$ permutating only the elements of Z.) We choose $y_t = y_{t,X\cup Y} \in \mathbf{S}_{X\cup Y}$ so that

$$w_t y_t(a+b) < \cdots < w_t y_t(a+b+c).$$

(In other words, $w_t y_t$ is of minimal length in the coset $w_t \cdot \mathbf{S}_{X\cup Y}$.) Suppose that $t \in \mathbf{T}(\lambda)$ is column increasing (i. e., increasing in each column from top to bottom). Then we have $y_t^{-1} \in G_{X,Y}$ since we see that

$$w_t(a+b) < \cdots < w_t(a+p), \; w_t(a+p+1) < \cdots < w_t(a+p+c).$$

from l-th and $l+1$-th columns of t.

Proposition 3 (The Garnir relations). *For any column increasing* $t\in\mathbf{T}(\lambda)$,

$$\sum_{w\in G_{X,Y}}(-q)^{-\ell(w)}d_{t*y_t w}=0.$$

Proof. This can be done by direct calculation using properties of quantum determinants (see below, for example). Note that this equality is just the generalized Plücker relations in [NYM; 1.2] (cf. [TT]).

The assumption "column increasing" is not essential here: For any t, there uniquely exist column increasing $t'\in\mathbf{T}(\lambda)$ and $y\in\mathbf{S}_\mu$ (the column stabilizer of the tableau t_0) such that $t=t'*y$. Then, by the property of the quantum determinant

$$\det{}_q(1,\dots,n;w(1),\dots,w(n))=(-q)^{-\ell(w)}\det{}_q(1,\dots,n;1,\dots,n)$$

(see, e.g., [TT]), we see that $d_t=(-q)^{-\ell(y)}d_{t'}$. Thus exactly as in the classical case (see, e.g., [P] or [G]), we can show that S_λ has R-free basis d_t ($t\in\mathbf{ST}(\lambda)$) by using the Garnir relations (cf. [DJ]).

Remark. (i) Let L_λ be the right A-comodule generated by d_t for all $t\in\mathbf{T}(\lambda,n)$ over R. We then have $(L_\lambda)^\delta=S_\lambda$. This is a q-analogue of the fact that each irreducible representation of $\mathbf{S}_n$ can be realized on the 0-weight space of some irreducible representation for $\mathbf{SL}_n$ over a field of characteristic 0. Actually, let $R\to K$ be a specialization map of R to some field K. We know that $K\otimes_R L_\lambda$(resp. $K\otimes_R S_\lambda$) is an irreducible $K\otimes_R A$-comodule (resp. irreducible $K\otimes_R H$-module) when the specialized $K\otimes_R H$ is semisimple, see e.g., [HH] (resp. [DJ]).

Incidentally it is known that L_λ has R-free basis d_t ($t\in\mathbf{SST}(\lambda,n)$) (see [TT], [HH] or [NYM]). Thus we see again that S_λ has R-free basis d_t ($t\in\mathbf{ST}(\lambda)$).

(ii) Let $A(m,n)=R[M_q(m,n)]$ be the q-analogue of the coordinate ring of the $m\times n$ matrix space over R defined as in the case of $A=A(n)$: $A(m,n)$ is the noncommutative R-algebra generated by x_{ij} ($1\le i\le m$, $1\le j\le n$) under the relations (1)–(4) given in Sect. 1. Then this $A(m,n)$ becomes a left $A(m)$- and right $A(n)$-comodule in a natural way. Take the right δ-eigenspace $A(m,n)^\delta$ of $A(m,n)$. This is a left $A(m)$- and right F^δ-comodule and has a free R-basis $x_{i_1 1}\cdots x_{i_n n}$ ($1\le i_1,\dots,i_n\le m$). Hence, as in the remark at the beginning of Sect. 2, we have a Hecke algebra action on $A(m,n)^\delta$ commuting with $A(m)$. Set $V=\sum_{i=1}^m R\cdot v_i$ and give a left $A(m)$-comodule structure by $v_i\longmapsto\sum_{j=1}^m x_{ij}\otimes v_j$. (This is a q-analogue of the vector representation of $\mathbf{GL}_n$.) Then we see easily that $V^{\otimes n}\simeq A(m,n)^\delta$

as left $A(m)$-comodules. In this way, we can see why the Hecke algebra H appears in $\mathrm{End}_{A(m)}(V^{\otimes n})$.

The q-analogue of Schur-Weyl reciprocity discovered by Jimbo [J] asserts that

$$\mathrm{End}_{K\otimes_R A(m)}(K\otimes_R V^{\otimes n})=K\otimes_R H \quad (\text{when } K\otimes_R H \text{ is semisimple}).$$

We would like to discuss this in [HKU].

DIVISION OF MATHEMATICS
DEPARTMENT OF FUNDAMENTAL SCIENCES
FACULTY OF INTEGRATED HUMAN STUDIES
KYOTO UNIVERSITY

References

[DJ] R. Dipper and G. James, Representations of Hecke algebras of general linear groups, Proc. London Math. Soc., **52** (1986), 20–52.

[D] V. G. Drinfeld, Quantum groups, in: Proc. ICM, Berkeley, California, U. S. A., 1986, pp. 798–820.

[J] M. Jimbo, A q-analogue of $U(\mathfrak{gl}(N+1))$, Hecke algebra and the Yang-Baxter equation, Lett. Math. Phys., **11** (1986), 247–252.

[G] J. E. Green, Polynomial representations of GL_n, Lect. Notes in Math. 830, Springer-Verlag, Berlin-Heidelberg-New York, 1980.

[HH] M. Hashimoto and T. Hayashi, Quantum multilinear algebra, Tôhoku Math. J., **44** (1992), 471–521.

[HKU] R. Howe, S. Kato and T. Umeda, in preparation.

[NYM] M. Noumi, H. Yamada and K. Mimachi, Finite dimensional representations of the quantum group $GL_q(n;\mathbf{C})$ and the zonal spherical functions on $U_q(n-1)\backslash U_q(n)$, Japan. J. Math., **19** (1993), 31–80.

[P] M. H. Peel, Specht modules and symmetric groups, J. Alg., **36** (1975), 88–97.

[RTF] N. Yu. Reshetikhin, L. A. Takhtajan and L. D. Faddeev, Quantization of Lie groups and Lie algebras, Leningrad Math. J., **1** (1990), 193–225.

[TT] E. Taft and J. Towber, Quantum deformation of flag schemes and Grassmann schemes I, A q-deformatiun of the shape-algebra for $GL(n)$, J. Algebra, **142** (1991), 1–36.