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Research Article

Generalized Differential Transform Method to Space-Time Fractional Telegraph Equation

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We use generalized differential transform method (GDTM) to derive the solution of space-time fractional telegraph equation in closed form. The space and time fractional derivatives are considered in Caputo sense and the solution is obtained in terms of Mittag-Leffler functions.

1. Introduction

Differential equations of fractional order have been successfully employed for modeling the so called anomalous phenomena during last two decades. As a consequence, there has been an intensive development of the theory of fractional differential equations [1–4]. Recently, various analytical and numerical methods have been employed to solve linear and nonlinear fractional differential equations. A few to mention are Adomian decomposition method [5–7], homotopy perturbation method [8, 9], homotopy analysis method [10], variational iteration method [11, 12], matrix method [13], and differential transform method [14–16]. The differential transform method was proposed by Zhou [17] to solve linear and nonlinear initial value problems in electric circuit analysis. This method constructs an analytical solution in the form of a polynomial. It is different from the traditional higher order Taylor series method, which requires symbolic computation of the necessary derivatives of the data functions and takes long time in computation, whereas the differential transform is an iterative procedure for obtaining analytic Taylor series solution. The method is further developed by Momani, Odibat, and Erturk in their papers [14–16] for solving two-dimensional linear and nonlinear partial differential equations of fractional order. Recently, Biazar and Eslami [18] applied differential transform method to solve systems of Volterra integral equations of the first kind, El-Said et al. [19] developed extended Weierstrass transformation method for nonlinear evolution equations, and Keskin and Oturanc [20] developed the reduced differential transform method to solve fractional partial differential equations.

In the present paper, we apply the method of generalized differential transform to solve space-time fractional telegraph equation. The classical telegraph equation is a partial differential equation with constant coefficients given by [21]

$$u_{tt} - c^2 u_{xx} + a u_t + bu = 0, \tag{1.1}$$

where a, b and c are constants. This equation is used in modeling reaction diffusion and signal analysis for propagation of electrical signals in a cable of transmission line [21, 22]. Both current I and voltage V satisfy an equation of the form (1.1). This equation also arises in the propagation of pressure waves in the study of pulsatile blood flow in arteries and in one-dimensional random motion of bugs along a hedge. Compared to the heat equation, the telegraph equation is found to be a superior model for describing certain fluid flow problems involving suspensions [23]. This equation is used in modeling reaction diffusion and signal analysis for transmission and propagation of electrical signals.

The classical telegraph equation and space or time fractional telegraph equations have been studied by a number of researchers namely Biazar et al. [24], Cascaval et al. [25], Kaya [26], Momani [5], Odibat and Momani [27], Sevimlican [12], and Yıldırım [9]. Orsingher and Zhao [28] have shown that the law of the iterated Brownian motion and the telegraph processes with Brownian time are governed by time-fractional telegraph equations. Orsingher and Beghin [29] presented that the transition function of a symmetric process with discontinuous trajectories satisfies the space-fractional telegraph equation. Several techniques such as transform method, Adomian decomposition method, juxtaposition of transforms, generalized differential transform method, variational iteration method, and homotopy perturbation method have been used to solve space or time fractional telegraph equation.

In the present paper, we make an attempt to solve homogeneous and nonhomogeneous space-time fractional telegraph equation by means of generalized differential transform method.

2. Preliminaries

Definition 2.1. Caputo fractional derivative of order α is defined as [30]:

$$D_a^{\alpha} f(x) = \frac{1}{\Gamma(m-\alpha)} \int_a^x \frac{f^{(m)}(\xi)}{(x-\xi)^{\alpha-m+1}} d\xi, \quad (m-1 < \alpha \le m),\ m \in \mathbb{N}. \tag{2.1}$$

Definition 2.2. The Mittag-Leffler function which is a generalization of exponential function is defined as [31]:

$$E_{\alpha}(z) = \sum_{n=0}^{\infty} \frac{z^n}{\Gamma(\alpha n + 1)} \quad (\alpha \in C,\ R(\alpha) > 0). \tag{2.2}$$

A further generalization of (2.2) is given in the form [32]

$$E_{\alpha,\beta}(z) = \sum_{n=0}^{\infty} \frac{z^n}{\Gamma(\alpha n + \beta)}; \quad (\alpha, \beta \in C,\ R(\alpha) > 0,\ R(\beta) > 0). \tag{2.3}$$

For $\alpha = 1, E_\alpha(z)$ reduces to e^z.

Definition 2.3. Generalized two-dimensional differential transform [14–16] is as given below: Consider a function of two variables $u(x, y)$ and suppose that it can be represented as a product of two single-variable functions, that is, $u(x, y) = f(x)g(y)$. If function $u(x, y)$ is analytic and differentiated continuously with respect to x and y in the domain of interest, then the generalized two-dimensional differential transform of the function $u(x, y)$ is given by

$$U_{\alpha,\beta}(k, h) = \frac{1}{\Gamma(\alpha k + 1)\Gamma(\beta h + 1)} \left[\left(D_{x_0}^{\alpha}\right)^k \left(D_{y_0}^{\beta}\right)^h u(x, y) \right]_{(x_0, y_0)}, \tag{2.4}$$

where $0 < \alpha, \beta \leq 1 (D_{x_0}^{\alpha})^k = D_{x_0}^{\alpha} \cdots D_{x_0}^{\alpha} \cdots D_{x_0}^{\alpha}$ (k times), $D_{x_0}^{\alpha}$ is defined by (2.1) and $U_{\alpha,\beta}(k, h)$ is the transformed function.

The generalized differential transform inverse of $U_{\alpha,\beta}(k, h)$ is given by

$$u(x, y) = \sum_{k=0}^{\infty} \sum_{h=0}^{\infty} U_{\alpha,\beta}(k, h)(x - x_0)^{k\alpha}(y - y_0)^{h\beta}. \tag{2.5}$$

Some basic properties of the generalized two-dimensional differential transform are as given below.

Let $U_{\alpha,\beta}(k, h)$, $V_{\alpha,\beta}(k, h)$ and $W_{\alpha,\beta}(k, h)$ be generalized two-dimensional differential transform of the functions $u(x, y)$, $v(x, y)$, and $w(x, y)$, respectively, then

(a) if $u(x, y) = v(x, y) \pm w(x, y)$, then $U_{\alpha,\beta}(k, h) = V_{\alpha,\beta}(k, h) \pm W_{\alpha,\beta}(k, h)$,

(b) if $u(x, y) = av(x, y)$, a is constant, then $U_{\alpha,\beta}(k, h) = aV_{\alpha,\beta}(k, h)$,

(c) if $u(x, y) = D_{x_0}^{\gamma} v(x, y)$ where $m - 1 < \gamma \leq m$, $m \in \mathbb{N}$ then $U_{\alpha,\beta}(k, h) = (\Gamma(\alpha k + \gamma + 1)/\Gamma(\alpha k + 1))\ V_{\alpha,\beta}(k + \gamma/\alpha, h)$,

(d) if $u(x, y) = D_{y_0}^{\mu} v(x, y)$ where $n - 1 < \mu \leq n$, $n \in \mathbb{N}$, then $U_{\alpha,\beta}(k, h) = (\Gamma(\beta h + \mu + 1)/\Gamma(\beta h + 1))\ V_{\alpha,\beta}(k, h + \mu/\beta)$.

3. Solution of Space-Time Fractional Telegraph Equations by Generalized Two-Dimensional Differential Transform Method

In this section, we consider space-time fractional telegraph equations in the following form:

$$c^2 D_x^{2\alpha} u(x, t) = D_t^{p\beta} u(x, t) + a D_t^{r\beta} u(x, t) + bu(x, t) + f(x, t), \quad 0 < x < 1,\ t > 0, \tag{3.1}$$

where $\beta = 1/q$, $p, q, r \in \mathbb{N}$, $1 < 2\alpha \le 2$, $1 < p\beta \le 2$, $0 < r\beta \le 1$, $D_x^{2\alpha} \equiv D_x^{\alpha} D_x^{\alpha}$, $D_t^{p\beta} \equiv D_t^{\beta} D_t^{\beta} \cdots D_t^{\beta}$ (p times), $D_t^{r\beta} \equiv D_t^{\beta} D_t^{\beta} \cdots D_t^{\beta}$ (r times), D_x^{α}, D_t^{β} are Caputo fractional derivatives defined by (2.1), a, b, and c are constants, $f(x, t)$ is given function.

Particularly for $\alpha = 1$, $q = 1$, $p = 2$, $r = 1$, $f = 0$, space-time fractional telegraph (3.1) reduces to classical telegraph (1.1).

To give a clear overview of the methodology, we have selected three illustrative examples, the first is a homogeneous space-time fractional telegraph equation with conditions involving ordinary derivative with respect to space, the second is a homogeneous space-time fractional telegraph equation with conditions involving fractional derivative with respect to space, and the third is a nonhomogeneous space-time fractional telegraph equation with conditions involving fractional derivative with respect to space.

Example 3.1. Consider the following homogeneous space-time fractional telegraph equation:

$$D_x^{3/2} u(x,t) = D_t^{p\beta} u(x,t) + D_t^{r\beta} u(x,t) + u(x,t), \quad 0 < x < 1,\ t > 0, \tag{3.2}$$

where $\beta = 1/q$, $p, q, r \in \mathbb{N}$, $1 < p\beta \le 2$, $0 < r\beta \le 1$, $D_x^{3/2} \equiv (D_x^{1/2})^3$, $D_t^{p\beta} \equiv D_t^{\beta} D_t^{\beta} \cdots D_t^{\beta}$ (p times), $D_t^{r\beta} \equiv D_t^{\beta} D_t^{\beta} \cdots D_t^{\beta}$ (r times), D_x^{α}, D_t^{β} are Caputo fractional derivatives defined by (2.1), $p + r$ is odd and

$$u(0,t) = E_{\beta}\left(-t^{\beta}\right), \qquad u_x(0,t) = E_{\beta}\left(-t^{\beta}\right). \tag{3.3}$$

Applying generalized two-dimensional differential transform (2.4) with $x_0 = 0 = y_0$, $\alpha = 1/2$, to both sides of (3.2) and (3.3) and using properties (c) and (d), we obtain

$$\begin{aligned} U_{1/2,\beta}(k+3,h) = &\frac{\Gamma((k/2)+1)}{\Gamma(((k+3)/2)+1)} \Bigg[\frac{\Gamma(\beta(h+p)+1)}{\Gamma(\beta h+1)} U_{1/2,\beta}(k,h+p) \\ &+ \frac{\Gamma(\beta(h+r)+1)}{\Gamma(\beta h+1)} U_{1/2,\beta}(k,h+r) + U_{1/2,\beta}(k,h) \Bigg], \end{aligned} \tag{3.4}$$

$$\begin{aligned} U_{1/2,\beta}(0,h) &= \frac{(-1)^h}{\Gamma(\beta h+1)}, \quad U_{1/2,\beta}(1,h) = 0, \\ U_{1/2,\beta}(2,h) &= \frac{(-1)^h}{\Gamma(\beta h+1)}, \quad h = 0,1,2,\ldots. \end{aligned} \tag{3.5}$$

Utilizing recurrence relation (3.4), the transformed conditions (3.5) and the condition $p + r$ is odd, we can easily obtain, for l, $h = 0,1,2,\ldots$

$$U_{1/2,\beta}(3l,h) = \frac{(-1)^h}{\Gamma((3/2)l+1)\Gamma(\beta h+1)}, \tag{3.6}$$

$$U_{1/2,\beta}(3l+1,h) = 0, \tag{3.7}$$

$$U_{1/2,\beta}(3l+2,h) = \frac{(-1)^h}{\Gamma((3/2)l+2)\Gamma(\beta h+1)}. \tag{3.8}$$

Now, from (2.5), we have

$$u(x,t) = \sum_{k=0}^{\infty}\sum_{h=0}^{\infty} U_{1/2,\beta}(k,h)x^{k/2}t^{\beta h}. \tag{3.9}$$

Using the values of $U_{1/2,\beta}(k,h)$ from (3.5)–(3.8) in (3.9), the exact solution of space-time fractional telegraph (3.2) is obtained as

$$u(x,t) = \left[E_{3/2}\left(x^{3/2}\right) + xE_{3/2,2}\left(x^{3/2}\right)\right]E_\beta\left(-t^\beta\right), \tag{3.10}$$

which is same as obtained by Garg and Sharma [33] using Adomian decomposition method.

Setting $p = 2$, $q = r = 1$, the space-time fractional telegraph (3.2) reduces to space fractional telegraph equation and the solution is same as obtained by Momani [5], Odibat and Momani [27], and Yıldırım [9] using Adomian decomposition method, generalized differential transform method, and homotopy perturbation method, respectively.

Further, setting $\alpha = 1$, it reduces to classical telegraph equation and the solution is same as obtained by Kaya [26] using Adomian decomposition method.

Example 3.2. Consider the following homogeneous space-time fractional telegraph equation:

$$D_x^{2\alpha}u(x,t) = D_t^{p\beta}u(x,t) + D_t^{r\beta}u(x,t) + u(x,t), \quad 0 < x < 1,\ t > 0, \tag{3.11}$$

where $\beta = 1/q$, $p,q,r \in \mathbb{N}$, $1 < 2\alpha \le 2$, $1 < p\beta \le 2$, $0 < r\beta \le 1$, $D_x^{2\alpha} \equiv D_x^\alpha D_x^\alpha$, $D_t^{p\beta} \equiv D_t^\beta D_t^\beta \cdots D_t^\beta$ (p times), $D_t^{r\beta} \equiv D_t^\beta D_t^\beta \cdots D_t^\beta$ (r times), D_x^α, D_t^β are Caputo fractional derivatives defined by (2.1), $p + r$ is odd and

$$u(0,t) = E_\beta\left(-t^\beta\right), \qquad D_x^\alpha u(x,t)|_{x=0} = E_\beta\left(-t^\beta\right). \tag{3.12}$$

Applying generalized two-dimensional differential transform (2.4) with $x_0 = 0 = y_0$ to both sides of (3.11), (3.12) and using properties (c) and (d) we obtain

$$\begin{aligned} U_{\alpha,\beta}(k+2,h) =& \frac{\Gamma(\alpha k+1)}{\Gamma(\alpha(k+2)+1)}\left[\frac{\Gamma(\beta(h+p)+1)}{\Gamma(\beta h+1)}U_{\alpha,\beta}(k,h+p)\right. \\ &\left.+\frac{\Gamma(\beta(h+r)+1)}{\Gamma(\beta h+1)}U_{\alpha,\beta}(k,h+r) + U_{\alpha,\beta}(k,h)\right], \end{aligned} \tag{3.13}$$

$$U_{\alpha,\beta}(0,h) = \frac{(-1)^h}{\Gamma(\beta h+1)}, \quad U_{\alpha,\beta}(1,h) = \frac{(-1)^h}{\Gamma(\alpha+1)\Gamma(\beta h+1)}, \quad h = 0,1,2,\ldots. \tag{3.14}$$

Utilizing the recurrence relation (3.13), the transformed conditions (3.14) and the condition $p+r$ is odd, we obtain

$$U_{\alpha,\beta}(k,h) = \frac{(-1)^h}{\Gamma(k\alpha+1)\Gamma(h\beta+1)}, \quad \text{for } k,h = 0,1,2,\ldots. \tag{3.15}$$

Now, from (2.5), we have

$$u(x,t) = \sum_{k=0}^{\infty}\sum_{h=0}^{\infty} U_{\alpha,\beta}(k,h) x^{\alpha k} t^{\beta h}. \tag{3.16}$$

Using the values of $U_{\alpha,\beta}(k,h)$ from (3.15) in (3.16), the exact solution of homogeneous space-time fractional telegraph (3.11) is obtained as

$$u(x,t) = E_\alpha(x^\alpha) E_\beta\left(-t^\beta\right). \tag{3.17}$$

Remark 3.3. (1) Setting $q=1, p=2, r=1$, (3.11) reduces to space fractional telegraph equation

$$D_x^{2\alpha} u(x,t) = D_t^2 u(x,t) + D_t u(x,t) + u(x,t), \quad 0 < x < 1,\ t > 0, \tag{3.18}$$

with solution

$$u(x,t) = E_\alpha(x^\alpha) e^{-t}. \tag{3.19}$$

(2) Setting $\alpha = 1$, (3.11) reduces to time fractional telegraph equation:

$$D_x^2 u(x,t) = D_t^{p\beta} u(x,t) + D_t^{r\beta} u(x,t) + u(x,t), \quad 0 < x < 1,\ t > 0, \tag{3.20}$$

with solution

$$u(x,t) = e^x E_\beta\left(-t^\beta\right). \tag{3.21}$$

(3) Setting $\alpha = 1, q = 1, p = 2, r = 1$, (3.11) reduces to classical telegraph equation:

$$D_x^2 u(x,t) = D_t^2 u(x,t) + D_t u(x,t) + u(x,t), \quad 0 < x < 1,\ t > 0, \tag{3.22}$$

with solution

$$u(x,t) = e^{x-t}, \tag{3.23}$$

which is same as obtained by Kaya [26] using Adomian decomposition method.

Example 3.4. Consider the following non-homogeneous space-time fractional telegraph equation:

$$D_x^{2\alpha}u(x,t) = D_t^{p\beta}u(x,t) + D_t^{r\beta}u(x,t) + u(x,t) - 2E_\alpha(x^\alpha)E_\beta\left(-t^\beta\right), \quad 0 < x < 1, \ t > 0, \tag{3.24}$$

where $\beta = 1/q$, $p, q, r \in \mathbb{N}$, $1 < 2\alpha \le 2$, $1 < p\beta \le 2$, $0 < r\beta \le 1$, $D_x^{2\alpha} \equiv D_x^\alpha D_x^\alpha$, $D_t^{p\beta} \equiv D_t^\beta D_t^\beta \cdots D_t^\beta$ (p times), $D_t^{r\beta} \equiv D_t^\beta D_t^\beta \cdots D_t^\beta$ (r times), D_x^α, D_t^β are Caputo fractional derivatives defined by (2.1), p and r are even and

$$u(0,t) = E_\beta\left(-t^\beta\right), \qquad D_x^\alpha u(x,t)|_{x=0} = E_\beta\left(-t^\beta\right). \tag{3.25}$$

Applying generalized two-dimensional differential transform (2.4) with $x_0 = 0 = y_0$ to both sides of (3.24), (3.25), and using properties (c) and (d) we obtain

$$\begin{aligned}U_{\alpha,\beta}(k+2,h) = &\frac{\Gamma(\alpha k+1)}{\Gamma(\alpha(k+2)+1)}\left[\frac{\Gamma(\beta(h+p)+1)}{\Gamma(\beta h+1)}U_{\alpha,\beta}(k,h+p) + \frac{\Gamma(\beta(h+r)+1)}{\Gamma(\beta h+1)}U_{\alpha,\beta}(k,h+r)\right.\\ &\left.+U_{\alpha,\beta}(k,h) - \frac{2(-1)^h}{\Gamma(\alpha k+1)\Gamma(\beta h+1)}\right],\end{aligned} \tag{3.26}$$

$$U_{\alpha,\beta}(0,h) = \frac{(-1)^h}{\Gamma(\beta h+1)}, \quad U_{\alpha,\beta}(1,h) = \frac{(-1)^h}{\Gamma(\alpha+1)\Gamma(\beta h+1)}, \quad h = 0,1,2,\ldots. \tag{3.27}$$

Utilizing the recurrence relation (3.26) and the transformed conditions (3.27), we obtain

$$U_{\alpha,\beta}(k,h) = \frac{(-1)^h}{\Gamma(k\alpha+1)\Gamma(h\beta+1)}, \quad k,h = 0,1,2,\ldots. \tag{3.28}$$

Now from (2.5), we have

$$u(x,t) = \sum_{k=0}^{\infty}\sum_{h=0}^{\infty}U_{\alpha,\beta}(k,h)x^{\alpha k}t^{\beta h}. \tag{3.29}$$

Using the values of $U_{\alpha,\beta}(k,h)$ from (3.28) in (3.29), the exact solution of non-homogeneous space-time fractional telegraph (3.24) is obtained as

$$u(x,t) = E_\alpha(x^\alpha)E_\beta\left(-t^\beta\right). \tag{3.30}$$

Remark 3.5. (1) Setting $q = 2$, $p = 4$, $r = 2$, (3.24) reduces to non-homogeneous space fractional telegraph equation:

$$D_x^{2\alpha}u(x,t) = D_t^2u(x,t) + D_tu(x,t) + u(x,t) - 2E_\alpha(x^\alpha)e^{-t}, \quad 0 < x < 1, \ t > 0, \tag{3.31}$$

with solution

$$u(x,t) = E_\alpha(x^\alpha)e^{-t}. \tag{3.32}$$

(2) Setting $\alpha = 1$, (3.24) reduces to non-homogeneous time fractional telegraph equation:

$$D_x^2 u(x,t) = D_t^{p\beta} u(x,t) + D_t^{r\beta} u(x,t) + u(x,t) - 2e^x E_\beta\left(-t^\beta\right), \quad 0 < x < 1,\ t > 0, \tag{3.33}$$

with solution

$$u(x,t) = e^x E_\beta\left(-t^\beta\right). \tag{3.34}$$

(3) Setting $\alpha = 1$, $q = 2$, $p = 4$, $r = 2$, (3.24) reduces to non-homogeneous telegraph equation:

$$D_x^2 u(x,t) = D_t^2 u(x,t) + D_t u(x,t) + u(x,t) - 2e^x E_{1/2}\left(-t^{1/2}\right), \quad 0 < x < 1,\ t > 0, \tag{3.35}$$

with solution

$$u(x,t) = e^x E_{1/2}\left(-t^{1/2}\right). \tag{3.36}$$

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