

Research Article

Extended Jacobi Elliptic Function Expansion Method to the ZK-MEW Equation

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The extended Jacobi elliptic function expansion method is applied for Zakharov-Kuznetsov-modified equal-width (ZK-MEW) equation. With the aid of symbolic computation, we construct some new Jacobi elliptic doubly periodic wave solutions and the corresponding solitary wave solutions and triangular functional (singly periodic) solutions.

1. Introduction

It is one of the most important tasks to seek the exact solutions of nonlinear equation in the study of the nonlinear equations. Up to now, many powerful methods have been developed such as inverse scattering transformation [1], Backlund transformation [2], Hirota bilinear method [3], homogeneous balance method [4], extended tanh-function method [5], Jacobi elliptic function expansion method [6] and Ma's transformed rational function method [7].

Recently [8], an extended-tanh method is used to establish exact travelling wave solution of the Zakharov-Kuznetsov-modified equal-width (ZK-MEW) equation. In this paper, an extended Jacobi elliptic function expansion method is employed to construct some new exact solutions of the Zakharov-Kuznetsov-modified equal-width (ZK-MEW) equation.

As known, the Zakharov-Kuznetsov (ZK) equation are given by

$$(u_t + auu_x + u_{xxx})_x + u_{yy} = 0, \quad (1.1)$$

$$u_t + auu_x + \left(\nabla^2 u\right)_x = 0, \quad (1.2)$$

where $\nabla^2 = \partial_x^2 + \partial_y^2 + \partial_z^2$ is the isotropic Laplacian. The ZK equation governs the behavior of weakly nonlinear ion-acoustic waves in a plasma comprising cold ions and hot isothermal electrons in the presence of a uniform magnetic field [9]. In [9], the ZK equation is solved by the sine-cosine and the tanh-function methods. In [10], the numbers of solitary waves, periodic waves, and kink waves of the modified Zakharov-Kuznetsov equation are obtained.

The regularized long wave (RLW) equation given by

$$u_t + u_x + \frac{1}{2}(u^2)_x - u_{xxt} = 0, \quad -\infty < x < +\infty, \quad t > 0, \quad (1.3)$$

appears in many physical applications and has been studied in [11]. Gardner et al. [12] solved the equal width equation by a Petrov Galerkin method using quadratic B-spline spatial finite-elements.

The modified equal width (MEW) equation given by

$$u_t + 3u^2u_x - \beta u_{xxt} = 0, \quad (1.4)$$

has been discussed in [11]. The MEW equation is related to the RLW equation. This equation has solitary waves with both positive and negative amplitudes. The two-dimensional ZK-MEW equation which first appeared in [13] is given by

$$u_t + a(u^3)_x + (bu_{xt} + ru_{yy})_x = 0, \quad (1.5)$$

where $u = u(x, y, t)$, a, b, r are constants. In [13], some exact solutions of the ZK-MEW equation (1.5) was obtained by using the tanh and sine-cosine methods. More detailed description for ZK-MEW equation (1.5) the reader can find in paper [13]. In this paper, we will give some new solutions of Jacobi elliptic function type of ZK-MEW equation by using an extended Jacobi elliptic function method.

The remainder of the paper is organized as follows. In Section 2, we briefly describe the extended Jacobi elliptic function expansion method. In Section 3, we apply this method to ZK-MEW equation to construct exact solutions. Finally, some conclusions are given in Section 4.

2. The Extended Jacobi Elliptic Function Expansion Method

In this section, the extended Jacobi elliptic function expansion method is proposed in [14]. Consider a given nonlinear wave equation, say in two variables

$$F(u, u_x, u_t, u_{xx}, u_{xt}, u_{tt}, \dots) = 0. \quad (2.1)$$

We make the transformation

$$u = u(x, t) = U(\xi), \quad \xi = k(x - ct), \quad (2.2)$$

where k is a constant to be determined later. Then (2.1) reduces to a nonlinear ordinary differential equation (ODE) under (2.2)

$$G(U, U_\xi, U_{\xi\xi}, U_{\xi\xi\xi}, \dots) = 0. \quad (2.3)$$

By the extended Jacobi elliptic function expansion method, introduce the following ansatz

$$U(\xi) = \sum_{j=-M}^N a_j Y^j(\xi), \quad (2.4)$$

where M, N, a_j ($j = -M, \dots, N$) are constants to be determined later, Y is an Jacobi elliptic function, namely, $Y = Y(\xi) = \operatorname{sn} \xi = \operatorname{sn}(\xi, m)$ or $\operatorname{cn}(\xi, m)$ or $\operatorname{dn}(\xi, m)$, m ($0 < m < 1$) is the modulus of Jacobian elliptic functions. Positive integer M, N can be determined by balancing the highest-order linear term with the nonlinear term in (2.3). After this, substituting (2.4) into (2.3), we can obtain a system of algebraic equations for a_j ($j = -M, \dots, N$). Solving the above-mentioned equations with the Mathematica Software, then a_j ($j = -M, \dots, N$) can be determined. Substituting these obtained results into (2.4), then a general form of Jacobi elliptic function solution of (2.1) can be given.

3. ZK-MEW Equation

In this section, we employ the extended Jacobi elliptic function expansion method to ZK-MEW equation that is given by (1.5). The transformation $u = U(\xi)$, $\xi = \lambda(x + \mu y - \sigma t)$ converts (1.5) into ODE

$$-\sigma U_\xi + a(U^3)_\xi + (r\lambda^2\mu^2 - b\lambda^2\sigma)U_{\xi\xi\xi} = 0. \quad (3.1)$$

Integrating (3.1) and setting the constant of integration to zero, we obtain

$$-\sigma U + a(U^3) + (r\lambda^2\mu^2 - b\lambda^2\sigma)U_{\xi\xi} = 0. \quad (3.2)$$

Substituting (2.4) into (3.2) to balance U^3 with $U_{\xi\xi}$, we find $M = N = 1$. Thus, the solution admits in the form

$$U(\xi) = a_{-1}Y^{-1}(\xi) + a_0 + a_1Y(\xi), \quad (3.3)$$

where a_{-1}, a_0, a_1 are constants to be determined later.

Notice that

$$\begin{aligned}\frac{d(sn\xi)}{d\xi} &= sn'\xi = cn\xi dn\xi, & \frac{d(cn\xi)}{d\xi} &= cn'\xi = -sn\xi dn\xi, \\ \frac{d(dn\xi)}{d\xi} &= dn'\xi = -msn\xi cn\xi, \\ cn^2\xi &= 1 - sn^2\xi, & dn^2\xi &= 1 - msn^2\xi.\end{aligned}\tag{3.4}$$

3.1. The Case of $Y = Y(\xi) = sn\xi = sn(\xi, m)$

Substituting $Y = Y(\xi) = sn\xi = sn(\xi, m)$ and (3.3) into (3.2), making use of (3.4), we obtain a system of algebraic equations, for a_{-1} , a_0 , a_1 , and λ of the following form:

$$\begin{aligned}2\lambda^2(r\mu^2 - b\sigma)a_{-1} + aa_{-1}^3 &= 0, \\ 3aa_{-1}^2a_0 &= 0, \\ -a_{-1}\left[(1+m)r\lambda^2\mu^2 - (-\sigma + b(1+m)\lambda^2)\sigma - 2aa_0^2\right] + 3aa_{-1}^2a_1 &= 0, \\ -\sigma a_0 + aa_0^3 + 6aa_{-1}a_0a_1 &= 0, \\ (-1-m)r\lambda^2\mu^2a_1 + \left[-1 + b(1+m)\lambda^2\right]\sigma a_1 + 3aa_0^2a_1 + 3aa_{-1}a_1^2 &= 0.\end{aligned}\tag{3.5}$$

Solving the system of the algebraic equations with the aid of Mathematica we can distinguish two cases, namely the following.

Case 1.

$$a_1 = 0, \quad a_0 = 0, \quad a_{-1} = \frac{\sqrt{2\sigma}}{\sqrt{a(1+m)}}, \quad \lambda = \frac{\sqrt{\sigma}}{\sqrt{(1+m)(b\sigma - r\mu^2)}}.\tag{3.6}$$

Case 2.

$$a_1 = \frac{\sqrt{\sigma(1+m)}}{\sqrt{a}}, \quad a_{-1} = \frac{\sqrt{\sigma}}{\sqrt{a(1+m)}}, \quad a_0 = 0, \quad \lambda = \frac{\sqrt{\sigma}}{\sqrt{2(1+m)(b\sigma - r\mu^2)}}.\tag{3.7}$$

Substituting (3.6), (3.7) into (3.3), respectively, yield, the following solutions of ZK-MEW equation:

$$u(x, y, t) = \frac{\sqrt{2\sigma}}{\sqrt{a(1+m)}} sn^{-1} \left[\frac{\sqrt{\sigma}}{\sqrt{(1+m)(b\sigma - r\mu^2)}} (x + \mu y - \sigma t), m \right], \quad (3.8)$$

$$\begin{aligned} u(x, y, t) = & \frac{\sqrt{\sigma(1+m)}}{\sqrt{a}} sn \left[\frac{\sqrt{\sigma}}{\sqrt{2(1+m)(b\sigma - r\mu^2)}} (x + \mu y - \sigma t), m \right] \\ & + \frac{\sqrt{\sigma}}{\sqrt{a(1+m)}} sn^{-1} \left[\frac{\sqrt{\sigma}}{\sqrt{2(1+m)(b\sigma - r\mu^2)}} (x + \mu y - \sigma t), m \right]. \end{aligned} \quad (3.9)$$

Notice that $m \rightarrow 1$, $sn\xi \rightarrow \tanh \xi$, and $m \rightarrow 0$, $sn\xi \rightarrow \sin \xi$, we can obtain solitary wave solutions and sin-wave solutions from (3.8) and (3.9), respectively,

$$u(x, y, t) = \frac{\sqrt{\sigma}}{\sqrt{a}} \coth \left[\frac{\sqrt{\sigma}}{\sqrt{2(b\sigma - r\mu^2)}} (x + \mu y - \sigma t) \right], \quad (3.10a)$$

$$u(x, y, t) = \frac{\sqrt{2\sigma}}{\sqrt{a}} \sin^{-1} \left[\frac{\sqrt{\sigma}}{\sqrt{b\sigma - r\mu^2}} (x + \mu y - \sigma t) \right], \quad (3.10b)$$

$$\begin{aligned} u(x, y, t) = & \frac{\sqrt{2\sigma}}{\sqrt{a}} \tanh \left[\frac{\sqrt{\sigma}}{2\sqrt{(b\sigma - r\mu^2)}} (x + \mu y - \sigma t) \right] \\ & + \frac{\sqrt{\sigma}}{\sqrt{2a}} \coth \left[\frac{\sqrt{\sigma}}{2\sqrt{(b\sigma - r\mu^2)}} (x + \mu y - \sigma t) \right], \end{aligned} \quad (3.11a)$$

$$\begin{aligned} u(x, y, t) = & \frac{\sqrt{\sigma}}{\sqrt{a}} \sin \left[\frac{\sqrt{\sigma}}{\sqrt{2(b\sigma - r\mu^2)}} (x + \mu y - \sigma t) \right] \\ & + \frac{\sqrt{\sigma}}{\sqrt{a}} \sin^{-1} \left[\frac{\sqrt{\sigma}}{\sqrt{2(b\sigma - r\mu^2)}} (x + \mu y - \sigma t) \right]. \end{aligned} \quad (3.11b)$$

Here we only give the graph of (3.8) (see Figure 1) and (3.10a) (see Figure 2) and the other graphs of equations are similar to discussing.

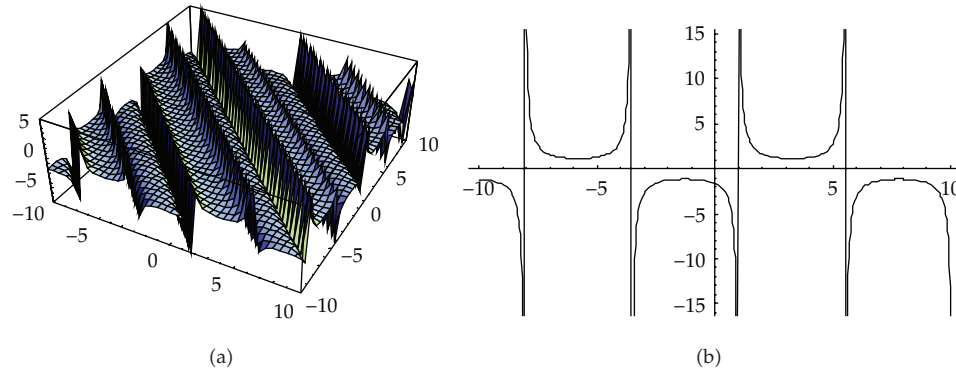


Figure 1: (a) The graph of (3.8) with $a = r = \sigma = 1, b = 2, m = 0.5$ at $t = 1$, and (b) is its plane when $y = 0$.

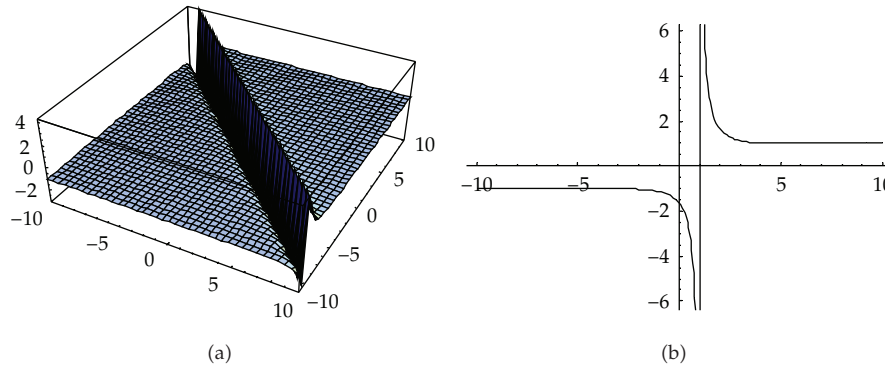


Figure 2: (a) The graph of (3.10a) with $a = r = \sigma = 1, b = 2$ at $t = 1$ and (b) is its plane when $y = 0$ (the case is the same to (3.8) when $m = 1$).

3.2. The Case of $Y = Y(\xi) = dn\xi = dn(\xi, m)$

The analysis proceeds of this case is as for Section 3.1. Substituting $Y = Y(\xi) = dn\xi = dn(\xi, m)$ and (3.3) into (3.2), making use of (3.4), we obtain a system of algebraic equations, for a_{-1}, a_0, a_1 and λ of the following form:

$$\begin{aligned}
 2(-1+m)\lambda^2(r\mu^2 - b\sigma)a_{-1} + aa_{-1}^3 &= 0, \\
 3aa_{-1}^2a_0 &= 0, \\
 -a_{-1}\left[(-2+m)r\lambda^2\mu^2 + \sigma + 2b\lambda^2\sigma - bm\lambda^2\sigma - 3aa_0^2\right] + 3aa_{-1}^2a_1 &= 0, \\
 -\sigma a_0 + aa_0^3 + 6aa_{-1}a_0a_1 &= 0, \\
 (2-m)r\lambda^2\mu^2a_1 + \left[-1 + b(-2+m)\lambda^2\right]\sigma a_1 + 3aa_0^2a_1 + 3aa_{-1}a_1^2 &= 0, \\
 a_1\left[2\lambda^2(-r\mu^2 + b\sigma) + aa_1^2\right] &= 0.
 \end{aligned} \tag{3.12}$$

Solving the system of the algebraic equations with the aid of Mathematica we can distinguish three cases, namely, The following.

Case 1.

$$a_{-1} = 0, \quad a_0 = 0, \quad a_1 = \frac{\sqrt{2\sigma}}{\sqrt{a(2-m)}}, \quad \lambda = \frac{\sqrt{\sigma}}{\sqrt{(2-m)(r\mu^2 - b\sigma)}}. \quad (3.13)$$

Case 2.

$$a_{-1} = \sqrt{\frac{2(1-m)\sigma}{a(2-6\sqrt{1-m}-m)}}, \quad a_0 = 0, \quad a_1 = \sqrt{\frac{2\sigma}{a(2-6\sqrt{1-m}-m)}}, \quad (3.14)$$

$$\lambda = \sqrt{\frac{\sigma}{(2-6\sqrt{1-m}-m)(b\sigma - r\mu^2)}}.$$

Case 3.

$$a_{-1} = \sqrt{\frac{2(1-m)\sigma}{a(2-m)}}, \quad a_0 = 0, \quad a_1 = 0, \quad \lambda = \frac{\sqrt{\sigma}}{\sqrt{(2-m)(r\mu^2 - b\sigma)}}. \quad (3.15)$$

Substituting (3.13), (3.14), and (3.15) into (3.3), respectively, yields the following solutions of ZK-MEW equation:

$$u(x, y, t) = \frac{\sqrt{2\sigma}}{\sqrt{a(2-m)}} dn \left[\frac{\sqrt{\sigma}}{\sqrt{(2-m)(r\mu^2 - b\sigma)}} (x + \mu y - \sigma t), m \right], \quad (3.16)$$

$$u(x, y, t) = \sqrt{\frac{2(1-m)\sigma}{a(2-6\sqrt{1-m}-m)}} dn^{-1} \left[\sqrt{\frac{\sigma}{(2-6\sqrt{1-m}-m)(b\sigma - r\mu^2)}} (x + \mu y - \sigma t), m \right] \\ + \sqrt{\frac{2\sigma}{a(2-6\sqrt{1-m}-m)}} dn \left[\sqrt{\frac{\sigma}{(2-6\sqrt{1-m}-m)(b\sigma - r\mu^2)}} (x + \mu y - \sigma t), m \right], \quad (3.17)$$

$$u(x, y, t) = \sqrt{\frac{2(1-m)\sigma}{a(2-m)}} dn^{-1} \left[\frac{\sqrt{\sigma}}{\sqrt{(2-m)(r\mu^2 - b\sigma)}} (x + \mu y - \sigma t), m \right]. \quad (3.18)$$

Notice that $m \rightarrow 1$, $dn\xi \rightarrow \text{sech}\xi$, thus we can obtain solitary wave of solutions of ZK-MEW equation from (3.16) and (3.17), respectively,

$$u(x, y, t) = \frac{\sqrt{2\sigma}}{\sqrt{a}} \text{sech} \left[\frac{\sqrt{\sigma}}{\sqrt{\pm(r\mu^2 - b\sigma)}} (x + \mu y - \sigma t) \right]. \quad (3.19)$$

Here we only give the graph of (3.17) (see Figure 3) and (3.19) (see Figure 4) and the other graphs of equations are similar to discussing.

3.3. The Case of $Y = Y(\xi) = cn\xi = cn(\xi, m)$

The analysis proceeds of this case is as for Sections 3.1 and 3.2. Substituting $Y = Y(\xi) = cn\xi = cn(\xi, m)$ and (3.3) into (3.2), making use of (3.4), we obtain a system of algebraic equations, for a_{-1} , a_0 , a_1 , and λ of the following form:

$$\begin{aligned} -2(-1+m)\lambda^2(r\mu^2 - b\sigma)a_{-1} + aa_{-1}^3 &= 0, \\ 3aa_{-1}^2a_0 &= 0, \\ a_{-1} \left[(-1+2m)r\lambda^2\mu^2 - \sigma + b\sigma(1-2m)\lambda^2 + 3aa_0^2 \right] + 3aa_{-1}^2a_1 &= 0, \\ -\sigma a_0 + aa_0^3 + 6aa_{-1}a_0a_1 &= 0, \\ (-1+2m)r\lambda^2\mu^2a_1 - \sigma a_1 + b\sigma(1-2m)\lambda^2a_1 + 3aa_0^2a_1 + 3aa_1^2a_{-1} &= 0, \\ 3aa_1^2a_0 &= 0, \\ a_1 \left[2m\lambda^2(-r\mu^2 + b\sigma) + aa_1^2 \right] &= 0. \end{aligned} \quad (3.20)$$

Solving the system of the algebraic equations (3.20) with the aid of Mathematica, we can distinguish three cases, namely, the following.

Case 1.

$$a_0 = 0, \quad a_1 = \sqrt{\frac{2m\sigma}{a(-1+2m)}}, \quad a_{-1} = 0, \quad \lambda = \frac{\sqrt{\sigma}}{\sqrt{(-1+2m)(r\mu^2 - b\sigma)}}. \quad (3.21)$$

Case 2.

$$a_0 = 0, \quad a_{-1} = \sqrt{\frac{2(1-m)\sigma}{-a(-1+2m)}}, \quad a_1 = 0, \quad \lambda = \frac{\sqrt{\sigma}}{\sqrt{(-1+2m)(r\mu^2 - b\sigma)}}. \quad (3.22)$$

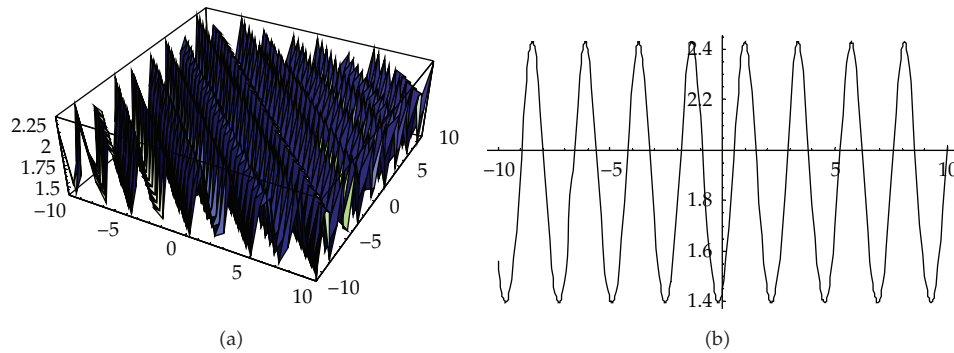


Figure 3: (a) The graph of (3.17) with $a = r = \sigma = 1$, $b = 2$, $m = 0.99$ at $t = 1$, and (b) is its plane when $y = 0$.

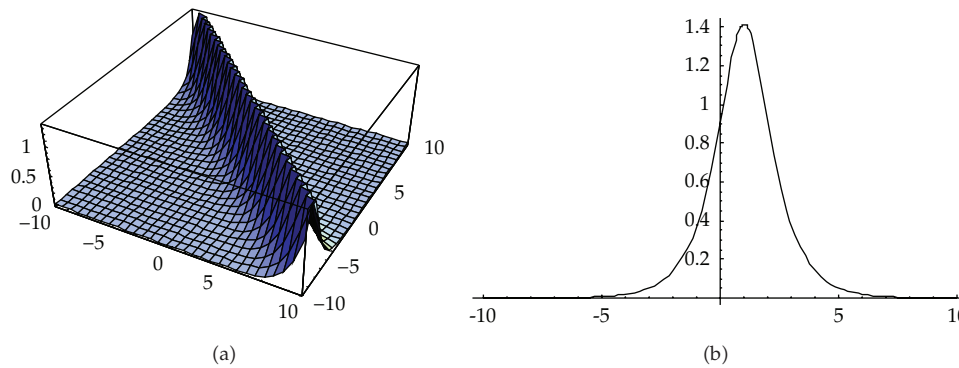


Figure 4: (a) The solitary wave graph of (3.19) with $a = r = \sigma = 1$, $b = 2$ at $t = 1$, and (b) is its plane when $y = 0$ (the case is the same to (3.17) when $m = 1$).

Case 3.

$$a_0 = \sqrt{\frac{\sigma}{a}}, \quad a_1 = -2\sqrt{\frac{m\sigma}{a(1-2m)}}, \quad a_{-1} = 0, \quad \lambda = \sqrt{\frac{2\sigma}{(1-2m)(r\mu^2 - b\sigma)}}. \quad (3.23)$$

Substituting (3.21), (3.22), and (3.23) into (3.3), respectively, yield the following solutions of ZK-MEW equation:

$$u(x, y, t) = \sqrt{\frac{2m\sigma}{a(-1+2m)}} \operatorname{cn} \left[\frac{\sqrt{\sigma}}{\sqrt{(-1+2m)(r\mu^2 - b\sigma)}} (x + \mu y - \sigma t), m \right], \quad (3.24)$$

$$u(x, y, t) = \sqrt{\frac{2(1-m)\sigma}{-a(-1+2m)}} \operatorname{cn}^{-1} \left[\frac{\sqrt{\sigma}}{\sqrt{(-1+2m)(r\mu^2 - b\sigma)}} (x + \mu y - \sigma t), m \right], \quad (3.25)$$

$$u(x, y, t) = \sqrt{\frac{\sigma}{a}} - 2\sqrt{\frac{m\sigma}{a(1-2m)}} \operatorname{cn} \left[\sqrt{\frac{2\sigma}{(1-2m)(r\mu^2 - b\sigma)}} (x + \mu y - \sigma t), m \right]. \quad (3.26)$$

Especially, when $m \rightarrow 1$, $cn\xi \rightarrow \operatorname{sech}\xi$ and $m \rightarrow 0$, $cn\xi \rightarrow \cos\xi$, thus we can obtain solutions of ZK-MEW equation from (3.24) and (3.25)

$$u(x, y, t) = \sqrt{\frac{2\sigma}{a}} \operatorname{sech} \left[\frac{\sqrt{\sigma}}{\sqrt{r\mu^2 - b\sigma}} (x + \mu y - \sigma t) \right], \quad (3.27)$$

$$u(x, y, t) = \sqrt{\frac{2\sigma}{a}} \cos^{-1} \left[\frac{\sqrt{\sigma}}{\sqrt{-r\mu^2 + b\sigma}} (x + \mu y - \sigma t) \right]. \quad (3.28)$$

Remark 3.1. In these solutions, (3.10a), (3.19) and (3.27) have been obtain in [8], the others solutions are new solutions for the ZK-MEW equation.

4. Conclusions

The extended Jacobi elliptic function expansion method was directly and effectively employed to find travelling wave solutions of the nonlinear ZK-MEW equation. Using the method, we found some new solutions of Jacobi elliptic function type that were not obtained by the sine-cosine method, the extended tanh-method, the mapping method, and other methods. In the limiting case of the Jacobi elliptic function (namely, modulus setting 0 or 1), we also obtained the solutions of sin-type, cos-type, tanh-type, and sech-type. The extended Jacobi elliptic function expansion method can be applied to some other nonlinear equation and gives more solutions.

The ZK-MEW equation was first appeared in Wazwaz's paper [13] in 2005. To my acknowledge, its many properties, such as integrability, Lax pairs, and multisoliton solutions, have not been studied. The study of these properties is a very signification work and is our task research in the future.

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