

# Correction to the article

## An infinite-rank summand of topologically slice knots

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We describe an error in the proof of a key proposition of our paper *An infinite-rank summand of topologically slice knots* (Geom. Topol. 19 (2015) 1063–1110), which was necessary for the proof of the main result. Alternative proofs of the main result are given by Ozsváth, Stipsicz and Szabó, and Dai, Hom, Stoffregen and Truong.

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In [4], we constructed a homomorphism

$$\mathcal{C} \rightarrow \mathcal{CFK},$$

where  $\mathcal{C}$  denotes the smooth knot concordance group and  $\mathcal{CFK}$  consists of knots Floer complexes modulo  $\varepsilon$ -equivalence. We then applied the Hahn embedding theorem to show that

$$\mathcal{CFK} \hookrightarrow \mathbb{R}^X.$$

These results still stand.

However, Proposition 1.3 of [4], which claimed that this embedding could be used to construct an infinite family of  $\mathbb{Z}$ -valued homomorphisms from  $\mathcal{CFK}$ , is false. The error is that we quoted a version of the Hahn embedding theorem for vector spaces — see Hausner and Wendel [3] — not groups. In particular, condition (2) above Definition 1.2 of [4] does not hold for groups.

To see that Proposition 1.3 is false for groups, consider the following example, due to B Gordon and found in Clifford [1]. Order  $\mathbb{Q}^2$  lexicographically. Let  $G$  be the (ordered) subgroup of  $\mathbb{Q}^2$  generated by  $(p_n^{-1}, np_n^{-1})$ , where  $p_n$  denotes the  $n^{\text{th}}$  prime. Then  $(0, 1) \in G$  satisfies the hypothesis of Proposition 1.3, but not the conclusion. In particular,  $(0, 1) \in G$  satisfies Property A but the image of projection from  $G$  onto its second coordinate is not isomorphic to  $\mathbb{Z}$ .

The main purpose of defining  $\mathbb{Z}$ -valued concordance homomorphisms was to prove the following result about  $\mathcal{C}_{TS}$ , the subgroup of the smooth concordance group generated by topologically slice knots.

**Theorem 1** *The group  $\mathcal{C}_{TS}$  contains a direct summand isomorphic to  $\mathbb{Z}^\infty$ .*

An independent proof of [Theorem 1](#) is given by Ozsváth, Stipsicz and Szabó [\[5\]](#), using their concordance homomorphism  $\Upsilon$ . (Ironically, Ozsváth, Stipsicz and Szabó state in their introduction that  $\Upsilon$  was inspired by [\[4\]](#).)

Recent work of Dai, Stoffregen, Truong and the author [\[2\]](#) provides a new infinite family of  $\mathbb{Z}$ -valued concordance homomorphisms  $\varphi_n$  that factor through  $\mathcal{CFK}$ . The definition of  $\varphi_n$  still relies on the total order of  $\mathcal{CFK}$ , although it does not rely on the Hahn embedding theorem. The homomorphisms  $\varphi_n$  give an alternative proof of [Theorem 1](#), using the same knots as in [\[4\]](#).

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## References

- [1] **A H Clifford**, *Note on Hahn's theorem on ordered abelian groups*, Proc. Amer. Math. Soc. 5 (1954) 860–863 [MR](#)
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- [3] **M Hausner, J G Wendel**, *Ordered vector spaces*, Proc. Amer. Math. Soc. 3 (1952) 977–982 [MR](#)
- [4] **J Hom**, *An infinite-rank summand of topologically slice knots*, Geom. Topol. 19 (2015) 1063–1110 [MR](#)
- [5] **P S Ozsváth, A I Stipsicz, Z Szabó**, *Concordance homomorphisms from knot Floer homology*, Adv. Math. 315 (2017) 366–426 [MR](#)

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