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SOME REMARKS ON STABILITY AND SOLVABILITY OF LINEAR FUNCTIONAL EQUATIONS

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To my friend Professor Themistocles M. Rassias
with the author's compliments and very good wishes

Submitted by T. Riedel

Abstract. In the present work we continue studying the solvability of the linear functional equations $\sum_{j=1}^{N} c_j F \circ a_j = H$ and also the strong and weak stability of the corresponding operator $\mathcal{P}$ (see the definition below). By analogy with the Cauchy and Jensen operators once more model operator $\widehat{\mathcal{P}}$ is considered, and the stability problems as well as some solvability problems for $\widehat{\mathcal{P}}$ are studied. Several unsolved problem of a general character are formulated.

1. Introduction.

In this work we continue studying the general linear functional operator

$$\mathcal{P}F := \sum_{j=1}^{N} c_j(x) F\big(a_j(x)\big),$$

where $x \in D \subset \mathbb{R}^n$, $n \geq 1$, $F \in C(I, B)$ is a compact supported Banach - valued continuous function of a single variable. The interest to this object of functional analysis is determined not only by its comparative novelty. The main reason is (at least for the author) the possibility to use the results obtained when studying various problems in such diverse fields of analysis as Integral geometry, Partial differential equations, Measure theory, Integral equations, Approximate

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calculation and some others. It is in this connection that the solvability questions related to the nonhomogeneous equations $\mathcal{P}F = H$ including some elements of the qualitative theory were studied for the first time in the author's papers [1, 2, 3, 4]. Along with the solvability questions of the equation $\mathcal{P}F = H$, we continue studying various form of stability of the operator $\mathcal{P}$ begun in the author's papers [5, 6].

The paper is organized in the following way. At first we remind to the reader the new notion of the stability of the operator $\mathcal{P}$ introduced by the author in [5] and define some function spaces suitable when studying this operator. Then, we formulate the problem we deal with, cite all the results obtained, and, finally, prove them. The concluding part of the paper is devoted to a short discussion of the above results, and to formulating some problems.

2. Main notions and definitions.

To present the results in a self contained form we remind the main definitions from [5]. The kernel and the range of any linear operator L are denoted by $\ker L$ and $\mathcal{R}(L)$, respectively. We denote by I the interval $\{t \mid 0 \le t \le 1\}$. Given a Banach space B with the norm $|\cdot|_B$ we denote by $|F| = \sup\limits_{t\in I} |F(t)|_B$ the norm in the space $C(I,B)$. The following function spaces introduced for the first time in [3] turned out to be very useful when working with the problems in question. By definition, if $\gamma > 0$, then

$$C_{\langle\gamma\rangle}(I,B) = \{F \mid F(t) = b_0 + \ldots + b_{[\gamma]}t^{[\gamma]} + t^\gamma f(t)\}$$

with $b_j \in B$ and $f(t) \in C(I,B)$ if $\gamma \notin \mathbb{N}$ with $[\gamma]$ being the integral part of the γ, and $f(0) = 0$, if $\gamma \in \mathbb{N}$. The space $C_{\langle\gamma\rangle}(I,B)$ endowed by the norm

$$|F|_{\langle\gamma\rangle} = \sum_{j=1}^{[\gamma]} |b_j| + |f(t)|$$

is a Banach space.

Let $D \subset \mathbb{R}^n$, $n \ge 2$, be a domain, and Γ one-dimensional submanifold (curve) in D. If $\zeta : I \to \Gamma$ is a one-to-one $C_{\langle\gamma\rangle}$ - map, then we denote by w_Γ and $\mathcal{P}_\Gamma F$ the restriction

$$w_\Gamma(s) = (w_\Gamma \circ \zeta)(s), \quad s \in I,$$

of an arbitrary function $w \in C(D)$ to Γ, and the operator

$$\mathcal{P}_\Gamma : F \to \sum_{j=1}^{N} c_{j\Gamma}(s)\, F\big(a_{j\Gamma}(s)\big), \quad s \in I,$$

respectively.

Definition 2.1. Given a $C_{\langle\gamma\rangle}$ - curve $\Gamma \subset D$, we say that the operator $\mathcal{P}$ is *strongly stable (along* Γ*)*, if the à priori estimate

$$\inf_{\varphi\in\ \ker\mathcal{P}} |\, F - \varphi\, |_{C_{\langle\gamma\rangle}} < c\, |\, \mathcal{P}_\Gamma F\, |_{C_{\langle\gamma\rangle}} \tag{2.1}$$

holds with a constant $c > 0$ not depending on $F \notin \ker\mathcal{P}$.

Definition 2.2. In the same situation the operator $\mathcal{P}$ is called *weakly stable (along* Γ*)*, if estimate (2.1) holds with $\ker \mathcal{P}_\Gamma$ instead of $\ker \mathcal{P}$.

It is clear that the strong stability implies the weak one.

3. Results: formulations and proofs.

The functional operator $\mathcal{P}$ we deal with in the present paper is

$$\mathcal{P}F := F(ax+by) - \alpha F(x) - \beta F(y),$$

where all number parameters satisfy the conditions

$$a > 1, \quad b > 1, \quad \alpha + \beta > 1, \tag{3.1}$$

the function F is in space $C(I, B)$, and the points (x, y) fill out the triangle

$$D = \{(x,y) \mid (ax+by) \leq 1, \quad 0 \leq x, y \leq 1\}.$$

Note that all the results of the work are trivially valid if $\alpha + \beta < 1$. If $\alpha + \beta = 1$ and $a = \alpha$, $b = \beta$, we deal with the Jensen operator $\mathfrak{J}$. If $\alpha = \beta = a = b = 1$, then $\mathcal{P}$ is the Cauchy operator $\mathfrak{C}$. The extensive class of functional operators $\mathcal{P}$, structurally associated with $\mathfrak{C}$ and $\mathfrak{J}$ has been studied in detail in [5] and [6].

The main problem we deal with and related to this operator is its stability. But the first result concerns with some solvability problems.

Let λ and μ be some positive reals such that

$$a\lambda + b\mu = 1, \tag{3.2}$$

and Γ the curve

$$\Gamma = \{(x,y) \in D \mid x = \lambda t, \ y = \mu t; \ \ t \in I\}.$$

Define γ as the unique τ- root of the equation

$$\alpha\lambda^\tau + \beta\mu^\tau = 1. \tag{3.3}$$

Theorem 3.1. 1° *The kernel of the operator*

$$\mathcal{P}_\Gamma : C_{\langle\gamma\rangle}(I, B) \to C_{\langle\gamma\rangle}(I, B)$$

consists of the functions

$$\varphi(t) = At^\gamma, \quad A \in \mathbb{R}.$$

2° *The function* $F = 0$ *is the unique solution of the homogeneous equation*

$$\mathcal{P}F = 0$$

in the space $C_{\langle\gamma\rangle}(I, B)$.

Note that the analogous fact of triviality of the kernel of the general operator $\mathcal{P}$ has been established for some Cauchy type and Jensen type operators (see [5] and [6], respectively).

Here is two examples of such operators quoted from [5, 6]. *The equations*

$$F(x^2+y+x)-F\left(\frac{3x}{2}\right)$$
$$-F\left(y^2-4xy-x^4+4x^2+\frac{3x}{2}\right)=0,\quad 0\le x,y\le 1,$$

and

$$F\Big(x\sqrt{1+x^2}e^{x^2y}+\sqrt{x^2y^3+1}\sin y\Big)-\frac{1}{3}e^{x^2y}F\Big(3x\sqrt{1+x^2}\Big)$$
$$-\frac{2}{3}\sqrt{x^2y^3+1}F(\frac{3}{2}\sin y)=0,\quad 0\le x,y\le\gamma.$$

with sufficiently small γ have only trivial solution $F=0$ in the space $C_{\langle 1+r\rangle}(I)$, $0\le r\le 1$.
Try to prove the triviality of their kernels. It would be interesting to describe as extensive as possible class of the operators $\mathcal{P}$ with the property in question.

Theorem 3.2. *The operator $\mathcal{P}$ is weakly stable (along Γ) in the space $C_{\langle\gamma+\delta\rangle}$ for an arbitrary $\delta>0$. In other words, the à priori estimate*

$$\inf_{\varphi\in\ker\mathcal{P}_\Gamma}|\,F-\varphi\,|_{\langle\gamma+\delta\rangle}<c\,|\,\mathcal{P}_\Gamma F\,|_{\langle\gamma+\delta\rangle},\quad F\in C_{\langle\gamma+\delta\rangle}(I,B),\tag{3.4}$$

holds with a constant $c>0$ not depending on F.

Corollary 3.3. 1° *If $\gamma\notin\mathbb{N}$, then, for all $\delta>0$, the à priori estimate*

$$|F|_{\langle\gamma+\delta\rangle}<c|\mathcal{P}_\Gamma F|_{\langle\gamma+\delta\rangle}$$

holds with a constant c not depending on F. Thus, the $\mathcal{P}$ is strongly stable in this case.
2° *If $\gamma\in\mathbb{N}$, then there is a constant $A\ge 0$ such that the à priori estimate*

$$|F-At^\gamma|_{\langle\gamma+\delta\rangle}<c|\mathcal{P}_\Gamma F|_{\langle\gamma+\delta\rangle}$$

holds with a constant c, as in 1°.

Proof of Theorem 3.1. To begin with, we prove the correctness of the definition of the number γ. Indeed, by (3.1) and (3.2) both numbers λ and μ are less then one. The function

$$\zeta(x)=\alpha\lambda^x+\beta\mu^x,\quad x\ge 0,$$

therefore, decreases in its domain from $\alpha+\beta$ to 0 and takes the value 1 at a single point.

1° To describe the kernel of the operator $\mathcal{P}_\Gamma$ consider the homogeneous equation $\mathcal{P}_\Gamma F=0$ or, in the detailed form,

$$F(t)-\alpha F(\lambda t)-\beta F(\mu t)=0.\tag{3.5}$$

Note that, by the definition of the space $C_{\langle\gamma\rangle}$, any solution $F\in C_{\langle\gamma\rangle}(I,B)$ of this equation admits the representation in the form

$$F(t)=t^\gamma f(t),$$

where f is a continuous function, satisfying the functional equation

$$f(t)-\alpha\lambda^{\gamma}f(\lambda t)-\beta\mu^{\gamma}f(\mu t)=0,\quad t\in I. \tag{3.6}$$

Indeed, by the definition of the space $C_{\langle\gamma\rangle}$,

$$F(t)=\sum_{j=0}^{[\gamma]}c_jt^j+t^{\gamma}f(t),\quad t\in I,$$

where $f(t)$ is a continuous function and, in addition,

$$f(0)=0,\quad \text{if }\ \gamma\in\mathbb{N}.$$

Substituting such an F in (3.5) leads to the asymptotic relation

$$\sum_{j=0}^{[\gamma]}c_j\Big(1-\alpha\lambda^j-\beta\mu^j\Big)t^j+t^{\gamma}\Big[f(t)-\alpha\lambda^{\gamma}f(\lambda t)-\beta\mu^{\gamma}f(\mu t)\Big]=0, \tag{3.7}$$

whence all c_j are equal to 0, if $\gamma\notin\mathbb{N}$, and

$$c_1=\ldots=c_{\gamma-1}=0,$$

as well as

$$1-\alpha\lambda^{\gamma}-\beta\mu^{\gamma}=0,$$

by the definition of γ, if $\gamma\in\mathbb{N}$. In both cases the function f satisfies equation (3.6). Show that among continuous functions the only constants solve this equation. The fact, that $f(t)$ =const is a solution to (3.6), follows from (3.3). To prove the part "only" we will use the general *Maximum principle for linear functional operators* proved in [1]. For the completeness, we cite it here in the simplest form.

Maximum principle. *Given an operator*

$$\mathcal{P}\colon F(t)\to F(t)-a_1(t)F\big(\delta_1(t)\big)-\ldots a_N(t)F\big(\delta_N(t)\big),\quad t\in I,$$

with continuous functions a_j and δ_j, satisfying conditions

$$\sum_{j=1}^{N}a_j=1, \tag{3.8}$$

$$\delta_k(t)<t\quad \textit{for some}\ \ k, \tag{3.9}$$

any solution F of the homogeneous equation $\mathcal{P}F=0$ takes its maximal value at the point $t=0$.

Indeed, let $M:=\max_I F(t)$ and $F(t_0)=M$. Then, by (3.8), $F\big(\delta_k(t_0)\big)=M$, and hence, $F\big(\delta_k^n(t_0)\big)=M$ for all $n\in\mathbb{N}$. By (3.9), $\delta_k^n(t_0)\to 0$, as $n\to\infty$, whence $F(0)=M$.

By the definition of γ, the operator in the left hand side of (3.6) satisfies conditions (3.8) and (3.9). In virtue of the Maximum principle, $M=\max_I f=f(0)$. But the above proof is equally true for the minimal value of the f. This leads to the relation $\max f=\min f=f(0)$, which means that $f(t)\equiv$const.

Thus,

$$F(t) = At^\gamma, \quad A \text{ is a constant},$$

as it was promised.

2° Let $F \in C_{\langle\gamma\rangle}(I, B)$ and

$$\mathcal{P}F = 0.$$

Let Γ be one of the above curves, generated by a pair (λ, μ). Then, for any such Γ,

$$\mathcal{P}_\Gamma F = 0,$$

and therefore,

$$F(t) - \alpha F(\lambda t) - \beta F(\mu t) = 0, \quad t \in I.$$

By the part 1° of the theorem, for some constants A and $\gamma \neq 1$, we have

$$F(t) = At^\gamma.$$

To prove the assertion 2° one need to show that $A = 0$. As F solves the equation $\mathcal{P}F = 0$, the relation

$$(ax + by)^\gamma - \alpha x^\gamma - \beta y^\gamma = 0 \tag{3.10}$$

holds for all $(x, y) \in D$. Substituting successively 0 for x and for y results in relations

$$a^\gamma = \alpha, \quad b^\gamma = \beta.$$

Together with (3.10) this leads to the relation

$$\left(\frac{ax}{ax + by}\right)^\gamma + \left(\frac{by}{ax + by}\right)^\gamma = 1, \quad (x, y) \in D. \tag{3.11}$$

However, the equality $\eta^\gamma + \nu^\gamma = 1$ under conditions $\eta + \nu = 1$, $\eta > 0$, $\nu > 0$ is possible only for $\gamma = 1$. Therefore, taking into account that the sum of values in the brackets in (3.11) is equal to 1, we arrive at the contradiction to the choice of F (see (3.10)). This completes the proof of the theorem.

Proof of Theorem 3.2. The proof is completely based on Proposition 1 in [5]. For the completeness, we remind it.

> *Let $L : B_1 \to B_2$ be a closed linear operator between Banach spaces and $\mathcal{K} = \ker L$. If the range $\mathcal{R}(L)$ is closed, then there is a positive constant c such that the à priori estimate*
>
> $$\inf_{\varphi \in \mathcal{K}} |F - \varphi|_{B_1} < c|LF|_{B_2}$$
>
> *holds for all elements $F \notin \mathcal{K}$.*

In our case the spaces B_1 and B_2 coincide with $C_{\langle\gamma+\delta\rangle}(I, B)$ and the role of L plays the operator

$$\mathcal{P}_\Gamma : F(t) \to F(t) - \alpha F(\lambda t) - \beta F(\mu t).$$

Consider first the situation with $\gamma \notin \mathbb{N}$. As the closedness of the operator $\mathcal{P}_\Gamma$ follows from its continuity, it suffices, by the above Proposition, to prove the solvability of the equation

$$F(t) - \alpha F(\lambda t) - \beta F(\mu t) = H(t) \tag{3.12}$$

in $C_{\langle\gamma+\delta\rangle}(I,B)$ for an arbitrary $H \in C_{\langle\gamma+\delta\rangle}(I,B)$. Comparing the asymptotic expansion of the left hand side in (3.12) (see (3.7)) with the analogous expansion

$$\sum_{j=0}^{[\gamma]} b_j t^j + t^{\gamma+\delta} h(t)$$

of the function H (with h being a continuous function) we obtain immediately all the values

$$c_j = b_j/(1-\alpha\lambda^j - \beta\mu^j\,), \quad j = 1,\dots,[\gamma]-1,$$

as well as

$$b_{[\gamma]} = c_{[\gamma]}\Big(1-\alpha\lambda^{[\gamma]} - \beta\mu^{[\gamma]}\Big) = 0, \tag{3.13}$$

and the relation

$$f(t) - \alpha\lambda^{\gamma+\delta} f(\lambda t) - \beta\mu^{\gamma+\delta} f(\mu t) = h(t), \tag{3.14}$$

where both functions f and h are in $C(I,B)$. Rewrite (3.14) in the operator form

$$f - Af = h,$$

where

$$A\colon f(t) \to \alpha\lambda^{\gamma+\delta} f(\lambda t) + \beta\mu^{\gamma+\delta} f(\mu t)$$

is the linear operator in $C(I,B)$ with the norm

$$\|A\| \le \alpha\lambda^{\gamma+\delta} + \beta\mu^{\gamma+\delta} < 1.$$

The latter inequality follows by the choice of the value γ and due to $\lambda,\mu < 1$. Applying the classical result in functional analysis (the invertibility of the operator $E - A$, E the identical operator) results in the unique solvability of equation (3.13) for an arbitrary function $h \in C_{\langle\gamma+\delta\rangle}(I,B)$. This completes the proof, when $\gamma \notin \mathbb{N}$.

Let now $\gamma \in \mathbb{N}$. Then, by (3.13), $b_{[\gamma]} = 0$. This means that now the range of the operator $\mathcal{P}_\Gamma$ coincides with the subspace of functions $H \in C_{\langle\gamma+\delta\rangle}$ with $H^{(n)}(0) = 0$. As such a situation is included in the above Proposition, we arrive at needed solvability of equation (3.12) repeating word for word the above proof. Theorem 3.2 is completely proved.

Proof of Corollary 3.3. 1° By Theorem 3.2, if $\gamma \notin \mathbb{N}$, then the kernel of the operator $\mathcal{P}_\Gamma$ consist of functions $\varphi = At^\gamma$, A a constant. But $t^\gamma \notin C_{\langle\gamma+\lambda\rangle}(I,B)$, so the only function $\varphi = 0$ lies in $\ker \mathcal{P}_\Gamma \cup C_{\langle\gamma+\lambda\rangle}(I,B)$, and the Corollary follows from (3.4).

2° If $\gamma \in \mathbb{N}$, then, in contrast to 1°, the subspace $\mathcal{K} = \ker \mathcal{P}_\Gamma \cap C_{\langle\gamma+\lambda\rangle}(I,B)$ consists of the functions $\varphi = At^\gamma$. Denote

$$m(\varphi) = |F - \varphi|_{\langle\gamma+\lambda\rangle}$$

with F being from (3.4), and let

$$\mu = \inf_{\varphi\in\mathcal{K}} m(\varphi).$$

If $\mu = m(\Phi)$ for some $\Phi \in \mathcal{K}$, then the proof is completed. Assume that $\mu < m(\varphi)$ for all elements $\varphi \in \mathcal{K}$. Let

$$\mu = c|\mathcal{P}_\Gamma F|_{\langle\gamma+\lambda\rangle} - \varepsilon$$

for some $\varepsilon > 0$ with c a constant in (3.4). Then, by the definition of μ, there is an element $\Phi \in \mathcal{K}$ such that

$$\mu < m(\Phi) < \mu + \varepsilon = c|\mathcal{P}_\Gamma F|_{\langle\gamma+\lambda\rangle}.$$

This completes the proof of the Corollary.

4. Concluding remarks.

1. The choice of the above operator $\mathcal{P}$ is not casual. On the one hand, it has a very simple structure, coinciding with that of the classical Cauchy and Jensen operators for some values of the parameters. These two are the main objects of investigation in innumerable papers dealing with the Ulam stability of functional operators. This makes it possible to the reader (using a very simple model) to compare the two different approaches to the stability from both quality (the character of the results obtained) and technical (the tools used in the proofs) points of view.

On the other hand, it is not seen how to prove the Ulam stability for the operator $\mathcal{P}$ in question in the space C. Therefore, Theorem 3.2 and Corollary 3.3 represent the best possible of today result related to the stability of the operator in question.

2. In the works [5, 6] the theory of the strong and weak stability has been work out for the operators

$$\widehat{\mathfrak{C}}\, F := F \circ a - \sum_{j=1}^{N} F \circ a_j$$

with the functions $a, a_j : D \subset \mathbb{R}^n \to I$ such that

$$a = \sum_{j=1}^{N} a_j \quad \text{or} \quad a_\Gamma = \sum_{j=1}^{N} a_{j\Gamma},$$

and

$$\widehat{\mathfrak{J}}\, F := F \circ a - \sum_{j=1}^{N} c_j\, F \circ a_j$$

with

$$a = \sum_{j=1}^{N} c_j a_j, \quad \sum_{j=1}^{N} c_j = 1 \quad \text{or} \quad a_\Gamma = \sum_{j=1}^{N} c_{j\Gamma} a_{j\Gamma}, \quad \sum_{j=1}^{N} c_{j\Gamma} = 1,$$

called by the author *the Cauchy and Jensen type operators*, respectively. These operators in principle can not be studied by the Hyer's method, as his machinery does not compatible with nonlinear arguments a_j nor with nonconstant coefficients c_j.

The operator $\widehat{\mathcal{P}}$, studied here, can also play a role of a model operator for different classes of operators of the type

$$\widehat{\mathcal{P}}f := F\bigl(a(x)\bigr) - \sum_{j=1}^{N} \alpha_j F\bigl(a_j(x)\bigr),$$

$x \in D \subset \mathbb{R}^n$, with the $a_j(x)$, $a(x)$ and α_j satisfying some specific conditions. One of such classes is already studied, and the corresponding results will be published elsewhere.

3. The reader acquainted with the Ulam stability knows that the hypothetical result related to the operator $\widehat{\mathcal{P}}$ should have the following form:

for all $\varepsilon > 0$, there is a function $\varphi(t) \in C(\mathbb{R}, B) \cap \ker\widehat{\mathcal{P}}$ such that if

$$|\widehat{\mathcal{P}}F| < \varepsilon \quad \text{for } \ F \in C(\mathbb{R}, B), \tag{4.1}$$

then the inequality

$$|F - \varphi| < c\varepsilon$$

holds with a constant $c > 0$ not depending on F or on ε.

The essential difference between such an result and Corollary 3.3 is as follows.

(i) Condition (4.1) dictates the restriction $|\widehat{\mathcal{P}}F(x, y)| < \varepsilon$ at all points (x, y) of the *plane*, whereas the analogous restriction in Corollary 3.3 reduces to the same inequality but only at points (x, y) of some *curve* Γ. This is equivalent to some overdeterminedness of the Ulam problem (if it is not solvable in the space more extensive than $C_{\langle\gamma\rangle}$).

(ii) The domain of F in (4.1) is the whole space $\mathbb{R}$, whereas it is compact in Theorem 3.2.

(iii) Theorem 3.2 guarantees the smoothness of the function φ provided that F is smooth (in applications this fact sometimes plays the crucial role). The above hypothetical result does not distinguish smooth and continuous cases.

It is possible to shorten such a gap between the two approaches by solving the following two problems.

Problem 1. To prove or to disprove that the operator $\widehat{\mathcal{P}}$ is strongly or weakly stable in the space $C(I, B)$.

Problem 2. Assume that the $\widehat{\mathcal{P}}$ is Ulam stable. To prove or to disprove that the $\widehat{\mathcal{P}}$ is strongly or weakly stable (along some curve Γ).

The same problems are waiting for their solvability in the case of all the above mentioned Cauchy type and Jensen type operators.

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