bifurcation (1942) for example lies deeper than CT. The Hopf theory shows how a stable equilibrium bifurcates to a stable oscillation in ordinary differential equations. Moreover, there is the reference *Theory of oscillations* by Andronov and Chaiken, 1937, with English translation in 1949 published by the Princeton University Press which is never referred by Thom or Zeeman. This book besides giving an early account of structural stability, gives a good account of dynamical systems in two variables with explicit development of discontinuous phenomena, quite close to Zeeman's use of the cusp catastrophe. Examples from physics and electrical engineering are studied in some depth.

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Bornologies and functional analysis, by Henri Hogbe-Nlend, Mathematics Studies no. 26, North-Holland, Amsterdam, New York, Oxford, 1977, xii + 144 pp., $19.50.

The author states in his introduction that functional analysis is analysis over infinite dimensional spaces. This is a fact. But concrete infinite dimensional spaces, e.g. function spaces, are more important than the reader would gather from the book.

Hard functional analysts evaluate and prove a priori inequalities. Their topologies are related to the problems they study, to the inequalities they prove. The solution of a concrete problem is the main emphasis. If this solution involves the consideration of a half dozen topologies on a given space, well it does, but the problem is solved.

The soft functional analyst does not find these proofs elegant. Some proofs may even be "clumsy", the hypotheses being too strong. Of course, the examples to which the "better" proof applies are fairly artificial, but that does not affect the general principle. Elsewhere, a "main theorem" can be proved, its proof involves one single topology or convergence on the space. The other topologies only serve to bridge the gap between the main result and the applications. The hard functional analyst does not appreciate the progress since the main result is only justified by its applications.

Bornology is a chapter of soft functional analysis.

Locally convex space theory is a well established subject. We all know a half dozen or more classes of examples of locally convex spaces. These examples put the flesh on the skeleton when we, or our students, read a textbook on locally convex space theory.

Bornology is not as well established. The reader of a text on bornology may not know how it can be used to help the hard analyst. It is the author's responsibility to lead his reader to the applications. These applications are the final test in judging the value of a soft analytic theory.

In this book, the author places more emphasis on the easy parts of bornology, or in the chapters where functional analysts used bornologies before they were invented than on the chapters where the consideration of bornologies really brings something to functional analysis. A senior functional analyst could prove all theorems to page 55, given the statement, a pencil, and scribbling paper. From page 62 to page 97, the reader will find an account of the classical duality theory of locally convex spaces, with a bornological terminology. Some important theorems are missing.

This leaves 26 pages for results which are not quite trivial, and where bornologies bring something. There are 58 exercises.

The book is readable by and aims at beginning graduate students. It can supplement a good text on locally convex space theory (such as references [1] to [4]). It cannot replace such a text. The reader will know that bornologies exist, he may even see some in the real world. It would be better if he could learn what bornology is good for.

An exercise section is a good place for leading the reader from the central theory, which must remain bornological, to the applications to less soft analysis. It is unfortunate that the 58 exercises are (nearly) all internal. They tend to make the reader a better bornologist rather than a better applier of bornology.

Two chapters of bornology are missing from this book.

Compactologies are not bornologies stricto sensu, but the considerations which lead to bornology also lead to compactology. Schwartz and infra-Schwartz bornological spaces have a unique compactology. Convex compactological spaces with a countable basis of compactology are separated by their dual, they are the duals of Fréchet spaces.

Computations within a bornological space often lead to the introduction of bornological subspaces, i.e. of a vector subspace with a bornology which is stronger than the induced one. This happens when we consider the subspace, the algebra, generated by one or by a countable family of bounded sets. The generated subspace will have a countable basis of bornology. If it happens that it is Schwartz, infra-Schwartz, or if it is compactological for other reasons, the generated subspace is the dual of a Fréchet space and we have a good grip on the situation.

Many applications of bornology can be obtained by these considerations. It is unfortunate that the author chose not to include them.

Next to nine standard references the book's bibliography contains fourteen "references for advanced studies". Unfortunately, ten of these references are unpublished theses, or are papers published in a university preprint series. These papers are difficult to get. By the way, the reader, or the prospective buyer of volumes 331 and 332 of the Springer Lecture Notes might appreciate knowing that these are Proceedings volumes, respectively of a Summer School on Topological Vector Spaces, or of the Lelong seminar.

It is J. Sebastião e Silva, not J. S. e Silva who defined Silva spaces.

The reader must expect a limited number of misprints, and will find some. Exercise 4.E.10 is only true if $'E$ is Hausdorff. A complete, convex bornological space with the property announced in exercise 5.E.5.b does exist; the hint yields a noncomplete convex bornological space. Exercise 5.E.5.e.vi is mistakenly labeled iv and does not make any sense if no relation is postulated between the given bornology and the new one. Just below this exercise, the remark does not make sense, again because no relation is postulated between $\mathfrak{B}$ and the new bornology.

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Transcendental number theory, by Alan Baker, Cambridge Univ. Press, New York, 1975, x + 147 pp., $13.95.

Lectures on transcendental numbers, by Kurt Mahler, Edited and completed by B. Diviš and W. J. LeVeque, Lecture Notes in Math., no. 546, Springer-Verlag, Berlin, Heidelberg, New York, 1976, xxi + 254 pp., $10.20.

Nombres transcendants, by Michel Waldschmidt, Lecture Notes in Math., no. 402, Springer-Verlag, Berlin and New York, 1974, viii + 277 pp., $10.30.

The last dozen years have been a golden age for transcendental number theory. It has scored successes on its own ground, while its methods have triumphed over problems in classical number theory involving exponential sums, class numbers, and Diophantine equations. Few topics in mathematics have such general appeal within the discipline as transcendency. Many of us learned of the circle squaring problem before college, and became acquainted with Cantor's existence proof, Liouville's construction, and even Hermite's proof of the transcendence of e well before the close of our undergraduate life. How can we learn more?

Sophisticated readers may profitably consult the excellent survey articles of N. I. Feldman and A. B. Shidlovskiĭ [9], S. Lang [12], and W. M. Schmidt [17]. I will begin by addressing the beginner who has a solid understanding of complex variables, basic modern algebra, and the bare rudiments of algebraic number theory (the little book of H. Diamond and H. Pollard [8] is more than enough). My first advice is to read the short book of I. Niven [14] for a relaxed overview of the subject. If the reader is impatient, he may take Chapter 1 of Baker for an introduction. Either way he will learn short proofs of the Lindemann-Weierstrass theorem, that if the algebraic numbers $\alpha_1, \ldots, \alpha_n$ are distinct, then

$$\beta_1 e^{\alpha_1} + \cdots + \beta_n e^{\alpha_n} \neq 0$$

for any nonzero algebraic numbers $\beta_1, \ldots, \beta_n$. As special cases of this e and π are transcendental. These proofs are unmotivated; Baker mentions that they stem from the problem of approximating e^x by rational functions of x, and refers the reader to Hermite's original papers. At this point the reader may also find it most enjoyable and enlightening to turn to the appendix of Mahler's book where a thorough discussion of most of the classical proofs for