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LOCALLY POLYNOMIAL ALGEBRAS ARE SYMMETRIC

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Quillen's recent solution [4] of Serre's problem (on projective modules over polynomial rings) is based on the following remarkable theorem.

Let K be a commutative ring, $\max(K)$ its set of maximal ideals, and T an indeterminate.

THEOREM 1 (QUILLEN [4, THEOREM 1]). *Let M be a finitely presented $K[T]$-module and put $M_0 = M/TM$. If $M_\mathfrak{m} \cong M_0[T]_\mathfrak{m}$ for all $\mathfrak{m} \in \max(K)$, then $M \cong M_0[T]$.*

We have developed an axiomatic version of Quillen's arguments, also using ideas of [1], which yields the following results, among others. Detailed proofs will appear elsewhere.

THEOREM 2. *Theorem 1 is valid with the word "module" replaced by "algebra".*

Theorem 1 follows from Theorem 2, applied to the symmetric algebra $S(M)$.

Call a commutative K-algebra A *invertible* if, for some K-algebra B, $A \otimes_K B$ is a polynomial algebra $K[X_1, \ldots, X_n]$. Then A admits an augmentation, $0 \to \overline{A} \to A \to K \to 0$, and the K-module $JA = \overline{A}/\overline{A}^2$ depends, up to isomorphism, only on A. We say A is *stably isomorphic* to a K-algebra B if $A \otimes_K C \cong B \otimes_K C$ for some invertible K-algebra C.

THEOREM 3. *Let A be a finitely presented K-algebra.*

(a) *If $A_\mathfrak{m}$ is a polynomial $K_\mathfrak{m}$-algebra for all $\mathfrak{m} \in \max(K)$ then A is a symmetric algebra $S(P)$ of a projective K-module P.*

(b) *If $A_\mathfrak{m}$ is an invertible $K_\mathfrak{m}$-algebra for all $\mathfrak{m} \in \max(K)$ then A is invertible.*

COROLLARY. *Let A and B be invertible K-algebras. If JA and JB are stably isomorphic, and if $A_\mathfrak{m}$ and $B_\mathfrak{m}$ are stably isomorphic for all $\mathfrak{m} \in \max(K)$, then A and B are stably isomorphic.*

REMARKS. The title of the paper refers to part (a), which solves a problem posed in [2, p. 67], [3], [5, §6], and [6, p. 3]. In geometric language it asserts that every affine space bundle over $\operatorname{spec}(K)$ arises from a vector bundle. Part (b)

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is proved by reducing it to part (a). Part (a) is proved by first constructing an augmentation $A \longrightarrow K$ and then applying the following general result, which has various other applications.

THEOREM 4. *Let A be a finitely presented (not necessarily commutative) K-algebra equipped with an augmentation,* $0 \longrightarrow \bar{A} \longrightarrow A \longrightarrow K \longrightarrow 0$, *and put* $\gamma A = \bigoplus_{n \geqslant 0} \bar{A}^n / \bar{A}^{n+1}$, *the associated graded algebra. If* $A_{\mathfrak{m}} \cong \gamma A_{\mathfrak{m}}$ *(as filtered algebras) for all* $\mathfrak{m} \in \max(K)$, *then* $A \cong \gamma A$.

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