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MAPPING ONTO 2-DIMENSIONAL SPACES

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In [**3**] the author showed that if X is a connected, locally connected, locally compact topological space and f is a 1-1 mapping of X onto E^2, then f is a homeomorphism. In this note, we present some significant extensions of the above result. An additional related result concerning 1-1 mappings is also given.

A topological space (not necessarily Hausdorff) will be said to be *locally compact* if each point has a compact neighborhood. By a *mapping* we will mean a continuous function. A mapping is said to be *open* if images of open sets are open. A mapping is *compact* if inverse images of compact sets are compact. If K is a 2-cell, then Bd K and Int K will denote, respectively, the boundary and interior of K, where K is regarded as a 2-manifold with boundary.

THEOREM 1. *Let Y be a locally connected, locally compact metric space such that for each simple closed curve J in Y there is a 2-cell K in Y with $J = \operatorname{Bd} K$ and $\operatorname{Int} K$ an open set in Y. If X is a connected, locally connected, locally compact topological space and f is a 1-1 mapping of X onto Y, then f is a homeomorphism.*

The proof of this theorem depends upon a theorem of G. T. Whyburn [**4**, Theorem 7, p. 1430] and the above cited result of the author in [**3**]. Details will appear in another paper.

In [**1**] Edwin Duda defines a mapping f to be *reflexive compact* if for each compact set H in the domain space, $f^{-1}(f(H))$ is compact. In [**2**] Duda and Jack W. Smith define a mapping f to be *reflexive open* if for each open set V in the domain, $f^{-1}(f(V))$ is open. The following theorem generalizes results of Duda and Smith in [**1**] and [**2**].

THEOREM 2. *Let X and Y be as in Theorem 1. If f is a reflexive compact (reflexive open) mapping of X onto Y then f is compact (open).*

The proof is obtained by using Theorem 1 and techniques from [**1**] and [**2**].

The following theorem establishes the existence of nontopological 1-1 mappings between certain kinds of spaces. The proof will appear elsewhere.

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THEOREM 3. *Suppose that a metric space* Y *is the union of* n 2-*cells* $K_1, K_2, \ldots, K_n$ *such that for* $1 \leqq i < j \leqq n$, $K_i \cap K_j = \mathrm{Bd}\, K_i = \mathrm{Bd}\, K_j$. *If* $n \geqq 3$, *then there is a connected, locally connected, locally compact metric space which can be mapped onto* Y *by a nontopological* 1-1 *mapping.*

It follows from Theorem 1 that if a metric space Y is the union of two 2-spheres whose intersection is a point, then Y is not the image, under a nontopological 1-1 mapping, of a connected, locally connected, locally compact topological space. In light of this, the following corollary to Theorem 3 is of interest.

COROLLARY. *Suppose that a metric space* Y *is the union of two* 2-*spheres having as their intersection a* 2-*cell. Then there is a connected, locally connected, locally compact metric space which can be mapped onto* Y *by a nontopological* 1-1 *mapping.*

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