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BOOK REVIEWS

Cohomology of Groups, by Edwin Weiss. Academic Press, New York, 1969. x+274 pp. $15.00.

When I first studied cohomology of groups, I disliked it. It seemed to consist of a batch of rather obscure definitions followed by theorems which were, if anything, even more obscure. The proofs generally consisted of formal manipulations which were simple enough individually, but which were heaped together in grotesque combinations, like a surrealist collage. And none of this ever seemed to be leading anywhere. There were virtually never any examples, and along the way there were almost no applications to anything else in mathematics. The only reason for persevering was the belief that eventually all of this would lead to something worthwhile—that there would be light at the end of the tunnel. It was not an image which inspired confidence.

The aim of Weiss' book is to provide the reader with all the cohomology theory that he needs to tackle class field theory. Thus he is concerned only with the cohomology of finite groups. Chapters I, II, and IV are devoted to the basic facts about cohomology: its definition, its properties under mappings, and cup products. The cohomology groups $H^n(G, A)$ (for all $n \in \mathbf{Z}$) are defined by means of G-complexes, rather than by some more abstract or axiomatic approach. Chapter III discusses an assortment of topics, including dimension shifting, the inflation-restriction sequence, cohomological equivalence, and the connections between the cohomology of a group and that of a Sylow subgroup. Chapter V is concerned with group extensions; it concludes with the group-theoretic principal ideal theorem. The last chapter begins with a discussion of formations and proceeds to prove the main theorems of abstract class field theory. Three of these chapters also include problems for the reader. In short, the book in its first four chapters provides the cohomological prerequisites for the Artin-Tate notes, and the last two chapters (which are similar in content to Chapters 13 and 14 of Artin-Tate) give a good introduction to the first part of those notes.

How successful is it? To begin with the easy part of the answer, the book is mathematically sound and is clearly written. Its approach is more "computational" than the other books on the cohomology of groups which I know; that feature makes it a good book to have around somewhere. The pace is more leisurely than in the other books, and I found it to be a book which lent itself, more than most books on the cohomology of groups, to being absorbed a few pages at a time. But despite the book's good points, I was unhappy with it. I think the reason is that it brought back forcibly to me how unpleasant a subject group cohomology really is. There are pages and pages of dreary calculations, and very few interesting patches as rewards for the effort expended.

This objection is at least in part directed toward the subject rather than the book. Perhaps a better way to judge Weiss' book, then, is to compare it with the competition. With this in mind, I looked at three other texts with comparable discussions of the cohomology of groups: Serre's *Corps Locaux* (Hermann, 1962), the Atiyah-Wall notes in *Algebraic Number Theory* (Cassels and Fröhlich, editors; Thompson, 1967), and Lang's *Rapport sur la cohomologie des groupes* (Benjamin, 1966). The first two works discuss a good deal more than just cohomology of groups; Lang's book is billed by the publishers as an introduction to the Artin-Tate notes, but it also contains further results in group cohomology (including an otherwise unpublished paper of Tate's). In what follows, I am considering only those parts of these books which are comparable to Weiss'.

Lang's texts often have the property of making every other book in the field seem like a pedagogical advance. The present case is a prime example. I suppose, that it is possible to learn about the cohomology of groups from the *Rapport*, just as it is possible for men to climb Mount Everest or to run the four-minute mile. Whether one tries it will depend in part on whether he regards it as a challenge to the human spirit. I would prefer to read Weiss.

The Atiyah-Wall treatment is extremely concisely written. The approach is somewhat different from that of Weiss; the cohomology functor is defined by a few axioms (short exact sequences turn into long ones, $H^q(G, A)=0$ for $q\geqq 1$ if A is co-induced, etc.) and then G-complexes are used to prove existence. Furthermore, homology and cohomology are separately defined and are later spliced together. I liked their treatment, in part because it made the subject go by faster. But I suspect that many people will prefer the greater detail of Weiss' account. Serre's treatment is more leisurely than that of Atiyah-Wall, but in broad outline it is similar. I found it to be the most enjoyable of the three, partly because it gave me a chance to brush up on my French. Besides that, Serre has a number of interesting asides (some put into appendices) on further results. But people who find French an obstacle will (obviously) prefer either of the other two books. Weiss' book, therefore, stands up creditably under this comparison.

There is one other consideration to be brought up: cost. My copy of Cassels-Fröhlich cost $16.00 (before discounts); *Corps Locaux* lists for 36 F (roughly $6.60, plus shipping costs, etc.). That means that for a dollar more than the cost of Weiss' book you can have all of class field theory thrown in, and that anyone willing to read French can get more mathematics than in Weiss for about half the price. Anyone who buys Weiss will need some justification other than cost-effectiveness.

LAWRENCE CORWIN

Studies in Geometry, by Leonard M. Blumenthal and Karl Menger. W. H. Freeman and Co., San Francisco, 1970. 512 pp.

The trend towards abstraction and generalization in modern mathematics at times seems to be at odds with the need for an intuitive insight into which concepts will prove fruitful and significant to the further development of the body of mathematics. In *Studies in Geometry* we find geometrical intuition at its best treated from an abstract point of view, which allows for significant generalization—notably the development of metric and topological properties of Boolean algebras and the development of projective geometry of any finite dimension without the introduction of new definitions for elements of each dimension.

Studies in Geometry is neither a systematic development of one particular geometry nor is it a survey of the main topics of geometry. Rather it presents essentially three theories—the theory of distance geometry, culminating with metric characterizations of Banach and Euclidean spaces, the theory of projective and related geometries, and the theory of curves, considered both from a lattice-theoretical viewpoint and from the traditional three-dimensional Euclidean point of view. These three topics seem quite unrelated and one might expect to find little continuity in the plan of the book. This, however, is not the case. For these theories are all developed from the common concepts of set and lattice, which provide an underlying unifying element to their study.

While both authors use the same algebraic structures in their studies, it is apparent that there is a definite difference between their basic ideas of what geometry is. Blumenthal takes a somewhat Kleinian point of view—a geometry is a system $\{S, E\}$ where S is a set and E an equivalence relation defined in the set of all subsets, called figures, of S. Fundamental, therefore, to the study of a geometry is a study of the properties shared by all the figures of an equivalence class. To Menger, however, a geometry cannot be divorced