translated and tried to palm off on the public. It seems unlikely that there will be many who will want to read the resulting volume when most of the same material and more is covered so well in J. Milnor's *Lecture notes on Morse theory.*

RICHARD SACKSTEDER

*Singularities of smooth maps* by James Eells, Jr. Gordon and Breach, New York, 1967. 104 pp. \$5.50; paper \$3.00.

This book is a reprinting of a set of lecture notes for the first half of a course given by James Eells in about 1960. The notes have essentially not been reworked and so maintain—as the author mentions in his preface—an "incomplete and definitely temporary character." The book is quite elementary and consists of three chapters. The first two require nothing more than calculus of several variables while the third uses a little algebraic topology.

The first chapter is a quick review of calculus of several variables leading to the definitions surrounding the notion of a finite dimension manifold (including the tangent bundle). The existence of the globalizing tool, the partition of unity, is proved completely.

The second chapter begins the study of singularities of smooth maps of compact manifolds with Whitney's theorems giving the open-density of imbeddings (immersons) among all $C^k$ maps of an $n$-manifold into $R^{2n+1}$ ($R^{2n}$). The weak form of the $C^\infty$ Sard-Dubovitsky-Morse theorem—that a $C^\infty$ map takes its critical set into a meager subset of the target—is proved and applied to show that most immersions of compact $n$-manifolds in $R^{2n}$ have only clean self-intersections—thus only isolated double points and no triple points.

The two simplest cases of maps which typically display some singular behavior are now discussed—maps of $n$-manifolds into $R^{2n-1}$ and into $R$. For each, the usual notion of nondegenerate singularity is defined in terms of local coordinates, local normal forms are given and the *generic* maps, those having only nondegenerate singularities, are shown to fill an open dense subset of the $C^k$-maps.

The general question of the existence of an open dense set of "generic" maps in $C^k(X, Y)$ is posed and some of the formalism of jets is introduced. Unfortunately the author does not develop quite enough of it to state the general transversality theorem of Thom, and so cannot even suggest that everything in the chapter except the Sard-type theorem is a corollary of this result.

The main object of the last chapter is the proof of the Morse-Pitcher inequalities which relate the critical points of a generic real-valued function on a compact manifold with the betti-numbers and torsion coefficients. Two applications of these inequalities are made. The first is to the function $f_A: X \to R$ where $A \in R^{n+1}$, $X$ is a compact $n$-manifold imbedded in $R^{n+1}$, and $f_A(x) = |x - A|$. The second applies the analysis of this distance function to prove that a Stein manifold has no integral homology past the middle dimension. This in turn yields the Lefschetz theorem relating the cohomology of a compact complex subvariety of complex projective space with that of its intersection with a complex hypersurface.

HAROLD I. LEVINE

*Foundations of constructive analysis* by Errett Bishop. McGraw-Hill, New York, 1967. xiii+370 pp. $12.00.

> For, compared with the immense expanse of modern mathematics, what would the wretched remnants mean, the few isolated results, incomplete and unrelated, that the intuitionists have obtained... (Hilbert, 1927)[1]

> While in a few cases one has succeeded in replacing certain intuitionistically void proofs by constructive ones, for the majority this has not been achieved nor is there a prospect of achieving it... (Fraenkel & Bar-Hillel, 1958)[2]

> L'école intuitionniste, dont le souvenir n'est sans doute destiné a subsister qu'à titre de curiosité historique... (Bourbaki, 1960)[3]

> Almost every conceivable type of resistance has been offered to a straightforward realistic treatment of mathematics. . . . It is time to make the attempt. (Bishop, 1967)[4]

Bishop's attempt has succeeded. Within a constructive framework intimately related to Brouwer's intuitionism—though with important differences—he has developed a substantial portion of abstract analysis, thereby arithmetizing it; and, moreover, he has done it in such a way as to establish the general feasibility and desirability of his constructivist program. He is not joking when he suggests that classical mathematics, as presently practiced, will probably cease to exist as an independent discipline once the implications and advantages of the constructivist program are realized. After more than two

[1] *The foundations of mathematics.* All quotes from Hilbert, Kolmogorov, Skolem, and Weyl are from the translations in J. van Heijenoort's *From Frege to Gödel, a source book in mathematical logic*, 1879–1931, Harvard Univ. Press, Cambridge, Mass., 1967.

[2] *Foundations of set theory*, North-Holland, Amsterdam.

[3] *Éléments d'histoire des mathématiques*, Hermann, Paris.

[4] From the first chapter of the book under review.