The book closes with an informal discussion of source material and references and a bibliography containing 52 entries. Approximately one-third of these are vital to the present undertaking, the others being either marginal in value or representing a caprice of the author. No errors major or minor were detected, a fact which is only one indication of the very careful way in which the booklet was prepared. Most pages exhibit a zest for play as well as work which is refreshing. Indeed, at times one may have a vague apprehension that the author is preparing a prank or baiting a trap; however it seldom turns out to be more than a friendly tweak given with a wink. Such an intimate style, in the present desert of works written with an unexceptionable scientific detachment, is warmly welcome. It is certainly a facet to the general success enjoyed by Halmos' previous books.

E. R. LORCH

A theory of formal deducibility. By H. B. Curry. (Notre Dame Mathematical Lectures, no. 6.) University of Notre Dame, 1950. 9+126 pp.

The monograph contains a detailed account of the predicate calculus as presented by Gentzen (Math. Zeit. vol. 39 (1934) pp. 176–210, 405–431) in a sequence calculus in which the rules of inference follow in a natural way from the intended meanings of the logical connectives. However, Curry's treatment differs in several major respects from earlier ones. The predicate calculus is approached as an episystem over a basic formal system of elementary propositions. Various portions of the classical and intuitionistic systems are studied separately. There is a discussion of alternative concepts of negation. And a final chapter suggests a new approach to modal logic.

A formal system is specified by a primitive frame which defines inductively terms, elementary propositions, and theorems. (The author develops here notions presented in a paper in Bull. Amer. Math. Soc. vol. 47 (1941) pp. 221–241.) In Curry's usage, the Hilbertian formal systems have as elementary propositions, propositions of the form "A is a provable formula," the formulas being terms in Curry's terminology. In studying a formal system it is customary and convenient not to limit attention only to elementary propositions, but to consider in addition compound propositions such as "Not for all formulas A, is A provable." These compound propositions are formed from the elementary ones by use of the logical connectives. Curry speaks of this broader system as an episystem over a formal system. (An episystem is not to be confused with a metasystem over a calculus.) The episystem may itself be formalized of course. Curry considers the predicate calculus as a formal system over an unspecified elementary formal system. In giving a detailed description of the notion of formal system, the author introduces a tentative theory of grammatics, relating formal systems to the actual language of communication in which they are imbedded, in the present case mathematical English.

In studying the predicate calculus, first the systems for the finite positive connectives ("and," "or," and "implies") are discussed. Systems corresponding to both Gentzen's L- and N-calculi are presented as well as axiom systems for the propositional algebras of these segments of the classical and intuitionistic systems. Gentzen's Hauptsatz receives a careful treatment, which differs in some details from the original one.

The chapter on quantification involves extensive detail designed to meet the well known difficulties in formulating this portion of logic. A notation is introduced to specify, for each deduction, the range of variables for which it is valid. It is shown that any theorem can be proved using only those free variables which appear in it.

A further chapter discusses three basic negation concepts, invalidity, refutability, and absurdity. A proposition A is said to be invalid in a system S just in case a proof for it does not exist in S. Such a negation has the disadvantage that, unlike the other logical connectives, it is not extensible, since A may be valid in some extension of S. A proposition is said to be refutable in S in case it implies R, where R is a proposition of a class defined to be directly refutable. A formula is said to be absurd in case it implies every formula of S. Curry considers four formal systems. The minimal, the intuitionistic, a system LD, and the classical one. Negation for all of the systems is of the refutability type. That for the intuitionistic one is particularized to absurdity. For the system LD the concept is not of the absurdity type, but it does satisfy a principle of excluded middle. Classical negation embodies all these features. This list of negation concepts is, of course, not exhaustive. Negation of yet another sort is described in the reviewer's note (Journal of Symbolic Logic vol. 14 (1949) pp. 16–26; Fitch has also used this type of negation in a system described in the same volume, pp. 209–218).

In a final chapter devoted to modal logic, Curry suggests that the proposition $\#A$ (A is necessary), where A is a proposition of a formal system S, be interpreted to mean that A is derivable by the rules of some specified subsystem $S\#$ of the system S. Rules for the operation $\#$ consistent with the interpretation are presented. The adjunction of these rules to the classical calculus results in a system equivalent to Lewis' system S4. A related notion for possibility is also suggested. Consider a family of systems; A is said to be possible in a system S_1 of the family in case it is provable in some stronger system of the family.

In making available in monograph form the important ideas of Gentzen, a definite need in the literature is filled. The work throughout gives meticulous attention to precise formulations and to detail in proof. It is almost entirely self-contained, and in spite of the great detail of the treatment should be of interest to the general mathematical public as well as to the specialist in foundations.

D. NELSON

Lezioni di analisi. By F. Severi and G. Scorza Dragoni. Bologna, Cesare Zuffi, 1951. Vol. 3, 6+255 pp. 2700 lire.

Volume 1 was written by the senior author alone and published by Zanichelli in 1933, it was reviewed by T. H. Hildebrandt in vol. 41, January 1935, of this Bulletin. A second edition appeared in 1938 and a third edition is in course of publication. Part 1 of volume 2 with G. Scorza Dragoni as coauthor appeared in 1943, but does not seem to have been reviewed in the Bulletin.

The present volume 3 contains a discussion of ordinary differential equations (existence and uniqueness theorem for single equations and for systems and problems in the large, mainly boundary value problems) followed by a chapter on trigonometric series and a brief discussion of the differential geometry of surfaces.

The main text gives a clear, fluent and rigorous account of the basic facts based on the Riemann integral. In each chapter, except the first, this exposition is supplemented by a section in smaller print labelled Complements and Exercises. Of exercises in our sense there are comparatively few, but the complements serve to open vistas to a multitude of more advanced questions. There is a wealth of historical material here and in the main text, over two hundred authors are quoted with dates though normally without textual references. The complement to the second chapter is particularly rich; it occupies a fourth of the book and the subject matter is kaleidoscopic. There is internal unity, however, and the brief sketches of the many topics are skilfully done. This section contains some interesting material on analytic functions of several variables and various extensions of Cauchy's theorem and Cauchy's integral to such functions. The Lebesgue integral is used in the complements. The discussion in the large in Chapter 3 takes the fixed point theorems of Brouwer and