BOOK REVIEWS

Activity analysis of production and allocation. Ed. by T. C. Koopmans. (Cowles Commission Monograph, no. 13.) New York, Wiley, 1951. $4.50.

This volume consists of twenty-five papers presented at a conference in Chicago in June 1949. The conference and the research leading to the papers presented were supported by The Rand Corporation. The theme of the conference was the application of mathematics to economics, and the papers are grouped by the editor under four headings: theory of programming and allocation, applications of allocation models, mathematical properties of convex sets, and problems of computation. Of these the third is partly expository, the fourth deals with problems of computing optimal points and situations some of which are discussed in the first part, and the second deals with some special applications to economic situations. (For recent results on the computing problem the reader is also referred to Brown and v. Neumann, pp. 73–80 of the volume *Contributions to the theory of games*, edited by H. W. Kuhn and A. W. Tucker.)

Several of the contributions to the first part deal with linear technological systems. A linear technological system is one such that the quantity of output of each commodity produced is multiplied by k if each input (of commodities and labor) is multiplied by k. The totality of points which correspond to possible outputs constitute a convex set in a Euclidean space. There is an obvious partial ordering of the points according to their "efficiency." Koopmans and Georgescu-Roegen discuss these efficient points, which obviously have to be on the surface of the convex body and must have suitably inclined support planes. These ideas have been current in mathematical statistics since Wald proved that the totality of Bayes solutions form a complete class. Basing himself on a paper by Wald and Wolfowitz (*Proceedings of the Second Berkeley Symposium on Mathematical Statistics and Probability*, University of California Press, pp. 149–158), Gerard Debreu has since discussed these ideas in an economic context in a paper in Econometrica (July, 1951, pp. 273–292).

Three contributions (by Samuelson, Koopmans, and Arrow) are devoted to proving a theorem enunciated and proved by Samuelson. Let X_i, $i=1, \cdots, n$, be the total production of the ith output, which is divided into a final output C_i and inputs X_{ji}, $j=1, \cdots, n$, which are used to produce the jth input. Thus

$$X_i = C_i + \sum_{j=1}^{n} X_{ji} \qquad (i = 1, \cdots, n).$$

Labor, the $(n+1)$st good, is thought of as the sole nonproduced good, and its given total X_{n+1} is allocated among the different industries so that

$$X_{n+1} = \sum_{j=1}^{n} X_{j,(n+1)}.$$

Let each good be subject to a production function F_i which is homogeneous of the first order. Equilibrium requires that any C, say C_1, be at a maximum subject to fixed values of X_{n+1} and the other C's. This means that

$$C_1 = F_1(X_{11}, X_{12}, \cdots, X_{1,(n+1)}) - \sum_{j=1}^{n} X_{j1}$$

is to be a maximum subject to

$$C_i = F_i(X_{i1}, X_{i2}, \cdots, X_{i,(n+1)}) - \sum_{j=1}^{n} X_{ji} \qquad (i = 2, \cdots, n),$$

$$X_{n+1} = \sum_{j=1}^{n} X_{j,(n+1)}.$$

Samuelson's theorem asserts that the maximizing values $\{X_{ij}\}$ are such that the $\{X_{ij}/X_i\}$ are independent of the fixed values $C_2, \cdots, C_n, X_{n+1}$.

Linear programming of an economy is a matter of allocating the available resources so as to maximize the utility of the economy. Mathematically the problem is one of maximizing a linear function of several variables constrained by linear inequalities. Dantzig proves that this problem is equivalent to the problem of solving a zero-sum two-person game. More general results are given by Gale, Kuhn, and Tucker, who prove general duality and existence theorems. (For nonlinear programming see Kuhn and Tucker, *Proceedings of the Second Berkeley Symposium on Probability and Statistics*, pp. 481–492.)

An introduction to the volume by Koopmans gives descriptions of the papers and their interrelations.

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Classical mechanics. By H. Goldstein. Cambridge, Addison-Wesley, 1951. 12+399 pp.

This book gives an advanced course in classical mechanics, with