$1 \leqq r < m_1$, in such a way that there are $m-r$ linearly independent omniconjugate directions at P with respect to the V_{m+r}. (The local R_{m+r} of the V_{m+r} at P is contained within the local R_{m+m_1}.) If there is a second normal space of m_2 dimensions, then $m_2 \leqq s(s+1)(s+2)/6+t(t+1)/2$, where s and t are integers determined by $s(s+1)/2 \leqq m_1 \leqq (s+1)(s+2)/2$, $t=m_1-s(s+1)/2$. Similar statements can be made about the vanishing of the third and other higher normal spaces. (Received March 19, 1946.)

192. Y. C. Wong: *Contributions to the theory of surfaces in a 4-space of constant curvature.*

A Riemannian 4-space of constant curvature and a surface in it are denoted by S_4 and V_2, respectively. The method of studying V_2 in S_4 in this paper is invariant and is similar to those of G. Ricci (*Lezioni sulla teoria della superficie*, Verona-Padova, 1898) and W. Graustein (Bull. Amer. Math. Soc. vol. 36 (1930)) for their studies of surfaces in a Euclidean 3-space. In essence, the method consists of setting up a suitable system of invariant fundamental equations for a V_2 in S_4, and expressing the required imbedding conditions of V_2 in S_4 in terms of the intrinsic properties of V_2. Curvature properties, especially those about the curvature conic, of a general V_2 in S_4 are first discussed. Then the V_2's whose curvature conic is of certain particular nature are studied. These include the minimal V_2, with the R-surface of K. Kommerell (Math. Ann. vol. 60 (1905)) as a special case, ruled V_2, and V_2 with an orthogonal net of Voss. The paper concludes with a complete determination of those V_2's in S_4 whose first fundamental form and one of whose second fundamental forms are respectively identical with the first and second fundamental forms of a surface in a 3-space of constant curvature. (Received March 11, 1946.)

193. Y. C. Wong: *Scale hypersurfaces for conformal-Euclidean space.*

This paper contains generalizations to n-space of some of the results obtained recently by E. Kasner and J. DeCicco (Amer. J. Math. vol. 67 (1945)) for the scale curves in conformal maps of a surface on a plane. The fundamental form $ds^2=e^{2\sigma}(dx_1^2+\cdots+dx_n^2)$, with $\sigma=\sigma(x_1, \cdots, x_n)$, represents a conformal-Euclidean n-space C_n, conformally mappable on the Euclidean n-space R_n with rectangular Cartesian coordinates $x_1, \cdots, x_n$. The hypersurfaces $\sigma=$constant in R_n are the scale hypersurfaces in the mapping of C_n on R_n, and any simple family of hypersurfaces in R_n is called quasi-isothermal if it represents the scale hypersurfaces of a conformal mapping of some C_n on R_n such that the scalar curvature of C_n is constant over each of the scale hypersurfaces. A few theorems are proved concerning the cases when a family of quasi-isothermal hypersurfaces is a family of (a) ∞^1 hyperplanes, (b) ∞^1 generalized cylinders of rotation. This subject is closely connected with the subject of the isoparametric hypersurfaces of T. Levi-Civita and B. Segre (Rendiconti della Reale Accademia Nazionale dei Lincei (6) vol. 26 (1937), vol. 27 (1938)) and incidentally connected with that of the subprojective Riemannian space of B. Kagan and H. Schapiro (*Abhandlung des Seminars für Vektor- und Tensoranalysis*, vol. 1, 1933). (Received March 11, 1946.)

Logic and Foundations

194. Ira Rosenbaum: *Hegel, mathematical logic, and the foundations of mathematics.*

Hegel, like Boole, DeMorgan, Pierce, Frege, Peano, Russell, Whitehead and others, sought a new logic more adequate than the traditional one to modern science and practice. Prior to Frege or Russell, Hegel possessed logistic intent which was determining factor in development of his logic. He was familiar with historic roots of modern symbolic logic—with work of Lully, Bruno, Leibniz, Ploucquet, Euler, and Bardili—rejecting their primitive efforts as inadequate. He knew the Sancho Panza dilemma, recognized by Church as closely related to Russell's paradox, and even proposes solution resembling theory of types. His denial of so-called "laws of thought" antedates denial of law $A \cdot A = A^2$ in Boole's logical algebra, of parallel-axiom in non-Euclidean geometry, of commutative law in quaternion theory. When mathematicians and mathematical philosophers were seeking to avoid the infinite, Hegel restored it, with new interpretation, to a place of central importance in foundations of analysis. He speaks of infinite as involving equality of whole and part, distinguishes "bad" infinite (Cantor's variable finite) from "good" infinite (recognized as reflexive) and objects to the phrase "and so on to infinity," shown eliminable by Frege. Once influential, he was known to Boole, DeMorgan, Bolzano, Pierce, G. Cantor, and Russell, while Frege refers to Fischer's Hegelian logic. (Received March 23, 1946.)

Statistics and Probability

195. Will Feller: *A limit theorem for random variables with infinite moments.*

Let $\{X_k\}$ be an arbitrary sequence of mutually independent random variables with the same distribution function $V(x)$. It is assumed that some moment of order less than two is infinite; the first moment may be infinite, but if it is finite it should be normed to zero. Let $S_n = X_1 + \cdots + X_n$ and let $\{a_n\}$ be a monotonic positive numerical sequence. It is shown that the probability that the inequality $|S_n| > a_n$ takes place for infinitely many n is the same as the probability that $|X_n| > a_n$ for infinitely many n; it is one or zero according as the series $\sum \{V(-a_n) + 1 - V(a_n)\}$ diverges or converges. (Received March 21, 1946.)

196. Will Feller: *The law of the iterated logarithm for identically distributed random variables.*

Let $\{X_n\}$ be a sequence of mutually independent random variables with the same distribution function $V(x)$ with vanishing first moment and unit variance. Suppose that (*) $\int_{|t|>x} t^2 dV(t) = O((\log \log x)^{-1})$, and let $\{\phi_n\}$ be an arbitrary monotonic sequence, $\phi_n > 0$. The probability that the inequality $X_1 + \cdots + X_n > n^{1/2}\phi_n$ will be satisfied for infinitely many n is shown to be zero or one according as the series $\sum \phi_n n^{-1} \exp(-\phi_n^2/2)$ converges or diverges. The condition (*) is in a certain sense the best possible. If it is not satisfied, the above exact analogue to the strict law of the iterated logarithm does not hold, but slightly more complicated necessary and sufficient conditions are given in the paper. (Received March 21, 1946.)

197. Casper Goffman: *Measures of fluctuation of a variable mean.* Preliminary report.

Suppose a finite order of random variables $x_1, \cdots, x_n$ is given, all normally distributed, with the same known standard deviation, but with unknown means $a_1, \cdots, a_n$, not necessarily alike. A measure of fluctuation of the means is defined as a function $f(a_1, \cdots, a_n)$ such that (1) for every real h, $f(a_1+h, \cdots, a_n+h) = f(a_1, \cdots, a_n)$, and (2) for every real c, $f(ca_1, \cdots, ca_n) = c^2 f(a_1, \cdots, a_n)$. It is