Vorlesungen über Zahlen- und Funktionenlehre. Erster Band, erste Abteilung. By ALFRED PRINGSHEIM. Leipzig, B. G. Teubner, 1916.

THIS is the first part of the first volume of an introductory treatise on the theory of functions of real and complex variables. The author proposes to include in the first volume, in addition to a detailed logical treatment of the rational numbers, a complete exposition of the material treated by him in his two articles in the Encyklopädie: "Irrationalzahlen und Konvergenz unendlicher Prozesse" and "Unendliche Prozesse mit komplexen Termen," the only part omitted being the theory of infinite determinants. The part of the first volume under review concerns itself with the following principal topics: the system of real numbers, convergent sequences of such numbers, limits, orders of infinity and of infinitesimals, and double sequences of real numbers.

The book is intended primarily for students who are beginning the study of higher mathematics, and the theory is built up throughout from first principles. In fact the author declares in his preface that no previous mathematical knowledge is in general necessary for the comprehension of the first volume. In spite of the elementary character of the exposition, rigor of proof is nowhere sacrificed and the reader is made acquainted with the modern viewpoints in the domain of analysis that is treated.

CHARLES N. MOORE.

I Numeri reali e l'Equazione esponenziale $a^x = b$ *per le Scuole Medie Superiori.* By Dr. GAETANO FAZZARI. Palermo, Libreria Scientifica D. Capozzi, 1918. 75 pp. Lire 1.80.

THIS little book, as its title indicates, is devoted mainly to the theory of real numbers. It has no scientific pretense, as the author states in the short preface, but contains an exposé of the theory as Dr. Fazzari has presented it for several years to his students at the R. Liceo Umberto I in Palermo.

The book is divided into two parts. In part I is found the theory of real numbers, the development being essentially that of Dedekind. It commences with the definition of a class of rational numbers; i. e., a series of rational numbers satisfying a given condition. Of two classes A and A', A is said to be inferior to A', and A' superior to A, if each element of A is less than every element of A'.

There follows the notion of contiguity. Two classes are contiguous if one is inferior to the other and if it is possible to find an element of one and an element of the other such that their difference is less than any given rational number, not zero, arbitrarily small. This definition is indicated symbolically by writing (A, A') with the conditions $a' > a$ and $a_n' - a_m < \epsilon$.

A number α is the limit of two contiguous classes (A, A') if $a \leqq \alpha \leqq a'$, and this is denoted by writing $\alpha = (A, A')$.

There follows the existence theorem: There exist contiguous classes of rational numbers which do not have a rational limit. The proof is essentially that of Dedekind. The converse is a very simple statement of the Dedekind postulate: If two contiguous classes do not have a rational limit, we shall say that there is one number, and but one, called an irrational number, which is their limit.

The class of all rational numbers and all irrational numbers constitute the class of real numbers.

Sections 2 to 9 inclusive of part I deal with the conditions for the equivalence of two real numbers $\alpha = (A, A')$ and $\beta = (B, B')$, and with the arithmetic operations of addition, subtraction, multiplication, division, involution, and evolution as applied to real numbers defined by contiguous classes.

The first two sections of part II deal with the theory of indices, the exponent being any real number. The last section is devoted to the solution of the exponential equation $a^x = b$.

A series of exercises is inserted after each section, dealing with the subject matter of that section, and there are numerous examples throughout the book to illustrate definitions and theorems.

The treatment is remarkably clear, but one would expect, even in an elementary treatise, some reference to the sources from which the material is drawn. No mention is made, however, of Dedekind or of others who have worked in this field.

Students of mathematics in this country would find no especial difficulty in mastering the contents of Dr. Fazzari's book, but would be unappreciative of the necessity or the beauty of the theory, at least until they were quite beyond the elementary mathematics of our secondary schools.

L. WAYLAND DOWLING.