of 4, 6, 9, 10, 11, 12, 13, 14, 18, 20, 22, 24, 30, or 72 sides, respectively, with appendices ranging in number from 0 up to more than 100. The distinct covariants for the two noncongruent systems on 13 elements were only nine in number and much simpler in form. We see that an increase in the number n of elements, which is probably always accompanied by an increase in the number of distinct systems, produces greater complexity in the form and a very rapid increase in the number of distinct covariants connected with the noncongruent systems.

To extend this method of trains to systems on 19 or more elements would be evidently too laborious, if the object is only to classify the different triad systems. Here the analogy of invariants of algebraic forms under linear transformation is instructive; the complete calculation of systems of invariants is always possible, but only desirable when it involves finite time, as in forms of very low order. Beyond that, it is only particular forms with special invariant characters that are of general interest. So here, it is obviously most interesting to give detailed study first to triad systems which have covariant trains ending in polygon-cycles containing the largest possible number of extraneous triads. This recalls Professor E. H. Moore's study of systems whose groups are cyclic and those might probably be found again early in the proposed research.

VASSAR COLLEGE.

A THEOREM ON AREAS.

BY PROFESSOR TSURUICHI HAYASHI.

THE relative area of two given convex ovals in the same plane, swept out by moving the join of two points lying on the peripheries of the two ovals respectively, so that the point of the join dividing it in a given ratio traces out the periphery of the area containing the totality of all the points which divide the joins of two points lying on and within the two ovals respectively, satisfies the relation

$$\sqrt{S} \leqq \sqrt{A} \sim \sqrt{B},$$

independent of the ratio, A, B, S being the areas of the two ovals and their relative area, respectively.

This can be proved by combining the two formulas in Elliott's paper in the *Messenger of Mathematics*, volume 7 (1878), page 151 and in Minkowski's paper in the *Mathematische Annalen*, volume 57 (1903), page 463.

But any similar formula for the relative volume of two convex ovoid bodies cannot be established.

March, 1918.

CONCERNING THE DEFINITION OF A SIMPLE CONTINUOUS ARC.

BY DR. GEORGE H. HALLETT, JR.

(Read before the American Mathematical Society October 26, 1918.)

IN a paper entitled "Curves in non-metrical analysis situs with an application in the calculus of variations," *American Journal of Mathematics*, volume 33 (1911), pages 285–326, N. J. Lennes gives the following definition of a simple continuous arc.*

"A continuous simple arc connecting two points A and B, $A \neq B$, is a bounded, closed, connected set of points $[A]$ containing A and B such that no connected proper subset of $[A]$ contains A and B."

I shall show that the word "bounded" in this definition is superfluous.†

Lennes proves the simpler properties of formal order on an arc without any use of the assumption that it is bounded. He also proves (§§ 4, 8) that "if A_0 is any point of an arc AB, and t_1 any triangle containing A_0 as an interior point, then (in case $A_0 \neq A$) there is a point A_1 on the arc AA_0 and (in case $A_0 \neq B$) a similar point B_1 on the arc BA_0 such that every point of the arc A_1B_1 lies within t_1."

The following theorem also follows readily without use of the assumption that an arc is bounded:

If a point A_0 of an arc AB is a limit point of a set of points $[S]$ of the arc AB, and C is A or (if $A_0 \neq A$) any point of the

* Loc. cit., p. 308.

† Since I wrote this paper it has been pointed out to me by Professor R. L. Moore that a modification of the argument used in the proof of Theorem 49 on p. 159 of his paper "On the foundations of plane analysis situs," *Transactions Amer. Math. Society*, vol. 17 (1916), pp. 131–164, would accomplish the same result.