The Annals of Probability
1988, Vol. 16, No. 1, 429–430

CORRECTION

COMPOUND POISSON APPROXIMATIONS FOR SUMS OF RANDOM VARIABLES

BY RICHARD F. SERFOZO

The Annals of Probability (1986) **14** 1391–1398

The proof of Theorem 1.1 contains the erroneous statement that

$$P\left(\sum_{i=1}^{n} X_i \in B\right) = E\left[F_1' * \cdots * F_n'(B)\right].$$

The following modifications to Theorem 1.1 and Lemma 2.2 will correct this error. To simplify the notation, we let $d(X|\mathscr{F}, Y|\mathscr{G})$ denote the random total-variation distance $d(P(X \in \cdot|\mathscr{F}), P(Y \in \cdot|\mathscr{G}))$.

(a) For the statement of Theorem 1.1, let $\zeta_i = I(X_i \neq 0)$ and redefine F_i as

$$F_i(B) = P(X_i \in B|\mathscr{F}_{i-1}, X_i \neq 0, \zeta_k, k \neq i).$$

Then in the bounds (1.3), (1.4), (1.5), replace each d_i by $\zeta_i d_i$. The error was that the original F_i did not contain the additional conditioning on ζ_k, $k \neq i$, which is needed in the proof.

(b) Replace assertion (iv) of Lemma 2.2 by the following:

(iv) *If $X_1, \dots, X_n$ are adapted to the increasing σ-fields $\{\mathscr{F}_i\}_{i=0}^n$ and $Y_1, \dots, Y_n$ are independent with respective distributions $G_1, \dots, G_n$, then*

$$D \leq E\left[\sum_{i=1}^{n} d(X_i|\mathscr{F}_{i-1}, Y_i)\right]. \tag{2.4}$$

(There is no conditioning on the Y_i.)

(c) Replace the last paragraph of the proof of Lemma 2.2 by the following:

We prove statement (iv) by induction. It is true for $n = 1$ since $d(X_1, Y_1) \leq E[d(X_1|\mathscr{F}_0, Y_1)]$ by Lemma 2.1. Now assume it is true for $n - 1$. Let $X^* = (X_1, \dots, X_{n-1}, Y_n)$, where Y_n is independent of $X_1, \dots, X_{n-1}$. Then by (i) and Lemma 2.1, we have

$$\begin{aligned} D \leq d(X, Y) &\leq d(X, X^*) + d(X^*, Y) \leq E\left[d(X|\mathscr{F}_{n-1}, X^*|\mathscr{F}_{n-1})\right] + d(X^*, Y) \\ &= E\left[d(X_n|\mathscr{F}_{n-1}, Y_n)\right] + d((X_1, \dots, X_{n-1}), (Y_1, \dots, Y_{n-1})), \end{aligned}$$

which equals the right-hand side of (2.4).

(d) In the proof of Theorem 1.1, replace the five lines after expression (3.1) with the following:

Received December 1986; revised May 1987.

We can write $Y = \sum_{i=1}^{n} \zeta_i Y_i$, where $Y_1, \ldots, Y_n$ are independent with the common distribution F and are independent of $\zeta_1, \ldots, \zeta_n$. Let $\mathscr{H}$ denote the σ-field generated by $\zeta_1, \ldots, \zeta_n$. Then applying (2.2) and Lemma (2.2)(iv) with X_i, Y_i, $P(\cdot)$ equal to $\zeta_i X_i$, $\zeta_i Y_i$, $P(\cdot|\mathscr{H})$, respectively, we have

$$\begin{aligned} d\left(\sum_{i=1}^{n} X_i, Y\right) &= d\left(\sum_{i=1}^{n} \zeta_i X_i, \sum_{i=1}^{n} \zeta_i Y_i\right) \\ &\leq E\left[d\left(\sum_{i=1}^{n} \zeta_i X_i|\mathscr{H}, \sum_{i=1}^{n} \zeta_i Y_i|\mathscr{H}\right)\right] \\ &\leq E\left\{\sum_{i=1}^{n} E\left[d(\zeta_i X_i|(\mathscr{F}_{i-1}, \mathscr{H}), \zeta_i Y_i|\mathscr{H})|\mathscr{H}\right]\right\} \\ &= E\left(\sum_{i=1}^{n} \zeta_i d_i\right). \end{aligned} \tag{3.2}$$

To obtain the last inequality, first note that the standard properties of conditional probabilities allow us to write

$$\begin{aligned} &P(\zeta_i X_i \in B|\mathscr{F}_{i-1}, \mathscr{H}) \\ &\quad = P(X_i \in B|\mathscr{F}_{i-1}, \zeta_i = 1, \zeta_k, k \neq i)I(\zeta_i = 1) + I(0 \in B)I(\zeta_i = 0). \end{aligned}$$

Then using this and a similar expression for $P(\zeta_i Y_i \in B|\mathscr{H})$, it follows from the definition of d that

$$d(\zeta_i X_i|(\mathscr{F}_{i-1}, \mathscr{H}), \zeta_i Y_i|\mathscr{H}) = I(\zeta_i = 1)d_i = \zeta_i d_i.$$

School of Industrial
and Systems Engineering
Georgia Institute of Technology
Atlanta, Georgia 30332