that is,

$$(dx_1 dx_2 \cdots dx_k)^{k+1} = J \cdot (dy_1 dy_2 \cdots dy_k)^{k+1}.$$

But $dx_1 dx_2 \cdots dx_k = A \cdot dy_1 dy_2 \cdots dy_k$, so that $J = A^{k+1}$. These formal operations show that in this case the differentials when multiplied in the usual way work like the determinants they signify.

(3) After obtaining that $J = A^{k+1}$ Rasch's functional equation is seen to hold good.

(4) When the constant in Wishart's distribution is evaluated in Rasch's notation using H. Cramér's method (see *Mathematical Methods of Statistics*, Princeton University Press, 1946, pp. 390–393), it will be found that a power of n is missing in the numerator. This is due to the fact that we have not considered the estimate $1/n \, || M_{ij} ||$ but worked with $|| M_{ij} ||$.

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A NOTE ON A TWO SAMPLE TEST[1]

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1. Summary. Mood ([1], p. 394) discusses a test for the hypothesis that two samples come from populations having the same continuous cumulative frequency distributions. It consists of arranging the observations from the two samples in a single group in order of size and then comparing the numbers in the two samples above the median. This technique is extended to using several order statistics from the combined samples, and to the case of several samples. The test is nonparametric and might be a good substitute for the single variable of classification analysis of variance in cases of doubtful normality. The application of the test would be the same as in Mood ([1], p. 398) except that there would be more than two rows in the table.

2. The distribution function. Suppose we have p populations all having the same continuous cumulative distribution function $F(x)$. Let X_{ij} ($i = 1, 2 \cdots, p$; $j = 1, 2 \cdots, n_i$) be the jth observation in a sample of size n_i from the ith population. Let $\sum_{i=1}^{p} n_i = N$.

Arrange these N observations in a single series according to size and rename them $z_1 \leq z_2 \leq \cdots \leq z_N$. We choose $k - 1$ of the z values, for example, $z_{\alpha_1}, z_{\alpha_2}, \cdots, z_{\alpha_{k-1}}$ (the α_i are integers and $1 \leq \alpha_1 < \alpha_2 < \cdots \leq N$). Denote by m_{ij} the number of observations X_{ih} such that $z_{\alpha_{j-1}} < X_{ih} \leq z_{\alpha_j}$ for $j = 2, 3, \cdots, k - 1$, by m_{i0} the number of $X_{ih} \leq z_{\alpha_1}$, and by m_{ik} the number of $X_{ih} > z_{\alpha_{k-1}}$. These can be illustrated by the following table.

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1st sample	2nd sample	$\cdots$	pth sample	
m_{1k}	m_{2k}	$\cdots$	m_{pk}	$\sum_{i=1}^{p} m_{ik} = N - \alpha_{k-1}$
$\vdots$	$\vdots$		$\vdots$	$\vdots$
m_{13}	m_{23}	$\cdots$	m_{p3}	$\sum_{i=1}^{p} m_{i3} = \alpha_3 - \alpha_2$
m_{12}	m_{22}	$\cdots$	m_{p2}	$\sum_{i=1}^{p} m_{i2} = \alpha_2 - \alpha_1$
m_{11}	m_{21}	$\cdots$	m_{p1}	$\sum_{i=1}^{p} m_{i1} = \alpha_1$
$\sum_{j=1}^{k} m_{1j} = n_1$	$\sum_{j=1}^{k} m_{2j} = n_2$	$\cdots$	$\sum_{j=1}^{k} m_{pj} = n_p$	

The joint distribution of the m_{ij} and z_i can be written as

$$\frac{\prod_{i=1}^{p} n_i!}{\prod_{i=1}^{p}\prod_{j=1}^{k} m_{ij}!} F(z_1)^{\alpha_1-1}\, dF(z_1)[F(z_2) - F(z_1)]^{\alpha_2-\alpha_1-1}\, dF(z_2) \cdots$$

$$\cdot [F(z_{k-1}) - F(z_{k-2})]^{\alpha_{k-1}-\alpha_{k-2}-1} [1 - F(z_{k-1})]^{N-\alpha_k}\, dF(z_{k-1})$$

$$\cdot \Sigma m_{\beta_1 1} m_{\beta_2 2} m_{\beta_3 3} \cdots m_{\beta_{k-1} k-1},$$

where the sum runs over all possible sets of values of $\beta_i = 1, 2, 3, \cdots p$. It is easy to show that this sum is equal to the product

$$\alpha_1(\alpha_2 - \alpha_1)(\alpha_3 - \alpha_2) \cdots (\alpha_{k-1} - \alpha_{k-2}).$$

Now the joint distribution of the m_{ij} is obtained by integrating the z's over their entire range $z_i \leq z_{i+1}$. This is a type of Dirichlet integral and we get

$$f(m_{11} \cdots m_{pk}) = \frac{\prod_{i=1}^{p} n_i! \prod_{j=1}^{k} (\alpha_j - \alpha_{j-1})!}{N! \prod_{i=1}^{p}\prod_{j=1}^{k} m_{ij}!},$$

where $\alpha_k = N$, $\alpha_0 = 0$. This is the distribution of cell frequencies in a p by k contingency table with all totals fixed when there is independence (see [1], p. 278) and thus, for large values of m_{ij} at least, the usual chi-square test can be used.

REFERENCE

[1] A. M. Mood, *Introduction to the Theory of Statistics*, McGraw-Hill Book Co., 1950.

AN OMISSION IN NORTON'S LIST OF 7 × 7 SQUARES

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1. In a previous paper the value 16,942,080 for the number of reduced 7 × 7 squares was obtained by the author by an exhaustive method, subject to a strict control ([4], Section 20). This number exceeds Norton's ([2], Table on p. 290) by 14,112. An attempt was made in Section 21 of [4] to show that this discrepancy in the total number does not affect Norton's conjecture ([2], p. 291) that the 146 species represent the whole of the universe of 7 × 7 Latin squares. However, R. A. Fisher has informed the author that the discrepancy cannot be explained away in this manner. It has therefore to be attributed to a gap in Norton's list.

2. Now, a 147th species containing 14,112 squares can arise only from an automorph type through an operator of the order 5^k. It is easy to construct a matrix Q corresponding to such an operator as, for example, $T = (34567)^3$. Here the cycle (34567) signifies a permutation [1] of columns, a permutation of rows and a substitution of elements.

The first two rows of Q are respectively identical with the first two columns and define the substitution (12) (34567). In the remaining 5 × 5 squares, it is necessary that the elements of the broken diagonals follow in the natural cyclic order, except the numbers 1 and 2, which each form a broken diagonal.

The square is given below:

$$Q = \begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 2 & 1 & 4 & 5 & 6 & 7 & 3 \\ 3 & 4 & 5 & 7 & 1 & 2 & 6 \\ 4 & 5 & 7 & 6 & 3 & 1 & 2 \\ 5 & 6 & 2 & 3 & 7 & 4 & 1 \\ 6 & 7 & 1 & 2 & 4 & 3 & 5 \\ 7 & 3 & 6 & 1 & 2 & 5 & 4. \end{matrix}$$

3. On replacing each row of Q by the conjugate permutation and rotating the figure through an angle of 180° about the diagonal, we obtain the square