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ABSTRACTS OF PAPERS

(Abstracts of papers to be presented at the Annual meeting, New York, New York, August 19–22, 1969. Additional abstracts will appear in future issues.)

10. Modification of Levene's comparison of variances to remove ρ. PETER NEMENYI, Virginia State College.

To test for inequality of variances, Levene t-tests or ANOVA-tests samples of absolute deviations $|x_1 - \bar{x}|, |x_2 - \bar{x}|, \cdots, |x_n - \bar{x}|$, or transforms of these. [Levene, H. (1960). Robust tests for equality of variances. *Contributions to Probability and Statistics* (I. Olkin *et al* eds.) 278–292.] The robust quality of t is relied on to neutralize the correlations $(-1/(n-1))$ between n deviations $(x_j - \bar{x})$, but does not quite. To get rid of the correlations, it is proposed to use $n - 1$ orthogonal contrasts, for example $x_j + n^{-\frac{1}{2}}x_1 - (1 + n^{-\frac{1}{2}})\bar{x}_{-1}$, $(j = 2, \cdots, n)$, instead of the n deviations. The samples of contrasts may be compared by t, F or nonparametric tests, and confidence intervals can be obtained. (Received 14 April 1969.)

11. Unbiased coin tossing with a biased coin. WASSILY HOEFFDING and GORDON SIMONS, University of North Carolina.

Procedures are exhibited and analyzed for converting a sequence $X_1, X_2, \cdots$ of iid Bernoulli variables with mean p into a (sequence of iid) Bernoulli variable(s) with mean $\frac{1}{2}$. von Neumann (1951) has suggested the procedure: *Sample* $X_1, X_2, \cdots$ *sequentially in pairs and stop the first time* $2m$ *for which* $X_{2m} \neq X_{2m-1}$. Then $Z = X_{2m}$ is a Bernoulli variable with mean $\frac{1}{2}$. His procedure is a special case within the class of "even procedures" (relatable to tests of Neyman structure) which are investigated by the authors. The best even procedure (in the sense that no other even procedure stop as soon as and sometime sooner than this one) is definable in terms of $S_n = \sum_1^n X_i$: *Sample* $X_1, X_2, \cdots$ *sequentially until time* N *for which the binomial coefficient* $\binom{N}{S_N}$ *is even. Set* $Z = S_{N-1}$ modulo 2. A better (noneven) procedure is found that has an expected sample size which is less than 4% larger than a theoretical lower bound at each value of $p \in (0, 1)$. (Received 15 April 1969.)

12. Analysis of factorial arrangements in unbalanced block designs. B. KURKJIAN and R. C. WOODALL, Headquarters Army Materiel Command and Harry Diamond Laboratories.

The unified theory of Kurkjian and Zelen (*Biometrika* 1963) for the analysis of factorial arrangements, as applied to balanced block designs, is extended to include the general case of unbalanced designs. Included are situations involving missing treatments, unequal number of treatments per plot per block and fractional factorial designs. The computations are programmed for an IBM 7094 computer. The Gauss-Markoff estimates for the treatment effects, as well as the usual quantities associated with the analysis of variance, are presented. Where appropriate, aliases of each treatment effect are provided as printer output. Confounded treatment effects are also identified. (Received 17 April 1969.)

13. Rank tests invariant only under linear transformations. ROBERT L. OBENCHAIN, University of North Carolina.

Nonparametric procedures appropriate for data measured on at least an ordinal scale utilize ranks invariant with respect to monotonically increasing transformations. By restricting attention to data measured on at least an interval scale and to procedures invariant only under the group of translations and nonsingular linear transformations, the univariate concept of rank order is generalized and extended to several dimensions as follows: Suppose $p \geqq 1$ variables are observed on individuals in random samples from $c \geqq 1$ populations. It is shown that the maximal invariant under the above group of transformations can be viewed as a scatter of points in Euclidean space of p or fewer dimensions. The concept of "difference ranks" of the distances between pairs of individuals in the combined sample is introduced and is shown to imply the ordinary ranks of univariate data. The assignment of general "linearly invariant rank scores" which preserve the dimensionality of the data is considered; conditional tests based on these scores are robust in small samples but are not asymptotically distribution-free because they can be asymptotically most powerful invariant. Use of polar coordinates leads to the concept of "radius rank" and a strictly distribution-free test with some power for detecting differences in dispersion. (Received 17 April 1969.)

14. Canonical analysis of several sets of variables. JON R. KETTENRING, University of North Carolina.

Five different techniques for the canonical analysis of several sets of variables are investigated. Each is such that it reduces to Hotelling's classical canonical analysis procedure when the number of sets is only two. A second important common feature is that each calls for the selection of (a number of stages of) canonical variables or linear composites (subject to appropriate restrictions), one from each set, according to a criterion of optimizing some function of their correlation matrix, $\mathbf{R} = ((r_{ij}))$. The criteria considered are the following: (i) maximize $\Sigma\Sigma r_{ij}$ [Horst, Paul. (1961). *Psychometrika* **26** 129–149]; (ii) maximize the largest eigenvalue of $\mathbf{R}$ [Horst, Paul. (1961). *J. of Clinical Psychology* (monograph supplement) **14** 331–347]; (iii) maximize $\Sigma\Sigma r_{ij}^2$; (iv) minimize the smallest eigenvalue of $\mathbf{R}$; (v) minimize $|\mathbf{R}|$ [Steel, Robert G. D. (1951). *Ann. Math. Statist.* **22** 456–460]. Models of the general principal component type are constructed for each of the five methods. The models serve to motivate and to interrelate the methods, as well as to reveal the types of effects which can be detected. (Received 24 April 1969.)

15. Asymptotic density of eigenvalues for a Gaussian ensemble of matrices. W. H. OLSON and V. R. RAO UPPULURI, Oak Ridge Associated Universities and Oak Ridge National Laboratory.

On a probability space $(\Omega, \mathfrak{F}, P)$ let $A_n = (a_{ij})_{i,j=1}^n$ be a random matrix such that: (i) $a_{ij} = a_{ji}$ a.s.; (ii) a_{ij}, $i \neq j$, is normal with mean 0 and variance 1, and a_{ii} is normal with mean 0 and variance $\frac{1}{2}$: (iii) $\{a_{ij}, i \leqq j\}$ is independent. Let $B_n = \frac{1}{2}n^{-\frac{1}{2}} A_n$ and denote by $\lambda_{1n}, \lambda_{2n}, \cdots, \lambda_{nn}$ the eigenvalues of B_n. Let $W_n(x)$ be the empirical distribution function of $\lambda_{1n}, \lambda_{2n}, \cdots, \lambda_{nn}$, i.e., $W_n(x) = N_n(x)/n$ where $N_n(x) =$ the number of $\lambda_{1n}, \lambda_{2n}, \cdots, \lambda_{nn}$ less than x. We shall prove that $E(W_n(x))$ converges to $W(x)$ as n approaches infinity where $W(x)$ is the distribution function with the semi-circle density, $w(x) = 2\pi^{-1}(1 - x^2)^{\frac{1}{2}}$, $|x| \leqq 1$, $w(x) = 0$, $|x| > 1$. Mehta [*Random Matrices*, Academic Press (1967)] outlines a proof in terms of convergence of density functions which are marginals of the joint density function of $(\lambda_{1n}, \lambda_{2n}, \cdots, \lambda_{nn})$. A corresponding theorem where the elements of the matrix take the values $+1$ and -1 with probabilities $\frac{1}{2}$ and $\frac{1}{2}$ respectively for off-diagonal elements and take the value 0 with probability 1 for diagonal elements was proved by Wigner [*Ann. Math.* **62** (1955) 548–564]. We shall make use of the combinatorial argument of Wigner's paper, and the method outlined in the general case by Arnold [*J. Math. Anal. Appl.* **20** (1967) 262–268]. (Received 18 April 1969.)

16. Moment inequalities for the maximum cumulative sum. R. J. SERFLING, Florida State University.

Let $\{X_i\}$ be a collection of random variables, not necessarily independent or identically distributed. Assume $E(X_i) \equiv 0$. Let $S_{a,n} = X_{a+1} + \cdots + X_{a+n}$ and $M_{a,n} = \max\{|S_{a,1}|, \cdots, |S_{a,n}|\}$. Let $r \geqq 2$. Bounds on the rth moment of $M_{a,n}$ are deduced purely from assumed bounds on the rth moment of $|S_{a,n}|$. The Rademacher-Mensov inequality, which pertains to the case of $r = 2$ and orthogonal X_i's, is generalized to allow $r \geqq 2$ and other types of dependent X_i's. For the case of $r > 2$, a further result is obtained which is considerably more useful for asymptotic applications of such inequalities. Applications to probability inequalities and to the tightness of sequences of random functions are considered. (Received 23 April 1969.)

17. A unified difference-equation approach to the study of problems in queues and in dams in discrete time. TOMÁS GARZA-HERNANDEZ. El Colegio de México.

A usual approach to the description of random walk processes is through the use of stochastic difference equations relating the states of the process in successive points in time. It is shown how to extend this treatment to other features of interest in the random walk, such as first-return times and limiting distributions. The present-day availability of methods of solution for a wide class of difference equations with strong boundary conditions opens the way to a unified approach to a number of problems in applied fields such as the theories of queues and dams. A systematic description of this approach is given in problems such as queue-length, busy period, and time-to-emptiness (in a dam), apart from the well-known ones of waiting time and storage content, and its relationship to other methods is discussed. Finally, some indications are given for obtaining explicit results in a number of particular cases. (Received 25 April 1969.)

19. Ladder phenomena for processes with stationary independent increments (preliminary report). M. RUBINOVITCH, Cornell University.

Let $\{X_t ; t \geqq 0\}$ be a separable process with stationary independent increments. Let $\mathcal{E} = \{E(t); t > 0\}$ where $E(t) = \{\omega: X_t(\omega) \geqq X_s(\omega); 0 \leqq s \leqq t\}$ and $\mathcal{E}^* = \{E^*(t); t > 0\}$ where $E^*(t) = \{\omega: X_t(\omega) \leqq X_s(\omega); 0 \leqq s \leqq t\}$. $\mathcal{E}$ and $\mathcal{E}^*$ are called the ascending and descending ladder phenomena for X_t. These are regenerative phenomena in the sense of Kingman [*Z. Wahrscheinlichkeitsth.* **2** (1964) 180–224]. Conditions for $\mathcal{E}$ and $\mathcal{E}^*$ to be standard or stable are given and their local time and limiting properties are characterized. It is found that there are three cases: (i) both $\mathcal{E}$ and $\mathcal{E}^*$ are standard. (ii) one, say $\mathcal{E}$, is standard and the other ($\mathcal{E}^*$) is degenerate (that is $Pr\{E^*(t)\} = 0$ identically) and (iii) both $\mathcal{E}$ and $\mathcal{E}^*$ are degenerate. It is shown that (i) is equivalent to the statement that both phenomena are stable, which is true iff X_t is a compound Poisson process. Ladder epochs as first passage times are introduced for processes X_t in cases (i) and (ii) and their behavior is characterized using the ladder phenomena. Finally, the connection between ladder epochs and the supremum functional of X_t is used to obtain limit theorems for the latter. (Received 1 May 1969.)

20. Certain properties of the positive Poisson distribution and the second type Stirling distribution. J. C. AHUJA, Portland State University. (By title)

The problem of estimating the parameter of the positive Poisson distribution (PPD) $f(x; \theta) = \alpha\theta^x/x!$, $x \,\varepsilon\, I$, where I is the set of positive integers, $\alpha = 1/(e^\theta - 1)$ and $0 < \theta < \infty$, has been considered by many authors. Among them, Tate and Goen [*Ann. Math. Statist.* **29** (1958) 755–765] have also obtained the distribution for the sum of n independent and identically distributed random variables having the PPD with parameter θ which we call the second type Stirling distribution (STSD). In this paper certain properties and characterizations of the PPD and the STSD are investigated following Patil and Wani [*Sankhyā, Ser. A.* **27** (1965) 271–280] and explicit expressions are given for the calculation of the crude and central moments of the PPD in terms of Stirling numbers of the second kind. The distribution for the sum of n independent random variables having the PPD with different parameters is derived, and from this the STSD is obtained as a special case. Recurrence relations for the probability function of the STSD are provided, and the distribution function of the STSD is expressed in terms of the sum of incomplete gamma functions. (Received 5 May 1969.)

21. Useful bounds in symmetrical factorial designs and error correcting codes. B. R GULATI, Eastern Connecticut State College.

Let $m_t(r, s)$ denote the maximum number of distinct points in a finite projective space $PG(r - 1, s)$ of $r - 1$ dimensions based on the Galois field $GF(s)$, where s is a prime or power of a prime, so that no t of these points are linearly dependent. It is well known that $m_t(r, s)$ also represents the maximum number of factors that can be accommodated in a symmetrical factorial design in which each factor is at $s = p^n$ levels, blocks are of size s^r, and no t-factor or lower order interaction is confounded. For an (n, k) group code, with k information symbols and fixed redundancy $r = n - k$, the maximum value of n for which u errors can be corrected in a channel capable of transmitting s distinct symbols with certainty is $m_{2u}(r, s)$. The maximum value of n for which u errors can be corrected with certainty and $u + 1$ can be detected is given by $m_{2u+1}(r, s)$. Bose has recently shown (*Bull. Inst. Internat. Statist.* **38** (1961)) that the theory of confounding and fractional replication due to Fisher, Finney, Bose and Kishan and theory of error correcting codes developed by Hamming and Slepian can be reduced to the problem of investigating $m_t(r, s)$. In this paper, we have established the following results: (i) $m_t(t + r, 2) = t + r + 1$ for $t \geqq 2(r + 1)$, $r \geqq 1$, (ii) $m_t(t + r, 2) = t + r + 2$ for $t = 2r, 2r + 1$, $r \geqq 2$, (iii) $m_t(t + r, 2) \leqq t + r + 5$ for $t = 2(r - 1), 2r - 1$, $r \geqq 4$. The bound is achieved for $r = 4$. (iv) $m_t(t + 1, 3) = t + 7$ for $3 \leqq t \leqq 5$ and $t + 2$ for $t \geqq 6$, and (v) $m_t(t + 2, 3) = t + 5$ for $6 \leqq t \leqq 8$ and $t + 3$ for $t \geqq 9$. These results generalize *some* of the previously reported results (*Ann. Math. Statist.* **40** (1969) 723). (Received 5 May 1969.)

22. Some approximations and uses of the Dirichlet distributions. (preliminary report). GEORGE C. TIAO and BIYI AFONJA, University of Wisconsin and University of Ife.

Several approximations to integrals of the types (1) $c\int_0^{a_1} \cdots \int_0^{a_k} (1 + \sum_{i=1}^k x_i)^{-\Sigma_{i=0}^k p_i} \prod_{i=1}^k x_i^{p_i-1}\, dx_i$ (2) $c\int_0^{a_1} \cdots \int_0^{a_k} (1 - \sum_{i=1}^k x_i)^{p_0-1} \prod_{i=1}^k x_i^{p_i-1}\, dx_i$ (3) $c\int_0^{a_1} \cdots \int_0^{a_k} (1 + \sum_{i=1}^k x_i)^{-\Sigma_{i=0}^k p_i} \prod_{i=1}^k x_i^{p_i-1}\, dx_i$ where $c = \Gamma(\sum_{i=0}^k p_i)/\prod_{i=0}^k \Gamma(p_i)$, are considered. Bayesian studies of variance component models in ANOVA (Tiao and Tan, *Biometrika* **52**, 37–53) suggest moment method of obtaining approximations to some incomplete gamma type integrals and hence to (1), (2) and (3). The resulting approximations involve only the calculations of incomplete beta functions which can be looked up in tables. Two of the approximations appear to be better than existing ones in both simplicity and accuracy. The good performance of a third and perhaps the simplest is limited to some values of the a_i. Some new bounds are also obtained for the bivariate case of (1) and (3), and simpler proofs are given for some other known bounds. Mention is made of various applications in such studies as ANOVA, homogeneity of variances, order statistics from gamma distributions, selection of populations with the smallest and largest variances, regression analysis, multinomial distributions and analysis of nonorthogonal designs. (Received 7 May 1969.)

23. A martingale analogue of Kolmogorov's law of the iterated logarithm.

WILLIAM F. STOUT, University of Illinois.

In the paper we establish a martingale analogue of Kolmogorov's law of the iterated logarithm for sums of bounded independent random variables. A modification of Kolmogorov's classical exponential bounds approach is used in conjunction with stopping rule theory for martingales in order to establish the result. Our law of the iterated logarithm is shown to include the martingale laws of the iterated logarithm established using different approaches by Strassen (*Proc. Fifth Berkeley Symp. Math. Statist. Prob.* **2** (1965) 315–343) and Levy (*Theorie de l'addition des variables aleatoires*, Paris (1965)). An application is made to sums of unbounded martingale differences. (Received 9 May 1969.)

24. A comparison of some parametric and non-parametric discrimination procedures in negative exponential populations.

JACK R. BORSTING and LT. JOSEPH L. LOCKETT, III, Naval Postgraduate School and Stanford University. (By title)

Several procedures based on the likelihood ratio for discrimination between two negative exponentially distributed populations are proposed. The small sample and asymptotic performance of these procedures is compared with that of non-parametric procedures and the classical linear discriminant function. Some guidelines for the use of the procedures discussed are presented. (Received 10 May 1969.)

25. Some distribution problems in life testing.

ROBERT R. READ, Naval Postgraduate School.

The paper treats distribution problems associated with truncated life testing of subsystems under conditions of limited test facilities. Specifically, there are c test chambers and i ($\geqq c$) subsystems available for test. The failure law is exponential and reliability estimates are to be based upon the total test time of all subsystems. The total idle time of the test center is also of interest and use is made of the fact that total idle time and total test time sum to a fixed quantity. The output process of the test center is also of interest for purposes of experimental design and scheduling. The paper characterizes the joint distribution of the output of the center, the total test time, and the total idle time under stopping rules based on either fixed time or fixed number of failures. Normal approximations are justified. (Received 10 May 1969.)

26. Limit laws for maxima of a sequence of random variables defined on a Markov chain (preliminary report).

SIDNEY I. RESNICK and MARCEL F. NEUTS, Purdue University and Cornell University.

Consider the bivariate sequence of r.v.'s $\{(J_n, X_n), n \geqq 0\}$ with $X_0 = 0$ a.s. The marginal sequence $\{J_n\}$ is an irreducible, aperiodic, m-state M.C., $m < \infty$, and the r.v.'s X_n are conditionally independent given $\{J_n\}$. Furthermore $P\{J_n = j, X_n \leqq x \mid J_{n-1} = i\} = p_{ij}H_i(x) = Q_{ij}(x)$, where $H_1(\cdot), \cdots, H_m(\cdot)$ are cdf's. Setting $M_n = \max\{X_1, \cdots, X_n\}$, we obtain $P\{J_n = j, M_n \leqq x \mid J_0 = i\} = [Q^n(x)]_{i,j}$, where $Q(x) = \{Q_{ij}(x)\}$. The limiting behavior of this probability and the possible limit laws for $\{M_n\}$ are characterized: THEOREM. *Let* $\rho(x)$

be the Perron-Frobenius eigenvalue of $Q(x)$ *for real* x, *then*: (a) $\rho(x)$ *is a* cdf; (b) *if for a suitable normalization* $\{Q^n_{ij}(a_{ijn}x + b_{ijn})\}$ *converges completely to a matrix* $\{U_{ij}(x)\}$ *whose entries are nondegenerate distributions, then* $U_{ij}(x) = \pi_j\rho_u(x)$, *where* $\pi_j = \lim_{n\to\infty} p^n_{ij}$ *and* $\rho_u(x)$ *is an extreme value distribution*; (c) *the normalizing constants need not depend on* i, j; (d) $\rho^n(a_nx + b_n)$ *converges completely to* $\rho_u(x)$; (e) *the maximum* M_n *has a nontrivial limit law* $\rho_u(x)$ *iff* $Q^n(x)$ *has a nontrivial limit matrix* $U(x) = \{U_{ij}(x)\} = \{\pi_j\rho_u(x)\}$ *or equivalently iff* $\rho(x)$ *or the* cdf $\prod_{i=1}^m H_i^{\pi_i}(x)$ *is in the domain of attraction of one of the extreme value distributions. Hence the only possible limit law for* $\{M_n\}$ *are the extreme value distributions which generalizes the results of Gnedenko for the* i.i.d. *case.* (Received 14 May 1969.)

(An abstract of a paper presented at the Central Regional meeting, Iowa City, Iowa, April 23–25, 1969. Additional abstracts appeared in earlier issues.)

44. Estimating the conditional probability of misclassification. MARILYN SORUM, Northwestern University.

An observation x is assumed to come from one of two p-dimensional normal populations Π_1, Π_2 with unknown mean vectors μ_1, μ_2 and common known covariance Σ. Letting $\bar{x}_1$, $\bar{x}_2$ denote sample mean vectors based on observations from Π_1, Π_2, x is classified as coming from Π_1 if $(\bar{x}_1 - \bar{x}_2)'\Sigma^{-1}x \geqq \frac{1}{2}(\bar{x}_1 - \bar{x}_2)'\Sigma^{-1}(\bar{x}_1 + \bar{x}_2)$. The quantity to be estimated is $P_2 = P_2(\bar{x}_1, \bar{x}_2, \mu_2)$, the conditional probability of misclassifying an x coming from Π_2, given the fixed rule based on $\bar{x}_1$, $\bar{x}_2$. Related quantities are $P_2^* = P_2^*(\Delta)$, the expectation of P_2, and $P_2^{**} = P_2^{**}(\Delta)$, the probability of misclassifying x from Π_2 when all parameters are known (i.e. the above rule with μ_1, μ_2 in place of $\bar{x}_1$, $\bar{x}_2$); $\Delta^2 = (\mu_1 - \mu_2)'\Sigma^{-1}(\mu_1 - \mu_2)$. The "original" observations on which the rule is based plus additional "test" observations from Π_2 are assumed available. Estimators considered include ones obtained by classifying the original or test observations from Π_2; ones of the form $P_2(\bar{x}_1, \bar{x}_2, \hat{\mu}_2)$, $P_2^*(\hat{\Delta})$, $P_2^{**}(\hat{\Delta})$, where $\hat{\mu}_2, \hat{\Delta}$ are estimators of μ_2, Δ; and the conditional (given $\bar{x}_1$, $\bar{x}_2$) UMVU estimator based on original and test observations. Looking at asymptotic mean square error (conditional moments), for all estimators studied the leading term is of order $o_p(N^{-1+\epsilon})$. Comparing estimators on the magnitude of the leading term, estimators based on normality are better; estimators based only on original samples are as good on the (unconditional) average as corresponding estimators requiring test observations; and all the estimators using normality and only original samples are equivalent. (Received 5 May 1969.)

(Abstracts of papers not connected with any meeting of the Institute.)

1. Comparison of translation experiments. ERIK NIKOLAI TORGERSEN, University of California, Berkeley.

In this paper we treat the problem of comparison of translation experiments. The "convolution divisibility" criterion for "being more informative" by C. Boll (1955, Ph.D. thesis Stanford Univ.) is generalized to a "ϵ-convolution divisibility" criterion for ϵ-deficiency. We also generalize the "convolution divisibility" criterion of V. Strassen (*Ann. Math. Statist.* **36** (1965) 423) to a criterion for "ϵ-convolution divisibility". It is shown, provided least favorable "ϵ-factors" can be found, how the deficiencies actually may be calculated. As an application we determine the increase of information—as measured by the deficiency—contained in an additional number of observations for a few experiments (rectangular with unknown scale parameter, exponential with unknown scale parameter, normal with known mean and unknown variance, multivariate normal with unknown mean and known covariance matrix, one way lay out with unknown means and known variances). Finally we consider the problem of convergence for the pseudo distance introduced by LeCam (*Ann. Math. Statist.* **35** (1964) 1419). It is shown that convergence for this distance is topologically equivalent with strong convergence of the individual probability measures up to a shift. (Received 2 May 1969.)

2. Maximum likelihood estimation of a unimodal density function. EDWARD J. WEGMAN, University of North Carolina.

Under the assumption of unimodality of the probability density function, a non-parametric maximum likelihood estimate of that density is given. Strong consistency of the density estimate is shown to depend on strong convergence of an estimate of the mode. The maximum likelihood estimate of the density generates an estimate of the mode which is shown to converge almost surely. (Received 7 May 1969.)

3. A characterization based on the absolute difference of two i.i.d. random variables. PREM S. PURI and HERMAN RUBIN, Purdue University.

Let X be a nonnegative random variable with X_1 and X_2 as its two independent copies. The problem considered here is to characterize all the nonnegative distributions with the property that the distribution of the absolute difference $|X_1 - X_2|$ is same as that of X. It is shown that in general such a distribution has to be either purely discrete, or purely absolutely continuous or singualr and that it cannot be their mixture. The result for the case when X is discrete was reported earlier (see Prem S. Puri, *Ann. Math. Statist.* **40** (1969) 725). For the case when X is absolutely continuous, it is shown under certain conditions that the only distribution that enjoys the above property is the exponential distribution with pdf $f(x) = \theta \exp(-\theta x)$, for $x \geqq 0$, and zero elsewhere, with $\theta > 0$. The case when X is singular is not considered. However, here X cannot be bounded as it is shown that the only bounded X satisfying the above property is either the one with distribution $Pr(x = 0) = Pr(X = a) = \frac{1}{2}$ for some $a > 0$ or with $Pr(X = 0) = 1$. (Received 12 May 1969.)

4. Estimation of the parameter of an exponential distribution on the basis of a preliminary test. J. SINGH, University of California at Berkeley.

Let $x_1, x_2, \cdots, x_m$ and $y_1, y_2, \cdots, y_n$ be two random samples from populations having pdf's $f(x, \theta_1)$ and $f(x, \theta_2)$ respectively where $f(x, \theta) = (1/\theta) \exp(-x/\theta)$. We are interested in estimating θ_1. We first make a test of the hypothesis $H: \theta_1 = \theta_2$. If the preliminary test accepts the hypothesis H we use both X and Y samples for estimating θ_1 and if the preliminary test rejects the hypothesis H we use only X sample for estimating θ_1. We thus describe an estimate $\hat{\theta}$ of θ_1 as below: $\hat{\theta} = (m\bar{x} + n\bar{y})/(m + n)$, if $1/F^{(\alpha)}_{(2n,2m)} \leqq (m/n)\Sigma y_j/\Sigma x_j \leqq F^{(\alpha)}_{(2n,2m)}$; $\hat{\theta} = \bar{x}$, otherwise, where α is some preassigned level of significance. The preliminary test used here is equivalent to the likelihood ratio test for the above hypothesis. The distribution of the estimate $\hat{\theta}$ has been derived and the bias and mean square error of $\hat{\theta}$, as an estimate of θ_1, has been investigated. We plan to do some empirical investigation for the comparison of the estimate $\hat{\theta}$ with the usual estimate $\bar{x}$. (Received 12 May 1969.)

5. The behavior of some robust estimators on dependent data II (preliminary report). JOSEPH L. GASTWIRTH and HERMAN RUBIN, The Johns Hopkins University and Purdue University.

This paper continues our study (Abstract, **39** (1968) 1087) of the behavior of robust estimators on dependent processes. In particular, we have proved the following theorems. THEOREM 1. *On stationary Gaussian sequences such that* $\rho_k \geqq 0$ *for all* k *and* $\sum \rho_k < \infty$, *the asymptotic efficiency of any linear combination of order statistics which is an unbiased esti-*

mator of the mean of the process relative to the sample mean is greater than or equal to its value in the case of independent observations. THEOREM 2. *For a normal first order autoregressive process with negative* ρ, *the asymptotic efficiency of any linear combination of order statistics which is an unbiased estimator of the mean of the process relative to the sample mean is less than or equal to its value in the case of independent observations.* THEOREM 3. *Analogous results hold for the Hodges-Lehmann estimator.* THEOREM 4. *Under the conditions of Theorem 2, the asymptotic efficiency of a finite linear combination of order statistics relative to the sample mean* $\rightarrow 0$ *as* $\rho \rightarrow -1$. (*This is not true for the Hodges-Lehmann estimator or the trimmed mean.*) (Received 20 May 1969.)