The Annals of Mathematical Statistics
1970, Vol. 41, No. 1, 326–327

CORRECTION NOTES

CORRECTION TO "OPTIMUM CLASSIFICATION RULES FOR CLASSIFICATION INTO TWO MULTIVARIATE NORMAL POPULATIONS"

BY S. DAS GUPTA

There are a few slips in the statement of Lemma 3.2 and its proof in my paper [*Ann. Math. Statist.* **36** (1965) 1174–1184] pointed out to me by Dr. M. S. Shrivastavha. The corrected and slightly modified form of Lemma 3.2 is stated below along with necessary changes in its proof. The main results of the paper are not affected by this change.

LEMMA. *Let* $\mathbf{u}$ *and* $\mathbf{v}$ *be two* $p \times 1$ *random vectors with mean vectors* $\boldsymbol{\delta} \neq \mathbf{0}$ *and* $\mathbf{0}$, *respectively, and having the identity matrix* I_p *as the common covariance matrix. Then, for any* $p \times p$ *positive definite matrix* $\mathbf{B}$,

$$\Pr[\mathbf{u}'\mathbf{B}\mathbf{u} < \mathbf{v}'\mathbf{B}\mathbf{v}] \leqq \frac{8\,\mathrm{tr}(\mathbf{B})}{\boldsymbol{\delta}'\mathbf{B}\boldsymbol{\delta}} \leqq \frac{8\,\mathrm{tr}(\mathbf{B})}{(\boldsymbol{\delta}'\boldsymbol{\delta})Ch_m(\mathbf{B})} \leqq \frac{8pCh_M(\mathbf{B})}{(\boldsymbol{\delta}'\boldsymbol{\delta})Ch_m(\mathbf{B})}$$

where $Ch_M(B)$ and $Ch_m(B)$ are the maximum and the minimum characteristic roots of B.

PROOF. It follows from the Cauchy–Schwarz inequality that

$$[(\mathbf{u}-\boldsymbol{\delta})'\mathbf{B}(\mathbf{u}-\boldsymbol{\delta}) < \tfrac{1}{4}\boldsymbol{\delta}'\mathbf{B}\boldsymbol{\delta}] \Rightarrow u'Bu \geqq \tfrac{1}{4}\boldsymbol{\delta}'\mathbf{B}\boldsymbol{\delta}$$

Thus

$$(\mathbf{u}-\boldsymbol{\delta})'\mathbf{B}(\mathbf{u}-\boldsymbol{\delta}) < \tfrac{1}{4}\boldsymbol{\delta}'\mathbf{B}\boldsymbol{\delta}, \qquad \mathbf{v}'\mathbf{B}\mathbf{v} < \tfrac{1}{4}\boldsymbol{\delta}'\mathbf{B}\boldsymbol{\delta} \Rightarrow \mathbf{u}'\mathbf{B}u \geqq \mathbf{v}'\mathbf{B}\mathbf{v}.$$

Hence

$$\begin{aligned}\Pr[\mathbf{u}'\mathbf{B}\mathbf{u} < \mathbf{v}'\mathbf{B}\mathbf{v}] &\leqq \Pr[(\mathbf{u}-\boldsymbol{\delta})'\mathbf{B}(\mathbf{u}-\boldsymbol{\delta}) \geqq \tfrac{1}{4}\boldsymbol{\delta}'\mathbf{B}\boldsymbol{\delta}] + \Pr[\mathbf{v}'\mathbf{B}\mathbf{v} \geqq \tfrac{1}{4}\delta'\mathbf{B}\delta] \\ &\leqq 8\,\mathrm{tr}(\mathbf{B})/\boldsymbol{\delta}'\mathbf{B}\boldsymbol{\delta}\end{aligned}$$

from Chebyshev's inequality. The other inequalities follow trivially.

CORRECTION TO "CYLINDRICALLY ROTATABLE DESIGNS OF TYPES 1, 2 AND 3"

BY AGNES M. HERZBERG

Imperial College, University of London

I am grateful to Dr. G. C. Derringer for pointing out that the example given on page 170, *Ann. Math. Statist.* **38** 167–176, is singular. A new example of a second order cylindrically rotatable design of type 1 in four dimensions is the following.

Consider the following point sets:

$$
\begin{array}{llrr}
(\pm a, & \pm a, & c, & e), \\
(\pm a, & \pm a, & -c, & -e), \\
(\pm 2^{\frac{1}{2}}a, & 0, & \pm g, & 0), \\
(0, & \pm 2a^{\frac{1}{2}}, & \pm g, & 0), \\
(0, & 0, & \pm d, & 0), \\
(0, & 0, & 0, & \pm f).
\end{array}
$$

For all values of a, c, d, e, f and g except zero, these point sets will form a second order cylindrically rotatable design of type 1 in four dimensions since the moments satisfy (2.16) and the variance-covariance matrix is nonsingular. The number of points involved is twenty. The moments of the design are, when multiplied by N,

$$
\begin{aligned}
&\sum x_{1u}^2 = \sum x_{2u}^2 = 16a^2, \\
&\sum x_{3u}^2 = 8(c^2+g^2)+2d^2, \\
&\sum x_{4u}^2 = 8e^2+2f^2, \\
&\sum x_{1u}^4 = \sum x_{2u}^4 = 3\sum x_{1u}^2 x_{2u}^2 = 24a^4, \\
&\sum x_{3u}^4 = 8(c^4+g^4)+2d^4, \\
&\sum x_{4u}^4 = 8e^4+2f^4, \\
&\sum x_{1u}^2 x_{3u}^2 = \sum x_{2u}^2 x_{3u}^2 = 8a^2(c^2+g^2), \\
&\sum x_{1u}^2 x_{4u}^2 = \sum x_{2u}^2 x_{4u}^2 = 8a^2e^2, \\
&\sum x_{1u}^2 x_{3u} x_{4u} = \sum x_{2u}^2 x_{3u} x_{4u} = 8a^2ce, \\
&\sum x_{3u}^2 x_{4u}^2 = 8c^2e^2, \\
&\sum x_{3u}^3 x_{4u} = 8c^3e, \\
&\sum x_{3u} x_{4u}^3 = 8ce^3, \\
&\sum x_{3u} x_{4u} = 8ce,
\end{aligned}
$$

and all other sums of powers and products up to and including order four are zero. The summation is taken over all design points.