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ABSTRACTS OF PAPERS

(*Abstracts of papers to be presented at the Eastern Regional meeting, University Park, Pennsylvania, April* 21–23, 1971. *Additional abstracts will appear in future issues.*)

129-3. Asymptotic properties of a certain estimator related to the maximum likelihood estimator (preliminary report). JAMES C. FU, The Johns Hopkins University.

Let the sequence $\{X_n\}$ be a sequence of i.i.d. random variables with common density $f(x \mid \theta)$, where θ is in an open interval Θ of R^1. An estimator $\{\hat{T}_n\}$ is called a maximum probability estimator of θ with respect to the (prior) density $g(\theta)$ of a positive measure on Θ if

$$\prod_{i=1}^n f(x_i \mid T_n(x_1, \cdots, x_n))g(T_n(x_1, \cdots, x_n)) = \max_{\theta \in \Theta} \prod_{i=1}^n f(x_i \mid \theta)g(\theta)$$

for all $n = 1, 2, \cdots$. In this report, we prove, under certain regularity conditions, that the estimator $\hat{T}_n$ is asymptotically efficient in the sense that $\lim_{\varepsilon \to 0} \lim_{n \to \infty} (1/\varepsilon^2 n) \log P_\theta\{|\hat{T}_n - \theta| > \varepsilon\} = -I(\theta)/2$, where $I(\theta)$ is Fisher's information. This proof also gives a direct method of verifying Bahadur's result [*Ann. Math. Statist.* **38** (1967) 303–324] that the maximum likelihood estimator $\hat{\theta}_n$ is asymptotically efficient in the above sense. The existence, consistency and asymptotic distribution of the estimator $\hat{T}_n$ are also discussed. (Received 20 October 1970.)

(*Abstracts of papers to be presented at the Central Regional meeting, Columbia, Missouri, May* 5–7, 1971. *Additional abstracts will appear in future issues.*)

130-1. Asymptotic theory for two estimators of the generalized failure rate function. B. L. S. PRAKASA RAO and J. VAN RYZIN, University of Wisconsin.

Let $Y_1 \leqq Y_2 \leqq \cdots \leqq Y_n$ be an ordered sample of n observations from an unknown distribution function F with density f. Let G be a known distribution function with density g. The, $r(x) = f(x)/g(G^{-1}(F(x)))$, defined for all x for which $gG^{-1}(F(x)) \neq 0$, is called the generalized failure rate function (GFR). Two estimators of $r(x)$ based on the sample of size n are proposed and studied. These are: (i) $r_n^*(x) = [G^{-1}(B_n/n) - G^{-1}(A_n/n)][Y_{B_n} - Y_{A_n}]^{-1}$ and (ii) $r_n^{**}(x) = [B_n - A_n][n\{Y_{B_n} - Y_{A_n}\}g(G^{-1}(T_n))]^{-1}$, where $\{A_n\}$, $\{B_n\}$ are such that $A_n = A_n(x)$, $B_n = B_n(x)$, $0 < B_n - A_n \leqq k_n$, $A_n \leqq n$, $B_n \leqq n$ are suitably chosen and specified integer valued random variables measurable with respect to the σ-field generated by the n observations, $T_n = (A_n + B_n)/(2n)$, and $\{h_n\}$ is a sequence of specified positive integers. Theorems giving sufficient conditions under which $r_n^*(x)$ and $r_n^{**}(x)$ are both weakly and strongly consistent estimators of $r(x)$ are presented. Furthermore, for each estimator three separate statements concerning the asymptotic distribution are developed. These results give conditions under which $k_n^{\frac{1}{2}}(r_n^*(x) - r(x))$ (or $k_n^{\frac{1}{2}}(r_n^{**}(x) - r(x))$), for different rate choices of the $\{k_n\}$ sequence, converge in distribution to a random variable which has a certain specified normal distribution. (Received 15 October 1970.)

130-2. Some recent results in the theory of symmetrical factorial designs and error correcting codes. BODH RAJ GULATI, Southern Connecticut State College.

It is well known that the problem of determining the alphabet of the code or the fundamental subgroup of the design can be reduced to the "packing problem." This problem deals with the investigation of the maximum possible number of points in $(t+r-1)$-dimensional projective space PG $(t+r-1, s)$ over a Galois field of order $s = p^h$ (where p and h are positive integers and p is the prime characteristic of the field), so that no t of the chosen points lie on a $(t-2)$-flat. In the language of experimental designs, the problem addresses itself to what is the maximum number of factors, each operating at s levels, that can be accommodated in a symmetrical factorial experiment with a block size of s^{t+r} so that no main effect or a t-factor ($t > 1$) or lower order interaction is confounded with block effects. In this paper, we have established that $m_t(t+r, s) = t+r+2$ for $f(r) \leqq t < s(r+1)$ and $m_t(t+r, s) \geqq t+r+3$ for $f(r) > t$, where $f(r) = (s-1)(r+2)+[(r+2)/(s+1)]$, $r = (s-2), (s-1) \bmod (s+1)$, $f(r) = (s-1)(r+1)+(j+1)+[(r+2)/(s+1)]$ for $r = j \bmod (s+1)$ and $j = 0, 1, \cdots, (s-3)$ and $f(r) = (s-1)(r+1)+[(r+2)/(s+1)]$ for $r = s \bmod (s+1)$ and $m_t(t+r, s)$ denotes the maximum number of distinct points in PG$(t+r-1, s)$, no t dependent. In the general case, upper bounds are obtained on t for which k points may be added to the basic set of E_i's, $i = 1, 2, \cdots, t+r$, where E_i is a point with a unity in ith position and zeros elsewhere. (Received 10 November 1970.)

(Abstracts of papers presented by title)

71T-17. Some asymptotic properties of a two-dimensional periodogram. M. PAGANO, The Johns Hopkins University.

The two-dimensional periodogram has been proposed as an estimator of the spectral density of a real, homogeneous, random field defined over a regular lattice on the plane. In the present paper, the asymptotic finite-dimensional distribution functions of the periodogram of an independent, orthonormal field which obeys the Lindeberg–Feller condition are found. This result is extended to cover the periodogram of random fields that may be represented as a moving average of such an orthonormal field. This extension is verified by showing that, save for a scale factor, the asymptotic finite-dimensional distribution functions of the two periodograms are equal. The rate of convergence to zero of the mean difference of the two random variables obtained by evaluating the above mentioned periodograms at any fixed point is obtained by paralleling Olshen's approach to a similar analysis of the one-dimensional periodogram (Olshen, R. A. (1967). Asymptotic properties of the periodogram of a discrete stationary process. *J. Appl. Probability* **4** 508–528). The analysis, however, requires defining a certain class of Lipschitz functions in two dimensions, and the derivation of some properties of this class of functions. (Received 15 October 1970.)

71T-18. Characterization of Wiener process by symmetry. B. L. S. PRAKASA RAO, Indian Institute of Technology, Kanpur.

Recently Heyde (*Sankhyā*, *Ser. A.* **32** 115–118) has proved that if the conditional distribution of a linear function of n independent random variables X_i, $1 \leqq i \leqq n$ on another linear function of the same set of random variables is symmetric, then each of the X_i is normally distributed or degenerate under some conditions. We obtain two characterization theorems for Wiener Processes extending the result of Heyde. Let $\{X(t), t \in T\}$, $T = [A, B]$ be a continuous homeogeneous process with independent increments with moments of all orders. Further suppose that the mean and covariance are of bounded variation. Suppose $a(\cdot)$ and $b(\cdot)$ are continuous functions on A, B. Let $Y = \int_A^B a(t)\,dX(t)$ and $Z = \int_A^B b(t)\,dX(t)$. Under some conditions on $a(\cdot)$ and $b(\cdot)$, we have shown that the process $\{X(t), t \in T\}$ is a Wiener process with linear mean function iff the conditional distribution of Y given Z is symmetric and in such as even $\int_A^B a(t)\,dt = 0$ and $\int_B^A a(t)b(t)\,dt = 0$. (Received 20 October 1970.)

71T-19. Characterization of stationary processes differentiable in mean square. B. L. S. PRAKASA RAO, Indian Institute of Technology, Kanpur.

Recently Mazo and Salz proved that if $\{Y(t), t \in T\}$ is a stationary random process with mean square derivative $\{\dot{Y}(t), t \in T\}$, then the conditional expectation of $\dot{Y}(t)$ given $Y(t)$ is zero almost everywhere with respect to the distribution of $Y(t)$. We extend this property and obtain a characterization of stationary processes differentiable in mean square. Suppose $\{Y(t), t \in T\}$ is a random process with mean square derivative $\{\dot{Y}(t), t \in T\}$. We have proved that $\{Y(t), t \in T\}$ is a stationary process if and only if the conditional expectation of $\dot{Y}(t_i)$ given $Y(t_j)$, $1 \leqq j \leqq k$ is zero for $1 \leqq i \leqq k$ for every collection t_i, $1 \leqq i \leqq k$ in T and for every positive integer k. (Received 20 October 1970.)

71T-20. Sequential estimation of a linear function of several means. RASUL A. KHAN, Columbia University.

A theorem of Anscombe (Rényi, A., *Acta Math. Acad. Sci. Hungar.* **8** 193–199) is generalized as follows. THEOREM. *Let* $\{x_{ij}, j = 1, 2, \cdots, n_i, \cdots\}(k \geqq i \geqq 1)$ *be* k *independent sequences of independent random variables with* $Ex_{ij} = 0$ *and* $0 < \sigma^2_{x_{ij}} = \sigma_i^2 < \infty$. *Define* $Y_{n_1, \cdots, n_k} = (\sum_{i=1}^k a_i \bar{x}_{in_i})/(\sum_{i=1}^k a_i^2\sigma_i^2/n_i)^{\frac{1}{2}}$ *where* $\bar{x}_{in_i} = n_i^{-1} \sum_{j=1}^{n_i} x_{ij}$ *and* a_i *are real finite constants for* $k \geqq i \geqq 1$. *Let further* $\{v_i(t_i)\}(k \geqq i \geqq 1)$ *be* k *sequences of positive integer-valued random variables such that* $v_i(t_i)/t_i \to c_i > 0$ *(positive constants) in probability as* $t_i \to \infty$ *for each* $i = 1, 2, \cdots, k$. Then $\lim_{t_i \to \infty, k \geqq i \geqq 1} P(Y_{v_1(t_1), \cdots, v_k(t_k)} \leqq x) = (2\pi)^{-\frac{1}{2}} \int_{-\infty}^x e^{-u^2/2}\, du$. Following Robbins et al. (*Ann. Math. Statist.* **38** 1384–1391), sequential procedures are set for making a fixed-width confidence interval of prescribed coverage probability for $\mu = \sum_{i=1}^k c_i\mu_i$ where μ_i are the means of k independent normal populations $N(\mu_i, \sigma_i^2)$. Using the results of Robbins et al. and the preceding theorem we prove asymptotic consistency and efficiency etc. As an application, the parameter μ could be a contrast in a fixed model. (Received 2 November 1970.)

71T-21. On a consistent two-dimensional spectrum estimator. M. PAGANO, The Johns Hopkins University.

Given a two-dimensional homogeneous random field which is Gaussian, regularity conditions for the spectral density of the field are found under which it is possible to construct uniformly strongly consistent sequences of estimators which are obtained by smoothing the two-dimensional periodogram. The finite-sample distribution of the random variables obtained by evaluating an estimator in the sequence at a finite number of points is unwieldly. It is shown that new random variables having scaled chi-square distributions can be defined (on the same sample space as the given random variables) which approximate the given random variables in mean square. An upper bound for this mean square approximation is found. Use is made of a toroidal field, and some of the properties of such a field are developed. (Received 9 November 1970.)

71T-22. Recurrence relation for the minimum variance unbiased estimator of parameter of a left-truncated Poisson distribution. J. C. AHUJA, Portland State University.

Let $x_1, x_2, \cdots, x_n$ be a random sample of size n drawn from the left-truncated Poisson distribution $p(x; \theta) = e^{-\theta}\theta^x/x!f_c(\theta)$, $x \in T$, where $T = \{c+1, c+2, \cdots, \infty\}$, $0 < \theta < \infty$, and $f_c(\theta) = 1 - \sum_{x=0}^c e^{-\theta}\theta^x/x!$. The problem of estimating the parameter θ in $p(x; \theta)$ has been considered among others by Rider (1953), Moore (1956), Tate and Goen (1958), Patil (1962), and Subrahmaniam (1965). Tate and Goen [*Ann. Math. Statist.* **29** (1958) 755–765] have obtained the minimum variance unbiased estimator $g_c(z, n)$ of θ, based on a sample size n and sample total z, in the form $g_c(z, n) = zD_c(z-1, n)/D_c(z, n)$ for $z > n(c+1)$, and $g_c(z, n) = 0$ otherwise, where $D_c(z, n)$ is the generalized Stirling number of the second kind too complicated for calculation even for small values of n and z. Recently, the author (1970) has investigated certain properties of the estimator $g_c(z, n)$ for the special cases $c = 0$ and $c = 1$, and provided their recurrence relations. In this paper, using the recurrence formula for the generalized Stirling number $D_c(z, n)$ given in Riordan (1958), we obtain the recurrence relation for $g_c(z, n)$ in the form

$$g_c(z+1, n) = (z+1)g_c(z-c, n-1)/ng_c(z-c, n-1) - ng_c(z, n) + z,$$

where $g_c(z, 1) = z$ for $z \geqq c+2$, and $g_c(z, n) = 0$ for $z = n(c+1)$. (Received 23 November 1970.)

71T-23. The distribution of frequency counts of the geometric distribution. C. J. PARK, University of Nebraska.

Let $X_1, X_2, \cdots, X_n$ be a random sample from the geometric distribution with parameter θ. Let $T = \sum_{i=1}^n X_i$ and S_j denote the number of X_i's equal to j, $j = 0, 1, 2, \cdots, T$. In this paper we establish the joint asymptotic normality of $(S_0, S_1, \cdots, S_k)$ given $T = t$, for a fixed k, as n and t tend to infinity with $n/(n+t) \to \theta$. In the proof we use the characteristic function. This result gives a simple alternative proof of the limiting normality of the cell frequency counts of non-parametric two sample problem proposed by S. Wilks (*Proc. Fourth Berkeley Symp. Math. Statist. Prob.* (1961) 707–717). (Received 24 November 1970.)

71T-24. Exact distribution of the sum of independent left-truncated logarithmic series variables. J. C. AHUJA, Portland State University.

Let $X_1, X_2, \cdots, X_n$ be n independent and identically distributed random variables having the left-truncated logarithmic series distribution $f(x; \theta) = \theta^x/xg_c(\theta)$, $x \in T$, where

$$T = \{c+1, c+2, \ldots, \infty\}, \quad 0 < \theta < 1, \quad \text{and} \quad g_c(\theta) = \textstyle\sum_{x=c+1}^{\infty} \theta^x/x.$$

Let $Z = X_1 + X_2 + \ldots + X_n$ The distribution of sum Z for $c = 0$ has been obtained by Patil and Wani [*Sankhyā*, *Ser. A* **27** (1965) 271–80] called the Stirling distribution of the first kind, while Wayland [Master's thesis (1970) Portland State University] has recently derived the distribution of Z for $c = 1$ in terms of associated Stirling numbers of the first kind. In this paper, the distribution of Z is provided for the general case in an explicit form in terms of what we call generalized Stirling numbers of the first kind. A recurrence relation is given for the generalized Stirling numbers and its use is made to obtain a recurrence relation for the probability function of Z. (Received 25 November 1970.)