

## Correction to the article Gorenstein duality for real spectra

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This paper reports and corrects a mistake in the authors' paper ([Algebr. Geom. Topol.](#) 17 (2017) 3547–3619).

55P91; 55Q91, 55U30

In this erratum we report a mistake in the authors' article [2] and record a corrected statement. Throughout this erratum we will use the notation of [2] and all numbering of lemmas and theorems refers to this source.

Theorem 1.4 is not correct as stated for  $n \geq 2$ , even on underlying nonequivariant spectra. Indeed, the homotopy groups of  $\Gamma_{J_{n-1}} E\mathbb{R}(n)$  are countable in every degree and this is not true for  $\mathbb{Z}_{(2)}^{E\mathbb{R}(n)}$ . Let us explain where the mistake in the proof lies and then how to correct the statement.

The mistake is in Lemma 5.8. Consider for example  $B = B\mathbb{P}\mathbb{R}\langle 2 \rangle$ , whose underlying homotopy groups are  $\mathbb{Z}_{(2)}[v_1, v_2]$ . Contrary to the statement of Lemma 5.8, the  $\mathbb{Z}[v_2]$ -module  $\mathbb{Z}_{(2)}[v_1^{\pm 1}, v_2]$  does not have bounded  $v_2$ -divisibility. Indeed, for every  $k$  the element  $v_2^k v_1^{-3k}$  is of degree 0 and divisible by  $v_2^k$ . Thus, the proof of Theorem 5.9 breaks down and the theorem is indeed wrong as stated. Note though that Example 5.10 (recovering a result of Ricka) remains unaffected.

What happens indeed is that  $\mathbb{Z}_{(2)}^{E\mathbb{R}(n)}$  is a kind of completion of a shift of  $\Gamma_{J_{n-1}} E\mathbb{R}(n)$ , at least on the level of underlying homotopy groups. Instead of making this precise, we show that Theorems 1.4 and 5.9 are true after a suitable cellularization. For an  $M\mathbb{R}$ -module  $K$ , we say that there is a  $K$ -cellular equivalence between two  $M\mathbb{R}$ -modules if there is an  $M\mathbb{R}$ -linear equivalence between their  $\mathbb{R}$ -cellularizations with respect to  $K$  in the sense of Section 2.B and Proposition 3.8.

**Theorem** (corrected form of Theorem 5.9) *Let the notation be as in Theorem 5.1 and assume for simplicity that only finitely many  $m_i$  are zero and that  $m_n = 0$ . Let  $K$*

be the  $M\mathbb{R}$ -module  $M\mathbb{R}/(\underline{v} \setminus \bar{v}_n)$ , where  $\underline{v} \setminus \bar{v}_n$  denotes the sequence of all  $\bar{v}_i$  such that  $m_i = 0$  and  $i \neq n$ . Then there is a  $K$ -cellular equivalence

$$\mathbb{Z}_{(2)}^{M[\bar{v}_n^{-1}]} \simeq_K \Sigma^{-m'+|\underline{v}|+(k-1)+4-2\rho} M.$$

**Proof** Modifying the proof of Theorem 5.9, it suffices to show that there is a  $K$ -cellular equivalence between  $\operatorname{holim}(\cdots \xrightarrow{\bar{v}_n} \Gamma_{\bar{v}_n}(\Gamma_{\underline{v} \setminus \bar{v}_n} M))$  and  $\Sigma^{-1} M[\bar{v}_n^{-1}]$ . For this, we claim first that the natural map

$$(1) \qquad \operatorname{holim}(\cdots \xrightarrow{\bar{v}_n} \Gamma_{\bar{v}_n}(\Gamma_{\underline{v} \setminus \bar{v}_n} M)) \rightarrow \operatorname{holim}(\cdots \xrightarrow{\bar{v}_n} \Gamma_{\bar{v}_n} M)$$

is a  $K$ -cellular equivalence. We have indeed a chain of equivalences

$$\begin{aligned} \operatorname{Hom}_{M\mathbb{R}}(K, \operatorname{holim}(\cdots \xrightarrow{\bar{v}_n} \Gamma_{\bar{v}_n}(\Gamma_{\underline{v} \setminus \bar{v}_n} M))) &\simeq \operatorname{holim} \operatorname{Hom}_{M\mathbb{R}}(K, \Gamma_{\bar{v}_n}(\Gamma_{\underline{v} \setminus \bar{v}_n} M)) \\ &\simeq \operatorname{holim} \operatorname{Hom}_{M\mathbb{R}}(K, \Gamma_{\bar{v}_n} M) \\ &\simeq \operatorname{Hom}_{M\mathbb{R}}(K, \operatorname{holim}(\cdots \xrightarrow{\bar{v}_n} \Gamma_{\bar{v}_n} M)). \end{aligned}$$

Thus, the map in (1) is a  $K$ -cellular equivalence. Moreover,

$$\operatorname{holim}(\cdots \xrightarrow{\bar{v}_n} \Gamma_{\bar{v}_n} M) \simeq \Sigma^{-1} M[\bar{v}_n^{-1}]$$

by Lemma 5.7. Combining this equivalence and the  $K$ -cellular equivalence (1) gives the result. □

In particular, we obtain:

**Theorem** (corrected form of Theorem 1.4) *For each  $n \geq 1$  set*

$$K = M\mathbb{R}/(\bar{v}_1, \dots, \bar{v}_{n-1}).$$

*Then we have a  $K$ -cellular equivalence*

$$\mathbb{Z}_{(2)}^{E\mathbb{R}(n)} \simeq_K \Sigma^{D_n\rho+(n-1)+2(1-\sigma)} E\mathbb{R}(n).$$

**Remark** We explain how classical algebra shows that cellularization should be expected in these kinds of examples.

Most basically, let  $R$  be a classical local Noetherian  $k$ -algebra  $R$  with residue field  $k$ . The  $R$ -module  $\operatorname{Hom}_k(R, k)$  plays the role of an Anderson dual, but it is not  $k$ -cellular. Its  $k$ -cellularization agrees with the injective hull  $I(k)$  of  $k$ ; see Dwyer, Greenlees and Iyengar [1, 7.1].

Dropping the  $k$ -algebra assumption, for a commutative Gorenstein local ring  $R$  with residue field  $k$  and Krull dimension  $d$ , we have  $\Gamma_{\mathfrak{m}} R \simeq \Sigma^{-d} I(k)$ . The statement

corresponding to the corrected form of Theorem 1.4 is  $R \simeq_k \Sigma^{-d} I(k)$ . The subscript  $k$  refers to the fact that the equivalence is only true after (derived) completion at the maximal ideal or after  $k$ -cellularization.

## References

- [1] **W G Dwyer, J P C Greenlees, S Iyengar**, *Duality in algebra and topology*, Adv. Math. 200 (2006) 357–402 [MR](#)
- [2] **J P C Greenlees, L Meier**, *Gorenstein duality for real spectra*, Algebr. Geom. Topol. 17 (2017) 3547–3619 [MR](#)

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Received: 28 May 2018

