Algebraic & Geometric Topology 13 (2013) 1733–1742

Milnor–Wood inequalities for products

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We prove Milnor–Wood inequalities for local products of manifolds. As a consequence, we establish the generalized Chern conjecture for products $M \times \Sigma^k$ of any manifold M and k copies of a surface Σ for k sufficiently large.

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1 Introduction

Let M be an n–dimensional topological manifold. Consider the Euler class $\varepsilon_n(\xi)$ in $H^n(M,\mathbb{R})$ and Euler number $\chi(\xi) = \langle \varepsilon_n(\xi), [M] \rangle$ of an oriented $\mathbb{R}^n$–vector bundle ξ over M. We say that the manifold M satisfies a Milnor–Wood inequality with constant c if for every *flat* oriented $\mathbb{R}^n$–vector bundle ξ over M, the inequality

$$|\chi(\xi)| \leq c \cdot |\chi(M)|$$

holds. Recall that a bundle is flat if it is induced by a representation of the fundamental group $\pi_1(M)$. We denote by

$$MW(M) \in \mathbb{R} \cup \{+\infty\}$$

the smallest such constant.

If X is a simply connected Riemannian manifold with closed quotients, we denote

$$\widetilde{MW}(X) := \sup\{MW(M) : M \text{ is a closed quotient of } X\}.$$

Milnor's seminal inequality [7] amounts to showing that the Milnor–Wood constant of the hyperbolic plane $\mathcal{H}$ is $\widetilde{MW}(\mathcal{H}) = 1/2$, and in [3], we showed that $\widetilde{MW}(\mathcal{H}^n) = 1/2^n$.

In this note we prove a product formula for the Milnor–Wood constants of general closed manifolds:

Theorem 1.1 *For any pair of compact manifolds M_1, M_2,*

$$MW(M_1 \times M_2) = MW(M_1) \cdot MW(M_2).$$

Published: 18 May 2013 DOI: 10.2140/agt.2013.13.1733

For the product formula for universal Milnor–Wood constant, we restrict to Hadamard manifolds:

Theorem 1.2 *Let X_1, X_2 be Hadamard manifolds. Then*

$$\widetilde{MW}(X_1 \times X_2) = \widetilde{MW}(X_1) \cdot \widetilde{MW}(X_2).$$

One important application of Milnor–Wood inequalities is to make progress on the generalized Chern conjecture.

Conjecture 1.3 (Generalized Chern conjecture) *Let M be a closed oriented aspherical manifold. If the tangent bundle TM of M admits a flat structure then $\chi(M) = 0$.*

This conjecture has been suggested by Milnor [7][1] and is a strong version of the famous Chern conjecture which merely predicts the vanishing of the Euler characteristic for affine manifolds, that is, for manifolds admitting a torsion-free flat connection.

As pointed out in [7], if $MW(M) < 1$ then the generalized Chern conjecture holds for M. Indeed, if $\chi(M) \neq 0$ the inequality

$$|\chi(M)| = |\chi(TM)| \leq MW(M) \cdot |\chi(M)| < |\chi(M)|$$

leads to a contradiction to the assumption that M has a flat structure.

One can use Theorem 1.1 to extend the family of manifolds satisfying the generalized Chern conjecture. For instance, we prove a stable variant of the generalized Chern conjecture:

Corollary 1.4 *For any manifold M, there exists $k_0 \geq 0$ such that the product $M \times \Sigma^k$, where Σ is a surface of genus ≥ 2, satisfies the generalized Chern conjecture for any $k \geq k_0$. If $\chi(M) = 0$, then $k_0 = 0$. If $\chi(M) \neq 0$, then one can take any $k_0 > \log_2(MW(M))$. In particular, in the latter case, the product $M \times \Sigma^k$ does not admit an affine structure.*

Remark (1) One can replace Σ^k in Corollary 1.4 by any $\mathcal{H}^k$–manifold.

(2) The corollary is somehow dual to a question of Yves Benoist [1, Section 3, page 19] asking whether for every closed manifold M there exists m such that $M \times (S^1)^m$ admits an affine structure. For example, if M is a hyperbolic manifold or a sphere, the product $M \times S^1$ admits an affine structure. On the other hand, if M admits a

[1] In [7] Milnor suggested the generalized conjecture without the assumption that M is aspherical, however Smillie [9] gave counterexamples, in any even dimension $\neq 2$, when this assumption is omitted.

quaternionic hyperbolic structure then $m = 1$ will not suffice, since the holonomy representation of $\pi_1(M)$ is superrigid in $\mathrm{Sp}(2,1)$ by Corlette's Theorem and the latter has no nontrivial 9–dimensional linear representations.

Note that since there are only finitely many isomorphism classes of oriented $\mathbb{R}^n$–bundles which admit a flat structure, it is immediate that the set

$$\{|\chi(\xi)| \mid \xi \text{ is a flat oriented } \mathbb{R}^n\text{–bundle over } M\}$$

is finite for every M. In particular, if $\chi(M) \neq 0$, there exists a finite Milnor–Wood constant $MW(M) < +\infty$. However, in general, the Milnor–Wood constant can be infinite, since the implication

$$\chi(M) = 0 \quad \Longrightarrow \quad \chi(\xi) = 0,$$

for a flat oriented $\mathbb{R}^n$–bundle ξ, does not hold in general as we will show in Section 6. Our example is inspired by Smillie's counterexample [9] of the generalized Chern conjecture for nonaspherical manifolds, and likewise this manifold is nonaspherical.

The following questions are quite natural:

(1) Does there exist a finite constant $c(n)$ depending on n only so that we have $MW(M) \leq c(n)$ for every closed aspherical n–manifold?

(2) Let X be a contractible Riemannian manifold such that there exists a closed X–manifold M with $MW(M) < \infty$. Is $\widetilde{MW}(X)$ necessarily finite?

(3) Does $\chi(M) = 0 \Rightarrow \chi(\xi) = 0$ for flat oriented $\mathbb{R}^n$–bundles ξ over aspherical manifolds M?

Acknowledgements The authors gratefully acknowledge support by the Institut Mittag-Leffler in Djursholm, Sweden. Michelle Bucher acknowledges support by the Swiss National Science Foundation grant PP00P2-128309/1. Tseachik Gelander acknowledges support by the Israel Science Foundation and the European Research Council.

2 Proportionality principles and vanishing of the Euler class of tensor products

Lemma 2.1 *Let X be a simply connected Riemannian manifold, $G = \mathrm{Isom}(M)$ and $\rho\colon G \to \mathrm{GL}_n^+(\mathbb{R})$ a representation. Then $\chi(\xi_\rho)/\mathrm{vol}(M)$, where $M = \Gamma\backslash X$ is a closed X–manifold and ξ_ρ is the flat vector bundle induced on M by ρ restricted to Γ, is a constant independent of M.*

Proof There is a canonical isomorphism $H_c^*(G) \cong H^*(\Omega^*(X)^G)$ between the continuous cohomology of G and the cohomology of the cocomplex of G–invariant differential forms $\Omega^*(X)^G$ on X equipped with its standard differential. (For a semisimple Lie group G, every G–invariant form is closed, hence one further has $H^*(\Omega^*(X)^G) \cong \Omega^*(X)^G$.) In particular, in top dimension $n = \dim(X)$, the cohomology groups are 1–dimensional, $H_c^n(G) \cong H^n(\Omega^*(X)^G)) \cong \mathbb{R}$, and contain the cohomology class given by the volume form ω_X.

Since the bundle ξ_ρ over M is induced by ρ, its Euler class $\varepsilon_n(\xi_\rho)$ is the image of $\varepsilon_n \in H_c^n(\mathrm{GL}^+(\mathbb{R}, n))$ under

$$H_c^n(\mathrm{GL}^+(\mathbb{R}, n)) \xrightarrow{\rho^*} H_c^n(G) \longrightarrow H^n(\Gamma) \cong H^n(M),$$

where the middle map is induced by the inclusion $\Gamma \hookrightarrow G$. In particular,

$$\rho^*(\varepsilon_n) = \lambda \cdot [\omega_X] \in H_c^n(G)$$

for some $\lambda \in \mathbb{R}$ independent of M. It follows that $\chi(\xi_\rho)/\mathrm{Vol}(M) = \lambda$. □

Lemma 2.2 *Let* $\rho_\otimes \colon \mathrm{GL}^+(n, \mathbb{R}) \times \mathrm{GL}^+(m, \mathbb{R}) \to \mathrm{GL}^+(nm, \mathbb{R})$ *denote the tensor representation. If* $n, m \geq 2$*, then*

$$\rho_\otimes^*(\varepsilon_{nm}) = 0 \in H_c^{nm}(\mathrm{GL}(n, \mathbb{R}) \times \mathrm{GL}(m, \mathbb{R})).$$

Proof The case $n = m = 2$ was proven in [3, Lemma 4.1], based on the simple observation that interchanging the two $\mathrm{GL}^+(2, \mathbb{R})$ factors does not change the sign of the top dimensional cohomology class in $H_c^4(\mathrm{GL}(2, \mathbb{R}) \times \mathrm{GL}(2, \mathbb{R})) \cong \mathbb{R}$, but it changes the orientation on the tensor product, and hence the sign of the Euler class in $H_c^4(\mathrm{GL}^+(4, \mathbb{R}))$.

Let us now suppose that at least one of n, m is strictly greater than 2, or equivalently, that $n + m < nm$. The Euler class is in the image of the natural map

$$H^{nm}(B\mathrm{GL}(nm, \mathbb{R})) \longrightarrow H_c^{nm}(\mathrm{GL}(nm, \mathbb{R})).$$

By naturality, we have a commutative diagram

$$\begin{array}{ccc} H^{nm}(B\mathrm{GL}^+(nm, \mathbb{R})) & \longrightarrow & H_c^{nm}(\mathrm{GL}^+(nm, \mathbb{R})) \\ \downarrow \rho_\otimes^* & & \downarrow \rho_\otimes^* \\ H^{nm}(B(\mathrm{GL}^+(n, \mathbb{R}) \times \mathrm{GL}^+(m, \mathbb{R}))) & \longrightarrow & H_c^{nm}(\mathrm{GL}^+(n, \mathbb{R}) \times \mathrm{GL}^+(m, \mathbb{R}))). \end{array}$$

Since the image of the lower horizontal arrow is contained in degree $\leq n+m$, it follows that $\rho_\otimes^*(\varepsilon_{nm}) = 0$. □

3 Representations of products

Lemma 3.1 *Let H_1, H_2 be groups and $\rho\colon H_1 \times H_2 \to \mathrm{GL}_n(\mathbb{R})$ a representation of the direct product and suppose that $\rho(H_i)$ is nonamenable for both $i = 1, 2$. Then, up to replacing the H_i by finite index subgroups, either*

- *$V = \mathbb{R}^n$ decomposes as an invariant direct sum $V = V' \oplus V''$, where the restriction $\rho|V' = \rho_1' \otimes \rho_2'$ is a nontrivial tensor representation, or*
- *$V = V_1 \oplus V_2$, where $\rho(H_i)$ is scalar on V_i.*

Proof This can be easily deduced from the proof of [3, Proposition 6.1]. □

Proposition 3.2 *Let $H = \prod_{i=1}^k H_i$ be a direct product of groups and let $\rho\colon H \to \mathrm{GL}_n^+(\mathbb{R})$ be an orientable representation, where $n = \sum_{i=1}^k m_i$. Suppose that $\rho(H_i)$ is nonamenable for every i. Then, up to replacing the H_i by finite index subgroups $H' = \prod_{i=1}^k H_i'$, either*

(1) *there exists $1 \le i_0 < k$ such that $V = \mathbb{R}^n$ decomposes nontrivially to an invariant direct sum $V = V' \oplus V''$ and the restricted representation*

$$\rho\,|_{(H_{i_0}' \times \prod_{i>i_0} H_i', V')}\colon H_{i_0}' \times \prod_{i>i_0} H_i' \longrightarrow \mathrm{GL}(V')$$

is a nontrivial tensor, or

(2) *the representation ρ' factors through*

$$\rho'\colon \prod_{i=1}^k H_i' \longrightarrow \Bigl(\prod_{i=1}^k \mathrm{GL}_{m_i'}(\mathbb{R})\Bigr)^+ \longrightarrow \mathrm{GL}_n^+(\mathbb{R}),$$

where the latter homomorphism is, up to conjugation, the canonical diagonal embedding, and $\rho'(H_i')$ restricts to a scalar representation on each $\mathrm{GL}_{m_j'}(\mathbb{R})$, for $i \neq j$.

Moreover, if all m_i are even then either $m_i' < m_i$ for some i or one can replace GL with GL^+ everywhere.

The notation $(\prod_{i=1}^k \mathrm{GL}_{m_i'}(\mathbb{R}))^+$ stands for the intersection of $\prod_{i=1}^k \mathrm{GL}_{m_i'}(\mathbb{R})$ with the positive-determinant matrices.

Proof We argue by induction on k. For $k = 2$ the alternative is immediate from Lemma 3.1. Suppose $k > 2$. If (1) does not hold, it follows from Lemma 3.1 that, up to replacing the H_i by some finite index subgroups, V decomposes invariantly to $V = V_1 \oplus V_1'$ where $\rho(H_1)$ is scalar on V_1' and $\rho(\prod_{i>1} H_i)$ is scalar on V_1. We now apply the induction hypothesis for $\prod_{i>1} H_i$ restricted to V_1'.

Finally, in case (2), since $\sum m_i = n$, either $m_i' < m_i$ for some i or equality holds everywhere. In the latter case, if all the m_i are even, given $g \in H_i$, since the restriction of $\rho(g)$ to each $V_{j\neq i}$ is scalar, it has positive determinant. We deduce that also $\rho(g)|_{V_i}$ has positive determinant. □

4 Multiplicativity of the Milnor–Wood constant for product manifolds: A proof of Theorem 1.1

Let M_1, M_2 be two arbitrary manifolds. We prove that

$$MW(M_1 \times M_2) = MW(M_1) \cdot MW(M_2).$$

First note that the inequality $MW(M_1 \times M_2) \geq MW(M_1) \cdot MW(M_2)$ is trivial. Indeed, let ξ_1, ξ_2 be flat oriented bundles over M_1 and M_2, respectively, of the right dimension such that $|\chi(\xi_i)| = MW(M_i) \cdot |\chi(M_i)|$ for $i = 1, 2$. Then $\xi_1 \times \xi_2$ is a flat bundle over $M_1 \times M_2$ with

$$|\chi(\xi_1 \times \xi_2)| = |\chi(\xi_1)||\chi(\xi_2)| = MW(M_1) \cdot MW(M_2) \cdot |\chi(M_1 \times M_2)|.$$

For the other inequality, let ξ be a flat oriented $\mathbb{R}^n$–bundle over $M_1 \times M_2$, where $n = \dim(M_1) + \dim(M_2)$. We need to show that

$$|\chi(\xi)| \leq MW(M_1) \cdot MW(M_2) \cdot |\chi(M_1 \times M_2)|\,.$$

Observe that if we replace M by a finite cover, and the bundle ξ by its pullback to the cover, then both sides of the previous inequality are multiplied by the degree of the covering.

The flat bundle ξ is induced by a representation

$$\rho\colon \pi_1(M_1 \times M_2) \cong \pi_1(M_1) \times \pi_1(M_2) \longrightarrow \mathrm{GL}_n^+(\mathbb{R}).$$

If $\rho(\pi_1(M_i))$ is amenable for $i = 1$ or 2, then $\rho^*(\varepsilon_n) = 0$ [3, Lemma 4.3] and hence $\chi(\xi) = 0$ and there is nothing to prove. Thus, we can without loss of generality suppose that, upon replacing $\pi_1(M_1 \times M_2)$ by a finite index subgroup, the representation ρ factors as in Proposition 3.2.

In case (1) of the proposition, we obtain that $\rho^*(\varepsilon_n) = 0$ by Lemma 2.2 and [3, Lemma 4.2]. In case (2) we get that ρ factors through

$$\rho\colon \pi_1(M_1)\times\pi_1(M_2) \longrightarrow \big(\mathrm{GL}_{m_1'}(\mathbb{R})\times \mathrm{GL}_{m_2'}(\mathbb{R})\big)^+ \xrightarrow{\ i\ } \mathrm{GL}_n^+(\mathbb{R}),$$

where the latter embedding i is up to conjugation the canonical embedding. Furthermore, up to replacing ρ by a representation in the same connected component of

$$\mathrm{Rep}\Big(\pi_1(M_1)\times\pi_1(M_2), \big(\mathrm{GL}_{m_1'}(\mathbb{R})\times \mathrm{GL}_{m_2'}(\mathbb{R})\big)^+\Big)$$

which will have no influence on the pullback of the Euler class, we can without loss of generality suppose that the scalar representations of $\pi_1(M_1)$ on $\mathrm{GL}_{m_2'}$ and $\pi_1(M_2)$ on $\mathrm{GL}_{m_1'}$ are trivial, so that ρ is a product representation. If m_1' or m_2' is odd, then $i^*(\varepsilon_n) = 0 \in H_c^n((\mathrm{GL}_{m_1'}(\mathbb{R})\times \mathrm{GL}_{m_2'}(\mathbb{R}))^+)$. If m_1' and m_2' are both even then Proposition 3.2 further tells us that either $m_i' < m_i$ for $i = 1$ or 2, or the image of ρ lies in $\mathrm{GL}_{m_1}^+(\mathbb{R})\times \mathrm{GL}_{m_2}^+(\mathbb{R})$. In the first case, the Euler class vanishes [3, Lemma 4.2], while in the second case, we immediately obtain the desired inequality. This finishes the proof of Theorem 1.1. □

5 Multiplicativity of the universal Milnor–Wood constant for Hadamard manifolds: A proof of Theorem 1.2

Theorem 1.2 can be reformulated as follows:

Theorem 5.1 *Let X be a Hadamard manifold with de Rham decomposition $X = \prod_{i=1}^k X_i$, then $\widetilde{MW}(X) = \prod_{i=1}^k \widetilde{MW}(X_i)$.*

Proof The inequality "$\geq$" is obvious. Let $M = \Gamma\backslash X$ be a compact X–manifold. We must show that $MW(M) \leq \prod_{i=1}^k \widetilde{MW}(X_i)$. Note that Γ is torsion-free. Let us also assume that $k \geq 2$. If M is reducible one can argue by induction using Theorem 1.1. Thus we may assume that M is irreducible. Observe that this implies that $\mathrm{Isom}(X)$ is not discrete. If Γ admits a nontrivial normal abelian subgroup then by the flat torus theorem (see [2, Chapter 7]), X admits an Euclidean factor which implies the vanishing of the Euler class. Assuming that this is not the case we apply Farb–Weinberger [4, Theorem 1.3] to deduce that X is a symmetric space of noncompact type. Thus, up to replacing M by a finite cover (equivalently, replace Γ by a finite index subgroup), we may assume that Γ lies in

$$G = \mathrm{Isom}(X)^\circ = \prod_{i=1}^k \mathrm{Isom}(X_i)^\circ = \prod_{i=1}^k G_i,$$

where the G_i are adjoint simple Lie groups without compact factors and $\Gamma \le G$ is irreducible in the sense that its projection to each factor is dense. Denote by $\widetilde{G}_i$ the universal cover of G_i, and by $\widetilde{\Gamma} \le \prod_{i=1}^k \widetilde{G}_i$ the pullback of Γ.

Let $\rho\colon \Gamma \to \mathrm{GL}_n^+(\mathbb{R})$ be a representation inducing a flat oriented vector bundle ξ over M. Up to replacing Γ by a finite index subgroup, we may suppose that $\rho(\Gamma)$ is Zariski connected. Let $S \le \mathrm{GL}_n^+(\mathbb{R})$ be the semisimple part of the Zariski closure of $\rho(\Gamma)$, and let $\rho'\colon \Gamma \to S$ be the quotient representation. By superrigidity, the map $\mathrm{Ad}\circ\rho'\colon \Gamma \to \mathrm{Ad}(S)$ extends to

$$\phi\colon \Gamma \le \prod_{i=1}^k G_i \longrightarrow \mathrm{Ad}(S)$$

(see [5], [6] and [8]). This map can be pulled back to $\widetilde{\phi}\colon \widetilde{\Gamma} \to S$. Recall also that $\prod_{i=1}^k \widetilde{G}_i$ is a central discrete extension of $\prod_{i=1}^k G_i$ and, likewise, $\widetilde{\Gamma}$ is a central extension of Γ. If

$$n_i = \dim X_i \quad \text{and} \quad n = \sum_{i=1}^k n_i$$

we deduce from Proposition 3.2 and Lemma 2.2 that either the Euler class vanishes or the image of $\widetilde{\phi}$ lies (up to decomposing the vector space $\mathbb{R}^n$ properly) in $(\prod_{i=1}^k \mathrm{GL}_{n_i})^+$.

Suppose that $\widetilde{MW}(X_i)$ is finite for all $i = 1, \dots, k$ and let M_i be closed X_i–manifolds. Let ξ' be the flat vector bundle on $\prod_{i=1}^k M_i$ coming from $\widetilde{\rho}$ reduced to $\prod_{i=1}^k M_i$, and let ξ_i' be the vector bundle on M_i induced by $\widetilde{\rho}_i$, $i = 1, \dots, k$. By Lemma 2.1, we have

$$\frac{\chi(\xi)}{\mathrm{vol}(M)} = \frac{\chi(\xi')}{\mathrm{vol}\big(\prod_{i=1}^k M_i\big)} = \prod_{i=1}^k \frac{\chi(\xi_i')}{\mathrm{vol}(M_i)} \le \prod_{i=1}^k \widetilde{MW}(X_i),$$

which finishes the proof. □

6 Example: a flat bundle with nonzero Euler number over a manifold with zero Euler characteristic

Recall that given two closed manifolds of even dimension, the Euler characteristic of connected sums behaves as

$$\chi(M_1 \sharp M_2) = \chi(M_1) + \chi(M_2) - 2.$$

The idea is to find $M = M_1 \sharp M_2$ such that M_1 admits a flat bundle with nontrivial Euler number which in turn induces such a bundle on the connected sum, and to choose then M_2 in such a way that the Euler characteristic of the connected sum vanishes. Take thus

$$M_1 = \Sigma_2 \times \Sigma_2, \quad M_2 = (S^1 \times S^3) \sharp (S^1 \times S^3) \quad \text{and} \quad M = M_1 \sharp M_2.$$

These manifolds have the following Euler characteristics:

$$\chi(M_1) = 4, \qquad \chi(M_2) = 2 \cdot \chi(S^1 \times S^3) - 2 = -2, \qquad \chi(M) = 0.$$

Let η be a flat bundle over Σ_2 with Euler number $\chi(\eta) = 1$. (Note that we know that such a bundle exists by [7].) Let $f\colon M \to M_1$ be a degree 1 map obtained by sending M_2 to a point, and consider

$$\xi = f^*(\eta \times \eta).$$

Obviously, since η is flat, so is the product $\eta \times \eta$ and its pullback by f. Moreover, the Euler number of ξ is

$$\chi(\xi) = \chi(\eta \times \eta) = 1.$$

Indeed, the Euler number of $\eta \times \eta$ is the index of a generic section of the bundle, which we can choose to be nonzero on $f(M_2)$, so that we can pull it back to a generic section of ξ which will clearly have the same index as the initial section on $\eta \times \eta$.

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Received: 14 March 2012