ment lies in the introduction from the outset of vectors and matrices. The treatment of the geometry of the triangle by vectors is very elegant, as is the application of matrices to the study of orthogonal coordinate transformations and the classification of quadrics.

T. H. GRONWALL

Methoden der Praktischen Analysis. By Fr. A. Willers. Göschens Lehrbücherei, Band 12. Berlin and Leipzig, de Gruyter, 1928. 344 pp.

This book treats numerical, graphical, and some instrumental methods of practical analysis. While the most important is the numerical, by which an approximation to any desired degree of accuracy may be reached, the author has felt the importance of treating the less precise methods because they frequently give results which are sufficiently accurate for the purpose in hand and in other cases provide easily a first approximation to more refined results by the numerical method. Lack of space has made it necessary to limit the applications in neighboring fields and the author regrets particularly that he has had to omit almost everything from mathematical statistics.

The first chapter considers the general problems involved in calculations with approximate numbers and describes the mechanical aids to computation, particular attention being given to various special types of coordinate paper. The second chapter, which is fundamental to those which follow, develops standard methods of interpolation including numerical differentiation and integration. In the third chapter we find a treatment of approximate integration and differentiation by various formulas and types of integrating machines. The fourth chapter is devoted to methods of approximating the roots of equations, including systems of linear equations and the complex roots of algebraic equations. The last paragraph gives a brief treatment of linear difference equations. Empirical formulas and curves, particularly those involving periodic functions, are described in Chapter Five. The sixth chapter explains graphical and numerical methods for obtaining approximate solutions of ordinary differential equations.

The last four chapters are independent and each is based on Chapter Two alone. This volume is to be recommended as a convenient reference book for any one confronted by a problem the solution of which requires the approximate methods listed above.

W. R. LONGLEY

Mathematische Existenz. By Oskar Becker. Untersuchungen zur Logik und Ontologie mathematischer Phänomene. Halle, Niemeyer, 1927. 370 pp.

This appears as a "Sonderdruck aus: Jahrbuch für Philosophie und phänomenologische Forschung, Band VIII," and as such has a double pagination. The work is a critical, searching study of the problem of mathematical existence in its present controversial status. It should be read in connection with the recent prior articles of Hilbert, Brouwer, Weyl and others. One is plunged in the first few pages into the present conflict concerning the foundations of mathematics as championed by Brouwer for Intuitionalism, and by Hilbert for Formalism, It is of significance that there is no present dispute between these contenders in regard to the subject of finite mathematics as distinguished from what Hilbert calls "metamathematics."

"The first point to be mentioned in this connection is the fundamental thesis of the necessity of a common establishment of logic, arithmetic (with combinatorial analysis) and the theory of aggregates, and indeed with limitation to the finite. This thesis is directed against the attempt of the 'Logistikers' (Peano, Frege, Russell, etc.) to derive arithmetic and the theory of aggregates from formal logic. That this is impossible and indeed worse than impossible since more closely viewed it is meaningless, such is the common conviction of Brouwer, Weyl and on the other side Hilbert." Thus concerning any countable sequence whose successive terms are given by an explicit rule of construction, no dispute here arises. The only features of disagreement occur for uncounted sets, or incompletely defined sequences. Brouwer insists upon the essential time element in any concept of unending sequence where no constructive rule is present. Hilbert admits that metamathematics can only be established by the use of a new order of ideas. He does not depend upon any principle of justification which requires explicitly time or human mental processes. As a consequence metamathematics can be thought of as studied purely formally, without regard to content, it being at least unnecessary in view of the other fruitful methods of approach, to follow so far the weary way of objective recognition of content.

For Brouwer, one cannot assert with regard to an unended sequence that the sequence (thought of as completed) either does or does not contain an element which has not yet been observed to occur in the sequence. The method of excluded middle, and in general reductio ad absurdum as a method of proof is not logically available in such cases. For Hilbert by means of an axiomatic approach the existence of an explicit construction reduces the subject to the realm of the finite or mathematics proper, but the logical existence of objects may be inferred whose construction remains undetermined. In particular Hilbert provides what is claimed as a proof that no contradiction can arise from metamathematical proofs of a certain sort. Hilbert 's proof itself makes free use of reductio ad absurdum, and so is unacceptable to the intuitionalists, even if they were to permit the use of the title "object" to what is known, as to its existence and properties, only by axioms.

The author may be regarded as using this conflict more or less as a text. The views of both sides are elaborated with clearness and sympathy but many trenchant comments (not all original of course) are directed at vulnerable points. Then the whole topic is studied historically, with special reference to Plato, Aristotle, Descartes, Leibniz, Kant, Cantor, Russell. No brief summary would be just to the writer.

Approximately half of the work (the second half) is devoted to the philosophical problems of valuation in terms of human activity as distinguished from ontology. This part may prove of minor interest to many mathematicians who can study the first half with profit and enjoyment.

The author settles no major issues, and champions no special cause, but he here provides an effective and discriminating presentation of some logical mathematical problems of present interest. If the style seems heavy, at least there is no hiding behind sonorous terms. Only the "logistikers" seem hope lessly outmoded. No special sympathy is offered to those who would justify arithmetic on a contentless axiomatic basis. The tone of the discussion is not adjusted to the level of anyone so naïve as to suppose that therefore the theorems of arithmetic are not assumed as logically derivable from a suitable system of postulates such as that framed by Hilbert himself.

A. A. BENNETT

Lezioni di Geometria Analitica e Proiettiva. By Annibale Comessatti. Padova, Casa editrice A. Milani, 1930. xv+462 pp.

The present volume, most of which had been used repeatedly by the author in mimeograph form previously, represents the work done in algebraic and projective geometry by the Italian university students during the first and second years. Probably no other country adheres so closely to the old Greek methods, nor guards so jealously the rich and varied traditions of its past. And in the present case, without forfeiting anything of this heritage, the book is decidedly down to date, not only in content, but also in method.

The subject is introduced as in our own elementary texts in plane analytic geometry, with numerous figures, and many examples left for the student, but presented at a much more rapid pace. Determinants are used freely from the beginning, and projective (affine) coordinate systems precede metrical ones. Properties of lines, planes, conics and quadric surfaces are derived in the first chapter of about 60 pages. This is followed by one on algebraic vector analysis in two and three dimensions, with a skillful blending of the method and results of the first chapter with those of the second.

Chapter three introduces projective geometry in the usual way, with the immediate extension to infinite points, lines and planes. After establishing the foundations synthetically, coordinates are employed: point coordinates, plane coordinates, line coordinates (Plücker), including complexes, congruences and ruled surfaces. Then follows a generous appendix on complex geometry.

A large chapter on curves and surfaces introduces the calculus and employs it to get metrical properties of plane algebraic and transcendental curves, of surfaces and of space curves, including an extensive treatment of quadric surfaces, reguli, asymptotic cones, etc.

The work is written in the peculiar limpid style of Comessatti, and is printed on thin opaque paper, making a convenient volume. One objection is that the exercises are printed in small type—pages of them at a time—and contain a wealth of valuable material that one can not afford to overlook.

This book would not be a suitable text for beginners in either analytic or projective geometry, according to the American methods of instruction, but is particularly valuable for reference and comparison after one has reached the maturity necessary to appreciate it. It is an eloquent commentary on the intense instruction given to the precocious Italian youth.

VIRGIL SNYDER