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NORMALIZERS OF p-SUBGROUPS IN FINITE GROUPS

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In a recent paper, Sims obtained some striking applications of graph theory to group theory. Using his work, Wong determined every finite primitive permutation group in which the stabilizer of a point has some orbit of length three. The techniques of Sims and Wong can be applied to other situations that occur in investigations of finite groups. In this paper we obtain some applications that we will use in studying weakly closed elements of Sylow 2-subgroups.

THEOREM 1. *Suppose P is a subgroup of a finite group G, $g \in G$, and $P \cap P^g$ is a normal subgroup of prime index p in P^g. Let n be a positive integer, and let $\tilde{G} = \langle P, P^g, \cdots, P^{g^n} \rangle$. Assume that:*

(1) *g normalizes no nonidentity normal subgroup of P, and*

(2) *$P \cap Z(\tilde{G}) = 1$.*

Then $|P| = p^t$ for some positive integer t for which $t \leqq 3n$ and $t \neq 3n-1$. Moreover, if $n = 2, p = 2$, and $t = 6$, then P contains a nonidentity normal subgroup of $\tilde{G}$.

THEOREM 2. *Suppose H is a subgroup of a finite group G, P is a Sylow 2-subgroup of H, $O_2(H) \neq 1$, and $H/O_2(H)$ is a dihedral group. Let S be a Sylow 2-subgroup of G that contains P, and let $|P| = 2^t$.*

(I) *Suppose $y \in S$-P, $y^2 \in PC(P)$, and y normalizes P. Assume y does not normalize any nonidentity normal subgroup of H contained in P. Then $2 \leqq t \leqq 4$, and $H/K(H)$ is isomorphic to $Z_2 \times Z_2$, S_4, or $S_4 \times Z_2$.*

(II) *Assume the hypothesis of* (I), *and suppose further that $N_S(O_2(H)) = P$. If $t = 2$ or $t = 3$, then S is a dihedral or a semi-dihedral group. It $t = 4$, then S is a group of order 32 generated by elements x, y, and z satisfying*

$$x^8 = y^2 = z^2 = [y, z] = 1 , \qquad [x, y] = x^6, \text{ and } [x, z] = x^4 .$$

(III) *Suppose $P < S$ and $N_S(Q) = P$ for every nonidentity normal subgroup Q of H that is contained in P. Then H satisfies the conclusions of* (I) *and* (II).

Throughout the paper, we will assume that G is a finite group. We will generally use the notation of [5] and [9]. In particular, if G denotes a permutation group on a set Ω and $\alpha \in \Omega, G_\alpha$ will denote the stabilizer of α in G. Also, Z_n, D_n, and S_n will denote the cyclic group of order n, the dihedral group of order n, and the symmetric group of degree n.

If H is an element or a subgroup of G and if $g \in G$, we let $H^g = g^{-1}Hg$. Suppose H is a subgroup of G. We let $O_2(H)$ denote the largest normal 2-subgroup of H and $K(H)$ denote the *2-regular core*, or largest normal subgroup of odd order, of H.

2. Applications of Sims' method. The proof of Theorem 1 depends on the arguments in Sims' paper [5]. The following result is essentially a restatement of his Proposition 2.6.

LEMMA 1. *Let x and g be elements of G. Put $x_i = x^{g^i} = g^{-i}xg^i$ for $i = 0, \pm 1, \cdots$, and define $H_i = < x_1, \cdots, x_i >$ for each $i \geqq 1$. Let $H_0 = 1$. Suppose that x has prime order p and that there exist positive integers t and n such that*

(1) $< H_t, g > = G$,

(2) $|H_i : H_{i-1}| = p, \quad 1 \leqq i \leqq t$, *and*

(3) *H_t contains no nonidentity normal subgroup of G and no nonidentity subgroup of $Z(H_{t+n})$.*

Then $t \leqq 3n$ and $t \neq 3n-1$. Moreover, if $n = 2$, $p = 2$, and $t = 6$, then H_t contains a nonidentity normal subgroup of H_8.

Proof. Let $s = t + 1$. Suppose $1 \leqq i \leqq t$. By (1) and (2), $G = < x_1, g > = < H_i, g >$ and $|H_i| = p^i$. By (3), g does not normalize H_i. Now the proof of Lemma 2.7 of [5] remains valid; that lemma states that H_i is Abelian whenever $1 \leqq i < (2s + 1)/3 = (2t + 3)/3$.

Assume that $t > 3n$ or that $t = 3n - 1$. Note that in the latter case, $t + n$ is odd. Let $i = (t + n + 1)/2$ if $t + n$ is odd, and $i = (t + n + 2)/2$ if $t + n$ is even. Then $1 \leqq i \leqq t$ and $2i - 1 \geq t + n$. Therefore $H_t \cap Z(H_{2i-1}) = 1$. Since

$$H_{2i-1} = < H_i, (H_i)^{g^{i-1}} > \text{ and } x_i \in H_i \cap (H_i)^{g^{i-1}} ,$$

H_i is not Abelian. By the previous paragraph, $i < (2t + 3)/3$, which yields a contradiction.

Now suppose $n = 2, p = 2$, and $t = 6$. On pages 85–86 of [5], Sims proves that

$$[x_i, x_j] = 1 \quad \text{for } |i - j| \leq 3$$

$$[x_1, x_5] = x_3 , \qquad [x_2, x_6] = x_4 , \qquad [x_3, x_7] = x_5 ,$$

and

$$[x_1, x_6] = x_3^c x_4^d , \qquad [x_2, x_7] = x_4^c x_5^d ,$$

for some integers c, d. Therefore,

$$[x_3, x_8] = [x_1, x_6]^{g^2} = x_5^c x_6^d , \qquad [x_4, x_8] = [x_2, x_6]^{g^2} = x_6 .$$

Thus $H_8 = < x_1, \cdots, x_8 >$ normalizes $< x_3, x_4, x_5, x_6 >$. This completes the proof of Lemma 1.

Suppose G is a transitive permutation group on a set Ω. Let $\alpha \in \Omega$, and let Λ be an orbit of G_α on Ω. By Propositions 4.1 and 4.2 of [5], there exist $\Lambda(\beta)$ and $\Lambda'(\beta)$ for each $\beta \in \Omega$ such that:

(1) $\Lambda(\beta)$ and $\Lambda'(\beta)$ are orbits of G_β on Ω and $\Lambda(\alpha) = \Lambda$,

(2) $\Lambda(\alpha)^g = \Lambda(\alpha^g)$ and $\Lambda'(\alpha)^g = \Lambda'(\alpha^g)$ for all $g \in G$, and

(3) $\beta \in \Lambda(\alpha)$ if and only if $\alpha \in \Lambda'(\beta)$.

Let $E = \{(\beta, \gamma) \mid \beta \in \Omega, \gamma \in \Lambda(\beta)\}$. Then (Ω, E) is called the *graph* of Λ. It is undirected if $\Lambda(\alpha) = \Lambda'(\alpha)$ and is directed if $\Lambda(\alpha) \neq \Lambda'(\alpha)$. By Proposition 4.3 of [5], G acts as a group of automorphisms of the graph of Λ and is transitive on both the points and the edges.

LEMMA 2. *Suppose $g \in G$ and $\alpha^g \in \Lambda$. Then the graph of Λ is connected if and only if $< G_\alpha, g > = G$.*

Proof. Let $H = < G_\alpha, g >$ and $\Gamma = \{\alpha^h \mid h \in H\}$. Then $\Lambda = \{\alpha^{gx} \mid x \in G_\alpha\}$. By the proof of Theorem 7.4 of [7], $\Gamma = \Omega$ if and only if $H = G$. For every $\alpha^h \in \Gamma$,

$$\Lambda(\alpha^h) = \Lambda(\alpha)^h = \{\alpha^{gxh} \mid x \in G_\alpha\} \subseteqq \Gamma .$$

Similarly,

$$\Lambda'(\alpha^h) = \Lambda'(\alpha)^h = \{\alpha^{g^{-1}xh} \mid x \in G_\alpha\} \subseteqq \Gamma .$$

Thus, if the graph of Λ is connected, then $\Gamma = \Omega$.

Conversely, assume that $H = G$. Let Φ be the connected component of α in Ω. Since G acts as a group of automorphisms on the graph of Λ, each element of G_α maps Φ onto itself. Similarly, $\Phi^g = \Phi$ because $\alpha^g \in \Phi^g \cap \Lambda(\alpha) \subseteqq \Phi^g \cap \Phi$. Thus, $\Phi = \Phi^H = \Phi^G = \Omega$. This completes the proof of Lemma 2.

Let us assume the notation of Lemma 2. Suppose $t \geqq 1$. Define a *t-arc* to be an ordered $(t+1)$-tuple of points of Ω, say, $X = (\alpha_0, \alpha_1, \cdots, \alpha_t)$, such that $\alpha_{i+1} \in \Lambda(\alpha_i)$, $0 \leqq i < t$. Any t-arc of the form $(\alpha_1, \cdots, \alpha_t, \gamma)$ is called a *successor* of X, and any t-arc of the form $(\gamma, \alpha_0, \cdots, \alpha_{t-1})$ is called a *predecessor* of X.

LEMMA 3. *Assume the hypothesis of Lemma 2. Suppose $t \geqq 1$ and the graph of Λ is connected. Let X and Y be t-arcs. Then there exists a sequence $X = X_0, \cdots, X_r = Y$ of t-arcs such that X_i is a successor of X_{i-1}, $1 \leqq i \leqq r$.*

REMARK. For $t = 1$, this is equivalent to Proposition 3.1 of [5].

Proof. Let $X = (\alpha_0, \alpha_1, \cdots, \alpha_t)$ and $Y = (\beta_0, \beta_1, \cdots, \beta_t)$. By

Proposition 3.1 of [5], there exists a finite sequence $\alpha_t = \gamma_0, \gamma_1, \cdots, \gamma_s = \beta_0$ such that $\gamma_i \in \Lambda(\gamma_{i-1})$ for $i = 1, 2, \cdots, s$. Let $r = s + t$, and let $\gamma_i = \beta_{i-s}$ for $i = s + 1, s + 2, \cdots, s + t$. Define $X_0 = X$,

$$X_i = (\alpha_i, \alpha_{i+1}, \cdots, \alpha_t, \gamma_1, \cdots, \gamma_i) \qquad (1 \leqq i \leqq t - 1) ,$$

and

$$X_{t+i} = (\gamma_i, \gamma_{i+1}, \cdots, \gamma_{i+t}) \qquad (0 \leqq i \leqq s) .$$

Then the sequence $X_0, X_1, \cdots, X_r$ satisfies the conclusion of the lemma.

Proof of Theorem 1. Clearly, we may assume that $G = < P, g >$. By condition (1) of the theorem, G is faithfully represented as a permutation group on the set of all cosets Px, $x \in G$, and P is the stabilizer in G of the point $\alpha = P1$. Let $h = g^{-1}$ and $\beta = \alpha^h$. Then

$$G_{\alpha\beta} = G_\alpha \cap G_\beta = G_\alpha \cap (G_\alpha)^{g^{-1}} = P \cap P^{g^{-1}} .$$

Since $P \cap P^g$ is normal and of index p in P^g, $G_{\alpha\beta}$ is normal and of index p in P. Let Λ be the orbit of P on Ω that contains β. Then $|\Lambda| = |\beta^P| = |P: P_\beta| = |G_\alpha: G_{\alpha\beta}| = p$. Since P_β is normal in P and P is transitive on Λ, P_β fixes every point of Λ. Thus P induces a regular group of permutations on Λ. We define $\Lambda(\gamma)$, $\gamma \in \Omega$, as above.

Suppose $t \geqq 1$. We define t-arcs as above. The arguments of §5 of [5] now give us the following results:

For some t_0, G is transitive on the set of all t_0-arcs but not on the set of all $(t_0 + 1)$-arcs (Lemma 5.7).

G acts regularly on the set of all t_0-arcs, and $|P| = |G_\alpha| = p^{t_0}$ (Lemma 5.12; see Lemma 3 above).

For any t_0-arc $X = (\alpha_0, \alpha_1, \cdots, \alpha_{t_0})$, the stabilizer of $\alpha_0, \alpha_1, \cdots,$ and α_{t_0-1} is generated by a single element x of order p. If X^k is a predecessor of X, then $< x^k, \cdots, x^{k^i} >$ has order p^i for $1 \leqq i \leqq t_0$. Moreover,

$$< x^k, \cdots, x^{k^{t_0}} > = P \qquad \text{(Lemma 5.13).}$$

Let $t = t_0$ and let X be the $(t+1)$-tuple given by $X = (\alpha, \alpha^h, \cdots, \alpha^{h^t})$. Since $\alpha^h = \beta \in \Lambda$, we have $\alpha^{h^{i+1}} = \beta^{h^i} \in \Lambda(\alpha)^{h_i} = \Lambda(\alpha^{h^i})$ for all i. Hence X is a t-arc and $X^{h^{-1}}$ is a predecessor of X. Let $k = g = h^{-1}$, and take x as in the last mentioned result above. Put $x_i = x^{g^i}$ for all $i \geqq 1$. Then

$$<x_1, \cdots x_t> = P \text{ and } < x_1, x_2, \cdots, x_{t+n} > = < P, P^g, \cdots, P^{g^n} > = \tilde{G} .$$

Now we may apply Lemma 1 to prove Theorem 1.

COROLLARY 1. *Suppose P and H are subgroups of G, $y \in N(P)$,* *and* $a \in H$. *Assume that* $H = < P, P^a >$ *and that* $P \cap P^a$ *is a normal subgroup of prime index* p *in* P. *Let* n *be a positive integer, and let* $\tilde{G} = < H, H^y, \cdots, H^{y^{n-1}} >$. *Assume that:*

(1) y *normalizes no nonidentity normal subgroup of* H *contained in* P, *and*

(2) $P \cap Z(\tilde{G}) = 1$.

Then $|P| = p^t$ *for some positive integer* t *for which* $t \leqq 3n$ *and* $t \neq 3n - 1$. *Moreover, if* $n = 2$, $p = 2$, *and* $t = 6$, *then* P *contains a nonidentity normal subgroup of* $\tilde{G}$.

Proof. We merely verify the hypothesis of Theorem 1 for $g = ya$. Now, $P \cap P^g = P \cap P^a$ and $< P, P^g > = < P, P^a > = H$. Let $H_i = < H, H^g, \cdots, H^{g^i} >$ for all $i \geqq 0$. Since

$$H_{i+1} = < H_i, (H_i)^g > = < H_i, (H_i)^{ya} > = < H_i, H_i^y > ,$$

we obtain $H_i = < H, H^y, \cdots, H^{y^i} >$ by induction. Therefore,

$$\tilde{G} = H_{n-1} = < H, H^g, \cdots, H^{g^{n-1}} > = < P, P^g, \cdots, P^{g^n} > .$$

Now Theorem 1 applies.

3. Applications of Wong's method.

To obtain Theorem 2 from Theorem 1, we use the methods of Wong's paper [9] and some known results about 2-groups.

LEMMA 4. *Suppose* S *is a Sylow* 2*-subgroup of* G *and* $P < S$. *Assume that* P *is a Klein four-group and that* $C_S(P) = P$. *Then* S *is a dihedral or a semi-dihedral group.*

Proof. Clearly, $Z(S) < P$, and $P = Z(S) \times < t >$ for some $t \in P$. Now $C_S(t) = C_S(P) = P$. By a result of Suzuki (Lemma 4, pp. 262–263 of [6]), S must be dihedral or semi-dihedral.

LEMMA 5. *Suppose a Sylow* 2*-subgroup* S *of* G *has order* 32 *and is generated by elements* x, y, *and* z *satisfying*

$$x^8 = y^2 = z^2 = [y, z] = 1 , \qquad [x, y] = z , \qquad [x, z] = x^4 .$$

Then G *has a normal* 2*-complement.*

Proof. For every subgroup H of G, let H^2 be the subgroup of H generated by the elements $h^2, h \in H$. Since every group of exponent one or two is Abelian, H/H^2 is Abelian.

In page 244 of [9], it is proved that G has a normal subgroup G_1 of index two such that

$$G_1 \cap S = < x^2, y, z > = < x^2, y > \times < z > \cong D_8 \times Z_2 .$$

Let $G_2 = (G_1)^2$. By a result of Wielandt (Lemma 5(a) of [2]), $z \notin G_2$. Now, G_2 is characteristic in G_1 and therefore normal in G. Since $[x, y] = z$, G/G_2 is a non-Abelian 2-group. Thus G/G_2 has at least two generators, and

$$| G/G^2 \; = | (G/G_2)/(G/G_2)^2 | \geqq 4 .$$

However, $S^2 \leqq S \cap G^2$ and $| S/S^2 | = 4$. Therefore,

$$G^2 \cap S = S^2 = < x^4, z > \cong Z_4 \times Z_2 .$$

Hence G^2 has a Sylow 2-subgroup, S^2, that is contained in the center of its normalizer. By a result of Burnside ([4], p. 203), G^2 has a normal 2-complement. This complement must be a normal 2-complement for G.

Proof of Theorem 2. (I) Since P is a Sylow 2-subgroup of H, $O_2(H) \subsetneqq P$. As $O_2(H/O_2(H)) = 1$, $| H/O_2(H) | = 2m$ for some odd number m. Therefore, $H/O_2(H)$ is generated by two conjugates of $P/O_2(H)$. Take $a \in H$ such that $H = < P, P^a >$. Then $P \cap P^a = O_2(H)$.

Let $\tilde{G} = < H, H^y >$, and let Q be the largest normal subgroup of $\tilde{G}$ contained in P. Take $b \in P$ and $c \in C(P)$ such that $y^2 = bc$. Then $H^{y^2} = H^{bc} = H^c$. Therefore,

$$H^c = H^{y^2} \leqq \tilde{G}^y \leqq N(Q^y), \text{ and } H \leqq N((Q^y)^{c^{-1}}) = N(Q^y) .$$

Hence $N(Q^y) \geqq < \tilde{G}^y, H > \geqq < H^y, H > = \tilde{G}$. Since $| Q^y | = | Q |$, $Q^y = Q$. By (I), $Q = 1$. Thus $P \cap Z(\tilde{G}) = 1$. In Corollary 1 we let $p = n = 2$ and obtain $2 \leqq t \leqq 4$. The proof of Lemma 5 of [9] shows that P is isomorphic to $Z_2 \times Z_2$, D_8, or $D_8 \times Z_2$.

Let M be a cyclic subgroup of order $\frac{1}{2} | H/O_2(H) |$ in H, and let $N = O_2(H)$. Then MN has index two in H and $C_{MN}(N) = Z(N) \times C_M(N)$, so $K(H) = C_M(N)$. Thus $M/K(H)$ is isomorphic to a group of automorphisms of N. If $t = 2$, then $| N | = 2, M = K(H)$, and $H/K(H) \cong P \cong Z_2 \times Z_2$.

Suppose $t > 2$. Then P is non-Abelian and $P' \leqq N$. Since P and y normalize P' and $H = PM$, M does not normalize P'. Therefore, $M/K(H) \neq 1$. If $t = 3$, then $| N | = 4$ and $| M/K(H) | = 3$; therefore $| H/K(H)N | = 6$ and $H/K(H) \cong S_4$. Suppose $t = 4$. Then $| N | = 8$ and the automorphism group of N is not a 2-group. Since $D_8 \times C_2$ has no quaternion subgroups, N must be an elementary Abelian group. The automorphism group of N contains a dihedral group of order $2m'$, with m' odd, only if $m' = 3$. Hence $| H/K(H)N | = 6$. The proof of Lemma 6 of [9] shows that $H/K(H) \cong S_4 \times Z_2$.

(II) Let $T = N_S(P)$. Then $P < T$. If $t = 2$, then $P = N_S(N) = C_S(N)$, and therefore $P = C_S(P)$. If $t=3$, then $N \cong Z_2 \times Z_2$ and $C_S(N) = N_S(N) \cap C_S(N) = C_P(N) = N$. By Lemma 4, S is a dihedral or semi-dihedral group in each of these cases.

Suppose $t = 4$. From (I), H does not have a normal 2-complement; hence, neither does G. Furthermore, $P \cong D_8 \times Z_2$ and $N \cong Z_2 \times Z_2 \times Z_2$. It follows that N has only two images under the automorphism group of P. Since $N_S(N) = P, |T/P| = 2$. Thus $T = < P, y >$. If $U \leqq S$ and $P < U$, then $P < N_U(P)$ and consequently $y \in T \leqq U$. Therefore, by hypothesis, $N_S(P_0) = P$ whenever $1 < P_0 < P$ and P_0 is a normal subgroup of H. Now by Lemma 8 of [9] and by Lemma 5, S has the desired form.

(III) In this case, there exists $y \in N_S(P) - P$ such that $y^2 \in P$, so the results of (I) and (II) may be applied. This completes the proof of Theorem 2.

COROLLARY 2. *Assume the hypothesis of part* (II) *of Theorem* 2. *If* $t = 2$ *or* $t = 3$, *then* G *satisfies one of the following conditions*:

(i) *G has a normal* 2-*complement.*

(ii) *G has a normal subgroup G_0 of index two, and G_0 has no normal subgroup of index two and has a dihedral Sylow* 2-*subgroup.*

(iii) *G has a normal subgroup G_0 of index two, and G_0 has no normal subgroup of index two and has a generalized quaternion Sylow* 2-*subgroup.* (*In this case, S must be a semi-dihedral group.*)

(iv) *G has no normal subgroup of index two, and the elements of order two in G are all conjugate in G.*

If $t = 2$ *and G satisfies* (i) *or* (iii), *then* $C_S(K(G)) = 1$. *If* $t = 3$, *then G cannot satisfy* (i) *or* (iii). *If* $t = 4$, *G satisfies one of the following conditions*:

(v) *G has a normal subgroup G_1 of index two, and G_1 has no normal subgroup of index two and has a semi-dihedral Sylow* 2-*subgroup.*

(vi) *G has a normal subgroup G_2 of index four, and G_2 has no normal subgroup of index two and has a dihedral Sylow* 2-*subgroup.*

Proof. If $t = 2$ or $t = 3$, then S is dihedral or semi-dihedral, by Theorem 2. A dihedral group has no generalized quaternion subgroups. Therefore, G satisfies one of the conditions (i) through (iv), by Lemma 8 of [3] (for S dihedral) and by Lemma 1 and Theorem 2 of [8] (for S semi-dihedral).

Let $K = K(G)$. Suppose $t = 2$, G satisfies (i) or (iii), and $C_S(K) \neq 1$. Then $P = N \times Z(S)$ and $Z(S) \nleqq Z(H)$. Since $|Z(S)| = 2$ and $C_S(K)$ is normal in S, $Z(S) \subseteqq C_S(K)$. However, $Z(S)K/K \leqq Z(G/K)$. (This requires the Brauer-Suzuki Theorem [1] if (iii) holds.) Therefore,

$Z(S)K = Z(S) \times K$, so $Z(S)$ is the unique Sylow 2-subgroup of $Z(S)K$. As $Z(S)K$ is normal in G, $Z(S)$ is normal in G. Since $|Z(S)| = 2$, $Z(S) \leqq Z(G)$. This is impossible, since $Z(S) \not\leq Z(H)$.

Suppose $t = 3$. Since H does not have a normal 2-complement, (i) is impossible. Since H' contains the four-group N, (iii) is impossible.

Suppose $t = 4$. The structure of S is given in Theorem 2. The argument for Case (V) of ([9], pp. 244–245) proves that G has a normal subgroup G_1 of index two for which $S \cap G_1$ is semi-dihedral. Now the structure of G_1 is given by one of the conditions (i) through (iv). Since H does not have a normal 2-complement, neither does G_1. Suppose G_1 has a normal subgroup G_2 of index two. Then G_2 is unique and $G_2 \cap S$ is a dihedral or a quaternion group. Therefore, G_2 is normal in G. Since $|G/G_2| = 4, G' \leqq G_2$. As $H' \cap N$ is a four-group, $G_2 \cap S$ is not a quaternion group. So $G_2 \cap S$ is a dihedral group of order eight. Now $G_2 \cap S \leqq N_S(H' \cap N) = P$. This completes the proof of Corollary 2.

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