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GROUPS OF ISOMETRIES OF A TREE AND THE KUNZE-STEIN PHENOMENON

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In this paper we prove that every group of isometries of a homogeneous or semihomogeneous tree which acts transitively on the boundary of the tree is a Kunze-Stein group. From this, we deduce a weak Kunze-Stein property for groups acting simply transitively on a tree (in particular free groups on finitely many generators).

1. Introduction. Let G be a locally compact group, then G is said to satisfy the "Kunze-Stein property" or sometimes G is called a "Kunze-Stein group" if $L^p(G) * L^2(G) \subset L^2(G)$ for every $1 < p < 2$.

This property was discovered by R. A. Kunze and E. M. Stein for the group $\mathrm{SL}_2(\mathbf{R})$ **[15]**. Later the same property was proved for every connected semisimple Lie group with finite center by M. Cowling **[6]**. In this paper we prove that every locally compact group of isometries of a homogeneous or semihomogeneous tree has the Kunze-Stein property provided that G acts transitively on the boundary of the tree. The proof of our Theorem is based on M. Cowling's proof of the Kunze-Stein phenomenon for $\mathrm{SL}_2(\mathbf{R})$ **[6]**. A weaker property is deduced for discrete groups acting simply transitively on the tree but not on the tree boundary.

It is known that the group $\mathrm{SL}_2(\kappa)$, where κ is a local field, may be realized as a closed subgroup of the group of all isometries of a homogeneous tree in such a way that $\mathrm{SL}_2(\kappa)$ acts transitively on the boundary **[17]**. In particular our result implies that $\mathrm{SL}_2(\kappa)$ is a Kunze-Stein group for every local field. This was proved by Gulizia **[13]** for a local field κ such that the finite residue class field associated with κ is not of characteristic 2.

We follow the terminology and definitions of **[6]**. In particular $A(G)$ is the Fourier algebra of G as defined in [7]; $C_{00}(G)$ denotes the space of continuous functions with compact support and $L^p(G), 1 \leq p \leq \infty$, the usual L^p-space with respect to a fixed left Haar measure. As observed in **[6]**, a locally compact group G is a Kunze-Stein group if and only if $A(G) \subset L^q(G)$ for every $q > 2$. We will also use the theory of representations for groups acting on a tree developed by P. Cartier

[3], A. Figà-Talamanca and M. A. Picardello [11, 12]. A convenient reference is [12]. In fact the results we quote and use from [12] are all valid with essentially the same proof when a discrete group acting simply transitively on a tree replaces the free group [1].

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2. Notations. We shall give a concise description of the tree and of the group of isometries. We refer the reader to [3, 17, 18] for undefined notions and terminology. Let X be a homogeneous tree of order r; the distance $d(x, y)$ is defined as the length of the unique geodesic $[x, y]$ connecting x to y. Let $\mathrm{Aut}(X)$ be the group of all isometries of X. We assume also $r \geq 3$ (otherwise, for $r = 2$, $\mathrm{Aut}(X)$ is amenable and noncompact, hence it is not a Kunze-Stein group). $\mathrm{Aut}(X)$ is a locally compact separable group and the stability subgroup K of a vertex of X is compact and open in $\mathrm{Aut}(X)$. A subgroup Γ of $\mathrm{Aut}(X)$ is called simply transitive if it acts transitively on the vertices and $\Gamma \cap K = \{1\}$. In other words, Γ acts simply transitively on X iff the map $\gamma \in \Gamma \to \gamma(x_0) \in X$ is bijective for a fixed vertex x_0 in X. It is known that every such group is isomorphic to the free product of t copies of the integers and s copies of the group of order 2 with $2t + s = r$ [1, 4]. Since K is open, Γ is discrete in $\mathrm{Aut}(X)$. Moreover $\Gamma \cdot K = \mathrm{Aut}(X)$ and Γ is a lattice. As usual, let $\langle f, h\rangle = \int f(g)h(g)\,dg$.

Let Ω be the boundary of the tree, that is the set of equivalence classes of sequences of distinct vertices $\{s_n : n = 0, 1, 2, \dots\}$ such that $d(s_i, s_{i+1}) = 1$ for every $i = 0, 1, 2, \dots$; two such sequences are said to be equivalent if they have infinitely many common vertices.

Ω is a compact metric space; if $x_0 \in X$ and $\omega_0 \in \Omega$ there exists a unique sequence of distinct vertices $\{s_n\}$ in the class ω_0 such that $s_0 = x_0$. In this way, Ω can be regarded as the set of infinite sequences starting from a fixed vertex x_0 in X. There exists a unique probability measure ν on Ω, $\mathrm{Aut}(X)$-quasi invariant and K-invariant. Let $P(g, \omega)$ be the Poisson kernel, that is, for $g \in \mathrm{Aut}(X)$ and $\omega \in \Omega$, $P(g, \omega) = d\nu_g/d\nu(\omega)$, with $\nu_g(\omega) = \nu(g^{-1}\omega)$.

For every complex number z, we define the following representation of $\mathrm{Aut}(X)$:

$$[\pi_z(g)f](\omega) = P^z(g, \omega)f(g^{-1}\omega).$$

It is known that, for $t \in \mathbf{R}$, $\pi_{1/2+it}$ are unitary irreducible representations on $L^2(\Omega)$; in fact even the restrictions to Γ are irreducible [**12**, pg. 76; **1**].

For a fixed vertex x_0 in X, let $X^+ = \{x \in X : d(x, x_0) \text{ is even }\}$ and $X^- = X \backslash X^+$. The partition X^+, X^- is independent of the choice of x_0. If G is a closed unimodular subgroup of $\mathrm{Aut}(X)$ acting transitively on X^+ but not on the tree, then the representations $\pi_{1/2+it}|_G$ are irreducible for $t \neq (2m+1)\pi/2lg(r-1)$, $m \in Z$ [**2**, pg. 39, pg. 62]. Let J be the interval $[0, \pi/lg(r-1)]$ and $c(z)$ the following complex function:

$$c(z) = [(r-1)^{2-2z} - 1]/[(r-1)^{1-2z} - 1].$$

Finally, let dm be the following measure:

$$dm(t) = [(r-1)lg(r-1)/4\pi r|c(\tfrac{1}{2}+it)|^2]\,dt.$$

3. The results. Let G be a closed noncompact subgroup of $\mathrm{Aut}(X)$ acting transitively on Ω, and $K_0 = G \cap K$. Since K_0 is compact open in G we can assume that its measure is one.

PROPOSITION 1. *K_0 acts transitively on Ω.*

Proof. Since G/K_0 is countable, Baire's theorem implies that every orbit of K_0 on Ω is open. By [**17**, Prop. 3.4], there exist $g \in G$, a sequence $\{s_n\} \subset X$, $n \in Z$ and $i_0 \in Z$ $i_0 \neq 0$, such that $d(s_n, s_{n+1}) = 1$ and $g(s_n) = s_{n+i_0}$ for every $n \in Z$. In this proof we realize Ω as the set of all infinite sequences $\{t_n\}$ issued from $t_0 = s_0$. Therefore the sets: $E(x) = \{\{t_n\} \in \Omega : t_j = x\}$ with $x \in X$ and $d(s_0, x) = j$ form a basis for the topology of Ω. Let $\omega_1 = \{s_0, s_1, \dots\}$ and $\omega_2 = \{s_0, s_{-1}, s_{-2}, \dots\}$.

Since $K_0\omega_1$ and $K_0\omega_2$ are open, it follows that there exists $j > 0$ such that $E(s_j) \subset K_0\omega_1$ and $E(s_{-j}) \subset K_0\omega_2$. Using the automorphism g, it is not hard to show that K_0 acts transitively on $\complement E(s_{-1})$ and $\complement E(s_1)$, respectively. Obviously, $\complement E(s_{-1}) \cap \complement E(s_1) \neq \varnothing$ and $\complement E(s_{-1}) \cup \complement E(s_1) = \Omega$. This means that K_0 acts transitively on Ω.

PROPOSITION 2. *Let G be a closed noncompact subgroup of* $\mathrm{Aut}(X)$ *acting transitively on Ω. Then either G acts transitively on the vertices of X, or G has two orbits X^+ and X^-.*

Proof. By Proposition 1, K_0 acts transitively on Ω, that is, K_0 acts transitively on the set $S_n^{s_0} = \{y \in X : d(s_0, y) = n\}$ for every $n \geq 0$. Moreover for every $g \in G$, gK_0g^{-1} acts transitively on S_n^x for every $n \geq 0$ and $g(s_0) = x$. In particular for every $x \in G(s_0)$, $G(s_0)$ is an infinite union of sets S_n^x. This implies that if $x, y \in G(s_0)$ $d(x, y) = m$, then $S_m^x \cup S_m^y \subset G(s_0)$. Therefore $S_m^x \subset G(s_0)$ implies that:

$$\bigcup_{j=0}^{+\infty} S_{jm}^x \subset G(s_0).$$

If $G(s_0)$ contains vertices x and y with $d(x, y) = 1$, then $G(s_0) = X$ and G is transitive on X. Suppose now $G(s_0) \neq X$; thus $G(s_0) \subset X^+$. Let $t = \min\{m > 0 : S_m^{s_0} \subset G(s_0)\}$. It follows that $G(s_0) \cap S_m^{s_0} = \varnothing$ for $0 < m < 2t$ $m \neq t$ and $\bigcup_{j=0}^{+\infty} S_{jt}^{s_0} \subset G(s_0)$. Let $x \in S_t^{s_0}$ and $[s_0, x] = \{s_0, x_1, x_2, \ldots, x_{t-1}, x\}$ the geodesic connecting s_0 to x; we can choose $y \in X$ in such a way that $d(y, x) = t$, $d(y, s_0) = 2t - 2$ and $[x, s_0] \cap [x, y] = [x, x_{t-1}] = \{x, x_{t-1}\}$. Since $d(x, y) = t$, $y \in G(s_0)$ but $y \in S_{2t-2}^{s_0}$ so that $S_{2t-2}^{s_0} \subset G(s_0)$. This implies that $2t - 2 = t$, that is, $t = 2$ and $G(s_0) = X^+$. Similarly, we can prove that $G(s_1) = X^-$, with $d(s_0, s_1) = 1$.

The aim of this note is to prove the following Theorem.

THEOREM 1. *Every closed subgroup G of* Aut(X) *acting transitively on Ω is a Kunze-Stein group.*

It is enough to prove the Theorem for noncompact groups.

First, we observe that:

$$\int_J \|\pi_{1/2+it}|_G(u)\|_{HS}^2 \, dm(t) \leq \|u\|_2^2 \quad \text{for every } u \text{ in } C_{00}(G).$$

Indeed (G, K_0) is a Gelfand pair because K_0 acts transitively on Ω and $g^{-1} \in K_0 g K_0$ for every g in G [**9**, Prop. 1.2]. The representations $\pi_{1/2+it}|_G$ are irreducible iff **1** (the function identically one on Ω) is a cyclic vector. By Proposition 2, we have two possibilities: if G is transitive on X, then the representations $\pi_{1/2+it}|_G$ are irreducible for every $t \in J$ [**12**, pg. 76; **1**]; otherwise for $t \in J$, $t \neq \pi/2lg(r-1)$ [**2**, pg. 39, pg. 62].

Since, for Gelfand pairs, the Plancherel measure on the irreducible unitary representations of G having a K_0-fixed vector depends only on the right K_0-invariant functions [**9**, Th. 4.2; **16**, pg. 65], to prove the inequality, it is enough to prove that

$$\int_J \|\pi_{1/2+it}|_G(u)\|_{HS}^2 \, dm(t) = \|u\|_2^2$$

for every right K_0-invariant function u in $C_{00}(G)$. To show this, let T be the following projection on $L^2(\Omega)$: $Tf = [\int_\Omega f(\omega)\, d\nu(\omega)]\mathbf{1}$ for

$f \in L^2(\Omega)$. We have $T = \int_{K_0} \pi_{1/2+it}(k)\,dk$ (recall that K_0 is transitive on Ω). Let $\mathrm{Aut}(X) = \Gamma K$; every function u right K_0-invariant on G corresponds to a function $\tilde{u}$ on Γ in such a way that $\|u\|_2 = \|\tilde{u}\|_2$ and $\pi_{1/2+it}|_G(u) = [\pi_{1/2+it}|_\Gamma(\tilde{u})]T$.

Therefore $\|\pi_{1/2+it}|_G(u)\|_{HS} = \|\pi_{1/2+it}|_\Gamma(\tilde{u})\mathbf{1}\|_{L^2(\Omega)}$; hence the equality follows from **[12**, pg. 86; **1]**. The proof of Theorem 1 is based on the following two Lemmas.

In the next Lemma, we denote by G a locally compact group and by $L_1^\infty(G)$ the space of all functions f in $L^\infty(G)$ such that $\|f\|_\infty \leq 1$; we assume ϕ to be a complex continuous function on the strip $S = [\alpha, \beta] \times \mathbf{R}$ with $0 < \alpha < \frac{1}{2} < \beta < 1$, analytic on $S^0 = (\alpha, \beta) \times \mathbf{R}$ and such that (1) ϕ is bounded on S; (2) $|\phi(x+it)| \geq h(x) > 0$ for every $t \in \mathbf{R}$ and $\alpha \leq x \leq \beta$, $x \neq \frac{1}{2}$. With these notations, we have:

LEMMA 1 (M. Cowling **[6]**). *Let $F\colon S \to L_1^\infty(G)$ be a continuous map, analytic on S^0 (i.e. $\langle F_z, u\rangle$ is an analytic function for every u in $C_{00}(G)$). If there exists a positive constant c such that*

$$\int_{\mathbf{R}} |\langle F_{1/2+it}, u\rangle|^2 |\phi(\tfrac{1}{2}+it)|\,dt \leq c\|u\|_2^2 \quad \textit{for every } u \textit{ in } C_{00}(G),$$

then the function $F_{1/2}$ is in $L^q(G)$ for every $q > 2$.

Proof. This Lemma is obtained from Lemma 2.1 of **[6**, pg. 215**]** where $S = [\alpha, \beta] \times \mathbf{R}$, $q = q' = 2$, $X = G$ and X_0 is a singleton, observing that the function $(z/z-2)^n$ could be replaced with a general analytic function ϕ with the properties (1) and (2).

LEMMA 2. *The coefficients of the quasi-regular representation on Ω, that is the functions:*

$$\langle \pi_{1/2}(g)\xi, \eta\rangle = \int_\Omega P^{1/2}(g,\omega)\xi(g^{-1}\omega)\overline{\eta(\omega)}\,d\nu(\omega) \qquad \textit{for } \xi, \eta \textit{ in } L^2(\Omega) \textit{ and } g \textit{ in } G$$

are in $L^q(G)$ for every $q > 2$.

Proof. Since $|\langle \pi_{1/2}(g)\xi, \eta\rangle| \leq \langle \pi_{1/2}(g)|\xi|, |\eta|\rangle$ it is enough to prove the Lemma for $\xi \geq 0$, $\eta \geq 0$ and $\|\xi\|_2 = \|\eta\|_2 = 1$. Define $\xi_z = \xi^{2z}$ and $\eta_z = \eta^{2-2z}$ for $\xi(\omega) \neq 0 \neq \eta(\omega)$, $\xi_z(\omega) = 0$ for $\xi(\omega) = 0$; similarly $\eta_z(\omega) = 0$ for $\eta(\omega) = 0$. In particular $\xi_{1/2} = \xi$ and $\eta_{1/2} = \eta$. Let $z = \delta + it \in S$ and $p = 1/\delta > 1$, $q = p/(p-1) = 1/(1-\delta)$ the conjugate index of p; it is easy to see that:

(1) $\xi_z \in L^p(\Omega)$, $\|\xi_z\|_p = 1$.

(2) $\eta_z \in L^q(\Omega)$, $\|\eta_z\|_q = 1$.

(3) $\|\pi_z(g)u\|_p = \|u\|_p$ for every u in $L^p(\Omega)$ and g in G.

Let $\psi(z) = \exp(z^2 - 1)$; $|\psi(z)| \leq 1$ on S and the map $F_z = \psi(z)\langle\pi_z(\cdot)\xi_z, \eta_z\rangle$ is a continuous map on S into $L_1^\infty(G)$, analytic on S^0. Since $F_{1/2} = \exp(-\frac{3}{4})\langle\pi_{1/2}(\cdot)\xi, \eta\rangle$, to prove the Lemma, it suffices to show that:

$$\int_{\mathbf{R}} |\langle F_{1/2+it}, u\rangle|^2 |\phi(\tfrac{1}{2} + it)|\, dt \leq c\|u\|_2^2 \quad \text{for every } u \text{ in } C_{00}(G)$$

and some analytic function ϕ.

Let $\phi(z) = (r-1)lg(r-1)/[4\pi r c(z)c(1-z)]$ where $c(z)$ is the function defined in the preliminaries. $\phi(z) = \phi(z + \pi i/lg(r-1))$ and so ϕ is bounded. Since $\phi(z) \neq 0$ for $\operatorname{Re} z \neq \frac{1}{2}$, it follows that: $|\phi(x+it)| \geq \min\{|\phi(x+it)|: t \in \mathbf{R}\} > 0$, for every $x \neq \frac{1}{2}$, $\alpha \leq x \leq \beta$. We have $|\phi(\frac{1}{2} + it)|\, dt = dm(t)$. Let J_k be the interval

$$J_k = [k\pi/lg(r-1), (k+1)\pi/lg(r-1)] \quad \text{for } k \in \mathbf{Z};$$

therefore $J_0 = J$. The functions $\|\pi_{1/2+it}(u)\|_{HS}$ and $dm(t)$ are periodic; hence, for every $k \in \mathbf{Z}$:

$$\int_{J_k} \|\pi_{1/2+it}(u)\|_{HS}^2\, dm(t) = \int_J \|\pi_{1/2+it}(u)\|_{HS}^2\, dm(t).$$

Let h_k be the maximum of the function

$$|\psi(\tfrac{1}{2} + it)|^2 = \exp(-3/2 - 2t^2) \text{ on } J_k \quad \text{and} \quad \sum_{-\infty}^{+\infty}{}_k\, h_k = c < +\infty.$$

Finally, we have:

$$\begin{aligned}
&\int_{\mathbf{R}} |\langle F_{1/2+it}, u\rangle|^2 |\phi(\tfrac{1}{2} + it)|\, dt \\
&= \sum_{-\infty}^{+\infty}{}_k \int_{J_k} |\psi(\tfrac{1}{2} + it)|^2 |\langle\pi_{1/2+it}(u)\xi_{1/2+it}, \eta_{1/2+it}\rangle|^2\, dm(t) \\
&\leq \sum_{-\infty}^{+\infty}{}_k\, h_k \int_{J_k} \|\pi_{1/2+it}(u)\|_{HS}^2\, dm(t) = c\int_J \|\pi_{1/2+it}(u)\|_{HS}^2\, dm(t) \\
&\leq c\|u\|_2^2,
\end{aligned}$$

(recall that $\|\xi_{1/2+it}\|_2 = \|\eta_{1/2+it}\|_2 = 1$).

Proof of Theorem 1. If G acts transitively on Ω, then $\Omega \simeq G/G_0$ where G_0 is the stability subgroup of a fixed point ω_0 in Ω. By the "principe de majoration" of C. Herz [**14**], for every f in $A(G)$ there exists a coefficient of $\pi_{1/2}$ such that: $|f(g)| \leq \langle \pi_{1/2}(g)\xi, \eta\rangle$ for every g in G. Hence, from Lemma 2, $A(G) \subset L^q(G)$ for every $q > 2$ and G is a Kunze-Stein group.

REMARK. We shall say that a vertex v of a tree is of homogeneity l if v belongs to exactly l edges. Let $X_{l,q}$ be a semihomogeneous tree, that is, a tree such that every vertex is of homogeneity l or q and two adjacent vertices are of homogeneity l and q, respectively. We suppose $l \neq q$, otherwise X is a homogeneous tree. Let S_l and S_q be the subsets of vertices of homogeneity l and q, respectively. Theorem 1 is true for semihomogeneous trees, with the same proof.

Indeed, if G is a closed noncompact subgroup of $\mathrm{Aut}(X_{l,q})$ acting transitively on the boundary of $X_{l,q}$, then $G \cap K_{v_0}$ acts transitively on the boundary for every vertex v_0. Moreover $G(v_0) = S_l$ and $G(w_0) = S_q$ for every $v_0 \in S_l$ and $w_0 \in S_q$. Hence, without loss of generality, we can suppose that $l < q$. The representations $\pi_{1/2+it}|_G$ are irreducible [**2**, pg. 62] and the Plancherel measure of the Gelfand pair $(G, G \cap K_{v_0})$ is a multiple of $|c(\frac{1}{2}+it)|^{-2}$ for an analytic function $c(z)$ [**10**, pg. 153]. The proof proceeds in the same fashion as for homogeneous trees.

4. Simply transitive subgroups. Let Γ be a simply transitive subgroup of $\mathrm{Aut}(X)$; for $\eta \in L^2(\Omega)$ we define, as in [**11; 1**], the Poisson transform of Γ: $\wp(\eta)(x) = \langle \pi_{1/2}(x)\mathbf{1}, \eta\rangle$.

COROLLARY. $\wp(L^2(\Omega)) \subset l^q(\Gamma)$ *for every* $q > 2$.

Proof. By Theorem 1, $f(g) = \langle \pi_{1/2}(g)\mathbf{1}, \eta\rangle \in L^q(\mathrm{Aut}(X))$ for every $q > 2$. Let $\mathrm{Aut}(X) = \Gamma K$ and $g = xk$ with $x \in \Gamma$ and $k \in K$; therefore $\pi_{1/2}(g)\mathbf{1} = \pi_{1/2}(x)\mathbf{1}$ because ν is K-invariant and so

$$|\wp(\eta)\|_{l^q(\Gamma)} = \|f\|_{L^q(\mathrm{Aut}(X))}.$$

The Corollary follows.

Γ is not a Kunze-Stein group (in a discrete Kunze-Stein group every amenable subgroup is finite); nevertheless, we can prove a "weak Kunze-Stein property":

$l_r^p(\Gamma) *_\Gamma l^2(\Gamma) \subset l^2(\Gamma)$ for every $1 < p < 2$, where l_r^p is the space of radial functions in l^p, that is, the functions which depend only on the length of the words of Γ and $*_\Gamma$ means the convolution product of

Γ. It is easy to see that the "weak Kunze-Stein property" is equivalent to the following: $A_r(\Gamma) \subset l^q(\Gamma)$ for every $q > 2$. This was proved in **[5]** for free groups on finitely many generators. Notice that $A_r(\Gamma) = l_r^2(\Gamma) *_\Gamma l_r^2(\Gamma)$.

THEOREM 2. *The following hold*:
(1) $l_r^2(\Gamma) *_\Gamma l^2(\Gamma) \subset l^q(\Gamma)$ *for every* $q > 2$.
(2) $l^p(\Gamma) *_\Gamma l_r^2(\Gamma) \subset l^2(\Gamma)$ *for every* $1 < p < 2$.

Proof. It is enough to prove (2); (1) follows by duality argument. Putting $\dot{f}(xk) = f(x)$ with $x \in \Gamma$ and $k \in K$, it is possible to identify the functions f on Γ with the right K-invariant functions $\dot{f}$ on $\operatorname{Aut}(X) = \Gamma K$. The radial functions on Γ correspond to the bi K-invariant functions on $\operatorname{Aut}(X)$. Let $f \in l^p(\Gamma)$ for $1 < p < 2$ and $\phi \in l_r^2(\Gamma)$, then the function $\dot{f} * \dot{\phi}$ is right K-invariant; hence, by Theorem 1, the restriction to Γ is in $l^2(\Gamma)$. Moreover: $(\dot{f} * \dot{\phi})|_\Gamma = f *_\Gamma \phi$ and, from this, Theorem 2 follows.

REFERENCES

[1] W. Betori and M. Pagliacci, *Harmonic analysis for groups acting on trees*, Boll. Un. Mat. It., **3-B** (1984), 333–349.

[2] F. Bouaziz-Kellil, *Representations spheriques des groupes agissant transitivement sur un arbre semi homogène*, Thèse de 3-*ème cycle*, Université de Nancy I.

[3] P. Cartier, *Fonctions harmoniques sur un arbre*, Symp. Math., **9** (1972), 203–270.

[4] F. Choucroun, *Groupes opérant simplement transitivement sur un arbre homogène et plongements dans* $\mathrm{PGL}_2(\kappa)$, C. R. Acad. Sci. Paris, **298** (1984), 313–315.

[5] J. M. Cohen and L. deMichele, *Radial Fourier-Stieltjes algebra on free groups*, Contemporary Math., **10** (Operator algebras and K-theory), 1982.

[6] M. Cowling, *The Kunze-Stein phenomenon*, Ann. of Math., **107** (1978), 209–234.

[7] P. Eymard, *L'algèbre de Fourier d'un groupe localment compact*, Bull. Soc. Math. France, **92** (1964), 181–236.

[8] P. Eymard et N. Lohouè, *Sur la racine carrée du noyau de Poisson dans les espaces symétriques et une conjecture de E. M. Stein*, Ann. Sci. Ecole Norm. Sup., Ser 4 (8) n. 2 (1975).

[9] J. Faraut, *Analyse harmonique sur les paires de Gelfand et les espaces hyperboliques*, Analyse harmonique Nancy 1980, CIMPA, Nice (1983).

[10] J. Faraut and M. A. Picardello, *The Plancherel measure for symmetric graphs*, Ann di Mat. Pura Appl., (IV) **138** (1984), 151–155.

[11] A. Figà-Talamanca and M. A. Picardello, *Spherical functions and harmonic analysis on free groups*, J. Funct. Anal., **47** (1982), 281–304.

[12] ———, *Harmonic Analysis on Free Groups*, Lecture notes in pure and applied mathematics, Marcel Dekker, New York, 1983.

[13] C. L. Gulizia, *Harmonic analysis of* SL(2) *over a locally compact field*, J. Funct. Anal., **12** (1973), 384–400.
[14] C. Herz, *Sur le phénomene de Kunze-Stein*, C. R. Acad. Sci. Paris, **271** (1970), 491–493.
[15] R. A. Kunze and E. M. Stein, *Uniformly bounded representations and harmonic analysis of the* 2×2 *real unimodular group*, Amer. J. Math., **82** (1960), 1–62.
[16] S. Lang, $SL_2(\mathbf{R})$, Addison-Wesley, Reading, Mass. 1975.
[17] J. P. Serre, *Arbres, amalgames*, SL_2, Astérisque, **46** (1977).
[18] J. Tits, *Sur le groupe des automorphismes d'un arbre*, Essays on topology and related topics, Mémoires dédiés a G. de Rham, Springer-Verlag, (1970), 188–211.

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