## Special Notations

Chapter I		$(\gamma)^n$	n-th component of a coded in-
_			finite sequence 11
$\operatorname{Dm} \varphi$	domain of $\varphi$ 7	ZF(ZFC)	Zermelo-Fraenkel set theory
Im φ	image of $\varphi$ 7		(with axiom of choice) 11
$\varphi(x)\downarrow$	$\varphi(x)$ is defined, $x \in \operatorname{Dm} \varphi$ 7	AC	axiom of choice 11
$\varphi(x)\uparrow$	$\varphi(x)$ is undefined, $x \not\in \operatorname{Dm} \varphi$	DC	axiom of dependent choice 11
	7	$AC_{\omega}$	axiom of countable choice 11
<b>≃</b>	strong equality 7	Or	class of ordinals 11
$\varphi \restriction X$	restriction of $\varphi$ to $X$ 7	$\inf X$	least element of $X$ 11
$\varphi$ " $X$	image of $X$ under $\varphi$ 7	$\sup X$	least ordinal ≥ all elements of
$\varphi: X \to Y$	function from $X$ into $Y$ 7	•	X 12
×Y	total functions $X \rightarrow Y$ 7	$\sup^+ X$	least ordinal > all elements of
$x \mapsto y_x$		•	X 12
$\lambda x \cdot y_x$	function which assigns 8	$\operatorname{Lim} X$	limit points of $X$ 12
$\langle y_x : x \in Z \rangle$	$y_x$ to x for each $x \in Z$	Card(X)	cardinal of $X$ 13
$\omega$	set of natural numbers 8	N <sub>a</sub>	$\sigma$ -th infinite cardinal 13
lg	length of a finite sequence 8	P(X)	power-set of $X$ 13
ıg x⊆y	y extends x 8	, ,	field of the relation $Z_1 \leq 13$ ,
<del>-</del>	x concatenated with y 8	110(2),110(7)	15
x * y	•	$  Z  ,   \gamma  $	order-type of the (pre-)wellor-
x * φ		2    ,    7	dering $Z_1 \leq 14$ , 15
$x \in Z$	$(\forall i < \lg(\mathbf{x})) \ x_i \in Z  8$	o(V)	
$\varphi(\mathbf{x})$	$(\varphi(x_0),\ldots,\varphi(x_{k-1})) \qquad 8$	o(X)	least ordinal not the type of a
<sup>k, l</sup> ω	$^{k}\omega\times^{\prime}(^{\omega}\omega)$ 8	_	pre-wellordering of X 14
F[ <b>m</b> , α]	$\lambda p \cdot F(p, \mathbf{m}, \boldsymbol{\alpha}) = 8$	≤ <sub>γ</sub>	binary relation coded by $\gamma$ 14
~ <b>R</b>	complement 8	W	codes for well-orderings of
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$^{k,l,l'}\omega$	$^{k}\omega \times ^{l}(^{\omega}\omega) \times ^{l'}(^{(\omega\omega)}\omega) \qquad 9$		≤ <sub>γ</sub> 15
∧,∨,¬,		$ p _{\gamma}$	ordinal represented by $p$ in
ightarrow, $ ightarrow$ , $ ightarrow$	logical symbols 9		≤ <sub>γ</sub> 15
$(\exists p < m),$		[m]	interval determined by <b>m</b> 16
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∃! <i>x</i>	exists exactly one $x = 10$	mes	Lebesgue measure 20
$\langle \mathbf{m} \rangle, \langle \boldsymbol{\alpha} \rangle$	codes for finite sequences 10	$ar{arGamma}$	set inductively defined by
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*	concatenation 10, 11		22
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[a]	primitive recursive functional indexed by $a \in Pri$ 34	W	(number) codes for recursive ordinals 140
{a}	partial recursive functional indexed by a 38	$\omega_1[oldsymbol{eta}]$	least ordinal not recursive in
Ω	codes of recursive computa-	$W[\beta]$	β 140 (number) codes for ordinals re-
12	tions 39	# [P]	cursive in $\beta$ 140
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