Special Notations

Chapter I

$\mathrm{Dm}\,\varphi$	domain of φ 7
$\mathrm{Im}\,\varphi$	image of φ 7
$\varphi(x)\downarrow$	$\varphi(x)$ is defined, $x \in \mathrm{Dm}\,\varphi$ 7
$\varphi(x)\uparrow$	$\varphi(x)$ is undefined, $x \notin \mathrm{Dm}\,\varphi$ 7
$\simeq$	strong equality 7
$\varphi \restriction X$	restriction of φ to X 7
$\varphi'' X$	image of X under φ 7
$\varphi : X \to Y$	function from X into Y 7
${}^{X}Y$	total functions $X \to Y$ 7
$x \mapsto y_x$, $\lambda x \,.\, y_x$, $\langle y_x : x \in Z \rangle$	function which assigns y_x to x for each $x \in Z$ 8
ω	set of natural numbers 8
lg	length of a finite sequence 8
$\mathbf{x} \subseteq \mathbf{y}$	$\mathbf{y}$ extends $\mathbf{x}$ 8
$\mathbf{x} * \mathbf{y}$	$\mathbf{x}$ concatenated with $\mathbf{y}$ 8
$\mathbf{x} * \varphi$	$\mathbf{x}$ concatenated with φ 8
$\mathbf{x} \in Z$	$(\forall i < \lg(\mathbf{x}))\ x_i \in Z$ 8
$\varphi(\mathbf{x})$	$(\varphi(x_0), \ldots, \varphi(x_{k-1}))$ 8
${}^{k,l}\omega$	${}^{k}\omega \times {}^{l}({}^{\omega}\omega)$ 8
$\mathsf{F}[\mathbf{m}, \boldsymbol{\alpha}]$	$\lambda p \,.\, \mathsf{F}(p, \mathbf{m}, \boldsymbol{\alpha})$ 8
$\sim\mathsf{R}$	complement 8
K_{R}	characteristic functional 9
$\mathsf{Gr}_{\mathsf{F}}, \mathsf{Gr}(\mathsf{F})$	graph 9
${}^{k,l,l'}\omega$	${}^{k}\omega \times {}^{l}({}^{\omega}\omega) \times {}^{l'}({}^{({}^{\omega}\omega)}\omega)$ 9
$\wedge, \vee, \neg, \to, \leftrightarrow, \forall, \exists$	logical symbols 9
$(\exists p < m)$, $(\forall \alpha \in \mathsf{A})$	bounded quantifier 10
$\exists!\, x$	exists exactly one x 10
$\langle \mathbf{m} \rangle, \langle \boldsymbol{\alpha} \rangle$	codes for finite sequences 10
$(\ )_i$	i-th component 10, 11
lg	length 10, 11
$*$	concatenation 10, 11
$\mathrm{Sq}, \mathrm{Sq}_1$	set of sequence codes 10, 11
$(\gamma)^n$	n-th component of a coded infinite sequence 11
ZF (ZFC)	Zermelo-Fraenkel set theory (with axiom of choice) 11
AC	axiom of choice 11
DC	axiom of dependent choice 11
AC_{ω}	axiom of countable choice 11
Or	class of ordinals 11
$\inf X$	least element of X 11
$\sup X$	least ordinal $\geq$ all elements of X 12
$\sup^{+} X$	least ordinal $>$ all elements of X 12
$\mathrm{Lim}\, X$	limit points of X 12
$\mathrm{Card}(X)$	cardinal of X 13
$\aleph_{\sigma}$	σ-th infinite cardinal 13
$\mathbf{P}(X)$	power-set of X 13
$\mathsf{Fld}(Z), \mathsf{Fld}(\gamma)$	field of the relation Z, $\leqslant_{\gamma}$ 13, 15
$\lVert Z \rVert, \lVert \gamma \rVert$	order-type of the (pre-)wellordering Z, $\leqslant_{\gamma}$ 14, 15
$o(X)$	least ordinal not the type of a pre-wellordering of X 14
$\leqslant_{\gamma}$	binary relation coded by γ 14
W	codes for well-orderings of ω 15, 81
$\gamma \restriction p$	code for initial segment of $\leqslant_{\gamma}$ 15
$\lvert p \rvert_{\gamma}$	ordinal represented by p in $\leqslant_{\gamma}$ 15
$[\mathbf{m}]$	interval determined by $\mathbf{m}$ 16
BIr	binary irrationals 19, 160
mes	Lebesgue measure 20
$\bar{\Gamma}$	set inductively defined by Γ 22
$\Gamma^{(\sigma)}, \Gamma^{\sigma}$	stages of an inductive definition 22
$\lvert \Gamma \rvert$	closure ordinal 23

Chapter II

$\mathrm{sg}^+, \mathrm{sg}^-$	signum functions 29
"least" $q < p$	bounded search 30
$\exists^0_<, \forall^0_<$	bounded number quantification 31
Pri	primitive recursive indices 34
$[a]$	primitive recursive functional indexed by $a \in$ Pri 34
$\{a\}$	partial recursive functional indexed by a 38
Ω	codes of recursive computations 39
Sb_i	substitution functions 41
"least" q	unbounded search 42
T, T	normal form relations 46, 49
$\exists^0, \forall^0$	type-0 (number) quantification 54
$\exists^1, \forall^1$	type-1 (function) quantification 57
cX	set of complements of members of X 59
$\mathrm{dg}(\alpha)$	degree of α 63
$\mathsf{R} \leq A$	R is (many-one) reducible to A 65
$A^{\mathrm{oJ}}, \beta^{\mathrm{oJ}}$	ordinary jump 65, 66

Chapter III

$\Sigma^0_r, \Pi^0_r, \Delta^0_r, \Delta^0_{(\omega)}$	arithmetical hierarchy 69, 77, 78
U^0_r, U^0_r	universal relations 73
$\Sigma^1_r, \Pi^1_r, \Delta^1_r, \Delta^1_{(\omega)}$	analytical hierarchy 80, 86, 87
U^1_r, U^1_r	universal relations 84
Δ^1_r-$\mathrm{dg}(\alpha)$	Δ^1_r-degree of α 86
$\mathscr{A}$	Suslin operation/quantifier 88
Z_α	zeros of α 89
P_Γ	relation which represents Γ 89
$U^0_{(\omega)}$	universal set for $\Delta^0_{(\omega)}$ 93
ω_1	least non-recursive ordinal 97
δ^1_r	least non-Δ^1_r ordinal 97, 208
$\mathfrak{N}$	standard model for arithmetic 114
$\sigma[\mathbf{m}, \boldsymbol{\alpha}]$	value of σ at $(\mathbf{m}, \boldsymbol{\alpha})$ in $\mathfrak{N}$ 115
$\models \mathfrak{A}[\mathbf{m}, \boldsymbol{\alpha}]$	$\mathfrak{A}$ is true at $(\mathbf{m}, \boldsymbol{\alpha})$ in $\mathfrak{N}$ 115
$\exists^i_r, \forall^i_r$	classes of arithmetic formulas 116
$\mathscr{T} \vdash \mathfrak{A}$	$\mathfrak{A}$ is a theorem of $\mathscr{T}$ 118
$\mathscr{T} \vdash^\omega \mathfrak{A}$	$\mathfrak{A}$ is a theorem of $\mathscr{T} + \omega$-rule 121
$\Vdash \mathfrak{A}[\mathbf{m}, s]$	$\mathfrak{A}$ is forced at $(\mathbf{m}, s)$ 126

Chapter IV

$\mathsf{R} \leq \mathsf{A}$	R is (many-one) reducible to A 136
$\trianglelefteq$	recursive dense linear ordering of Sq 136
$\trianglelefteq^p_{\mathbf{m}, \boldsymbol{\alpha}}$	restriction of $\trianglelefteq$ 136
$\leq_\Sigma, <_\Sigma, \leq_\Pi, <_\Pi$	ordinal comparison on W 138, 144
W_σ	codes for ordinals $< \sigma$ 140
W	(number) codes for recursive ordinals 140
$\omega_1[\beta]$	least ordinal not recursive in β 140
$W[\beta]$	(number) codes for ordinals recursive in β 140
$\leq_\beta$	reducible recursively in β 141
$\lessdot$	reducible by a continuous functional 141
$\Sigma^{1,\mathrm{Hyp}}_1$	$(\exists \beta \in \Delta^1_1[\boldsymbol{\alpha}])\mathsf{P}(\mathbf{m}, \boldsymbol{\alpha}, \beta)$ 147
$A \leq^1_1 B$	$A \in \Delta^1_1[B]$ 149
$\mathrm{hydg}(A)$	hyperdegree of A 149
A^{hJ}	hyperjump 155
$\boldsymbol{\Sigma}^0_\rho, \boldsymbol{\Pi}^0_\rho, \boldsymbol{\Delta}^0_\rho$	Borel hierarchy 157
U^0_ρ	universal relations 159
$N^{k,l}, N^{k,l}_\rho$	indices for the effective Borel hierarchy 163
$\Sigma^0_\rho, \Pi^0_\rho, \Delta^0_\rho$	effective Borel hierarchy 164
$O, <_O, +_O, \vert\,\vert_O$	notations for recursive ordinals 173–174
D_u	Hyperarithmetic hierarchy 173

Chapter V

$\leq^{\mathsf{W}}_\Sigma, <^{\mathsf{W}}_\Sigma, \leq^{\mathsf{W}}_\Pi, <^{\mathsf{W}}_\Pi$	new notations for $\leq_\Sigma, <_\Sigma, \leq_\Pi, <_\Pi$ 203
$\boldsymbol{\delta}^1_r$	least non-$\boldsymbol{\Delta}^1_r$ pre-wellorder type 208
$\mathscr{L}_{\mathrm{ZF}}$	language of set theory 214
$\mathfrak{M} \models \mathfrak{A}[\mathbf{u}]$	$\mathfrak{A}$ is true at $\mathbf{u}$ in $\mathfrak{M}$ 214
L_ρ	ρ-th level of the hierarchy of constructible sets 215
$\mathrm{V} = \mathrm{L}$	Hypothesis of Constructibility: all sets are constructible 215
$<_{\mathrm{L}}$	well-ordering of constructible functions 215
$\varepsilon_{\mathrm{I}}, \varepsilon_{\mathrm{II}}$	sequence of moves of player I (player II) 222
$\gamma \# \delta$	play resulting from two strategies 222
$\mathrm{Det}(X)$	all sets in X are determined 222
PD	Hypothesis of Projective Determinacy: all projective sets are determined 222
$\Phi\langle \mathsf{P}_p : p \in \omega\rangle$, $\Phi\langle \mathsf{P}_p\rangle$	application of Φ to a countable family 237