

## Editors' Preface

In the Winter Quarter of the academic year 1984-1985, Raj Bahadur gave a series of lectures on estimation theory at the University of Chicago. The role of statistical theory in Chicago's graduate curriculum has always varied according to faculty interests, but the hard and detailed examination of the classical theory of estimation was in those years Raj's province, to be treated in a special topics course when his time and the students' interests so dictated. Winter 1985 was one of those times. In ten weeks, Raj covered what most mathematical statisticians would agree should be standard topics in a course on parametric estimation: Bayes estimates, unbiased estimation, Fisher information, Cramér-Rao bounds, and the theory of maximum likelihood estimation. As a seasoned teacher, Raj knew that mathematical prerequisites could not be taken entirely for granted, that even students who had fulfilled them would benefit from a refresher, and accordingly he began with a review of the geometry of  $L^2$  function spaces.

Two of us were in that classroom, WHW who was then a junior member of the statistics faculty and DX who was then an advanced graduate student. We both had previously studied parametric estimation, but never from Raj's distinctive perspective. What started as a visit motivated by curiosity (just how would one of the architects of the modern theory explain what were by then its standard elements?) soon became a compelling, not-to-be-missed pilgrimage, three times a week. Raj's approach was elegant and turned what we thought were shop-worn topics into polished gems; these gems were not only attractive on the surface but translucent, and with his guidance and insight we could look deeply within them as well, gaining new understanding with every lecture. Topics we thought we understood well, such as locally unbiased estimates and likelihood ratios in Lecture 11, and the asymptotic optimality of maximum likelihood estimators in Lectures 28-30, were given new life and much more general understanding as we came to better understand the principles and geometry that underlay them. The two of us (WHW and DX) took detailed notes, reviewing them after the lectures to recover any gaps and smooth the presentation to a better approximation of what we had heard. Some time after the course, Raj was pleased to receive from us an edited, hand-copied version of his lectures.

In these lectures, Raj Bahadur strived towards, and in most cases succeeded in deriving the most general results using the simplest arguments. After stating a result in class, he would usually begin its derivation by saying "it is really very simple...", and indeed his argument would often appear to be quite elementary and simple. However, upon further study, we would find that his arguments were far from elementary — they

appear to be so only because they are so carefully crafted and follow such impeccable logic. Isaac Newton's *Principia* abounds with "simple" demonstrations based upon a single well-designed diagram, demonstrations that in another's hands might have been given through pages of dense and intricate mathematics, the result proved correctly but without insight. Others, even the great astrophysicist S. Chandrasekhar (1995) would marvel at how Newton saw what could be so simply yet generally accomplished. So it is with some of these lectures. Raj focused on the essential and core aspect of problems, often leading him to arguments useful not only for the solutions of the immediate problems, but also for the solutions of very large classes of related problems. This is illustrated by his treatment of Fisher's bound for the asymptotic variance of estimators (Lectures 29-30). Here he used the Neyman-Pearson lemma and the asymptotic normal distribution under local alternatives to provide a remarkably elegant proof of the celebrated result (which he attributed to LeCam) that the set of parameter points at which the Fisher bound fails must be of measure zero. Although Raj developed this theory under the assumption of asymptotic normality, his approach is in fact applicable to a much wider class of estimators (Wong, 1992; Zheng and Fang, 1994).

In this course, Raj Bahadur did not use any particular text and clearly did not rely on any book in his preparation for lectures, although he did encourage students to consult other sources to broaden their perspective. Among the texts that he mentioned are Cramér (1946), Pitman (1979), Ibragimov and Hasminskii (1981), and Lehmann (1983). Raj's own papers were useful, particularly his short and elegant treatment of Fisher's bound (Bahadur, 1964, which is (23) in the appended bibliography of Raj's works). Raj did lecture from notes, but they were no more than a sketchy outline of intentions, and none of them survive. And so these lecture notes are exactly that, the notes of students.

A couple of years after these notes were recorded Raj gave the course again, but we know of no record of any changes in the material, and he may well have used these very notes in preparing for those lectures. While Raj at one time considered using these notes as a basis for a monographic treatment, that was not to be. As a consequence they reflect very much the pace and flavor of the occasion the course was given. Some topics occupied several lectures; others were shorter. Homework problems were stated at the time he thought them most appropriate, not at the end of a topic or chapter. Notation would shift occasionally, and the one change that we have made in this published version is to attempt to make the notation consistent.

Unfortunately, many memorable aspects of Raj's teaching style cannot be conveyed in these notes. He had a great sense of humor and was able to amuse the class with a good number of unexpected jokes that were not recorded. Raj also possessed a degree of humility that is rare among scholars of his stature. His showed no outward signs of self-importance and shared his time and insight without reservation. After class his door was always open for those who needed help and advice, except, of course, immediately after lunch hour, when he was needed in the Billiard Room of the Quadrangle Club.

For many years these notes were circulated as xeroxed copies of the handwritten originals. The repeated requests for copies we received over many years led us to prepare them for publication. These lectures are now superbly typed in Tex by Loren Spice, himself an expert mathematician, and in the process he helped clarify the exposition in

countless small ways. They have been proofread by George Tseng, Donald Truax, and the editors. We are grateful for the support of the University of Chicago's Department of Statistics and its Chairman Michael Stein, and for the assistance of Mitzi Nakatsuka in their final preparation. These lecture notes are presented here with the permission of Thelma Bahadur, in the hope of helping the reader to appreciate the great beauty and utility of the core results in estimation theory, as taught by a great scholar and master teacher Raj Bahadur.

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