LECTURE 10

Open Questions

There are a number of open questions that arise naturally. In Lectures 8 and 9, results on long-range dependence were discussed. However, all these were discussed for processes subordinate to Gaussian stationary processes. It would be of some interest to obtain appropriate limit theorems in a domain broader than that of processes subordinate to Gaussian processes, particularly when dealing with a continuous time parameter. One should note that all the results discussed in these notes have been for a discrete time parameter.

In Lecture 3, results of a global character for probability density estimates were obtained making use of the result of Komlós, Major and Tusnády when dealing with independent observations. It is natural to ask whether one can get a sufficiently broad version of a strong invariance principle with an error term good enough so as to get comparable results in the case of short-range dependence.

Lecture 7 considered conditions for asymptotic normality of spectral density estimates. However, these conditions still seem rather far from what one might think are natural boundaries for the domain of validity of asymptotic normality for these estimates. Herrndorf (1983) gave an interesting example of a class of strongly mixing sequences (with exponentially decaying mixing coefficients) that do not satisfy the central limit theorem. One would like to know whether a similar phenomenon could occur for spectral density estimates (just as for partial sums in Herrndorf's example).

The extent to which corresponding results hold for random fields as well as sequences is a natural question. It is clear that one has to formulate questions in terms of an appropriate version of a strong mixing condition when dealing with dependence, since otherwise results will possibly have a trivial character.

In Lecture 2, questions relating to boundary behavior were mentioned. Such questions are of special interest when dealing with the asymptotic behavior of smoothing splines of any fixed order as the basis for regression estimates. In this respect, it would be worthwhile elaborating the methods used in Messer and Goldstein (1989). These questions are more complicated and interesting in the multidimensional context. What can one say about the boundary behavior of smoothing splines (though it is not clear whether they should be called splines) in the multidimensional case? In Lecture 2, some brief comments were made on how one might compensate for boundary behavior in the case of one-dimensional probability density kernel estimates. One would like to consider simple methods of compensating for boundary behavior when dealing with any type of reasonable multidimensional probability density or regression estimate.

In the last lecture, limited results for probability density kernel estimates were obtained when one had long-range dependence. One would like to obtain broader results for both regression and density estimates when the observed sequence is long-range dependent.

Some mention was made of nonminimum phase non-Gaussian linear processes in Lecture 7. Such processes typically have nonlinear best predictors in the sense of minimum mean least squares error. It is natural to ask how to estimate parameters efficiently and predict efficiently.