Index of main notations

Chap. 1

$\Omega = \mathcal{C}(\mathbb{R}_+ \to \mathbb{R})$: the space of continuous functions from $\mathbb{R}_+$ to $\mathbb{R}$
$(X_t,\ t \geq 0)$: the set of coordinates on this space
$(\mathcal{F}_t,\ t \geq 0)$: the natural filtration of $(X_t, t \geq 0)$
$\mathcal{F}_\infty = \underset{t\geq 0}{\vee} \mathcal{F}_t$
$b(\mathcal{F}_t)$: the space of bounded real valued $\mathcal{F}_t$ measurable functions
$(W_x,\ x \in \mathbb{R})$: the set of Wiener measures on $(\Omega, \mathcal{F}_\infty)$
$W = W_0$
$W_x(Y)$: the expectation of the r.v. Y with respect to W_x
$(L_t^y,\ y \in \mathbb{R},\ t \geq 0)$: the bicontinuous process of local times
$(L_t := L_t^0,\ t \geq 0)$ the local time at level 0
$\big(\tau_l := \inf\{t \geq 0\ ;\ L_t > l\},\ l \geq 0\big)$: the right continuous inverse of $(L_t,\ t \geq 0)$
q : a positive Radon measure on $\mathbb{R}$
$\mathcal{I}$: the set of positive Radon measures on $\mathbb{R}$ s.t. $0 < \displaystyle\int_{-\infty}^{\infty} \big(1 + |x|\big) q(dx) < \infty$
δ_a : the Dirac measure at a
$\left(A_t^{(q)} := \displaystyle\int_0^t q(X_s)ds = \int_{\mathbb{R}} L_t^y q(dy),\ t \geq 0\right)$: the additive functional associated with q
$(W_{x,\infty}^{(q)},\ x \in \mathbb{R})$: the family of probabilities on $(\Omega, \mathcal{F}_\infty)$ obtained by Feynman-Kac penalisation
$(M_{x,s}^{(q)},\ s \geq 0)$: the martingale density of $W_{x,\infty}^{(q)}$ with respect to W_x
γ_q : a scale function
$\varphi_q, \varphi_q^{\pm}$: solutions of the Sturm-Liouville equation $\varphi'' = q\varphi$
$(\mathbf{W}_x,\ x \in \mathbb{R})$: a family of positive σ-finite measures on $(\Omega, \mathcal{F}_\infty)$
$L^1(\Omega, \mathcal{F}_\infty, \mathbf{W})$ $\big($resp. $L_+^1(\Omega, \mathcal{F}_\infty, \mathbf{W})\big)$: the Banach space of
$\mathbf{W}$-integrable r.v.'s (resp. the cone of positive and $\mathbf{W}$-integrable r.v.'s)
$\big(M_t(F),\ t \geq 0\big)$: a martingale associated with $F \in L^1(\Omega, \mathcal{F}_\infty, \mathbf{W})$
$g_a := \sup\{s \geq 0\ ;\ X_s = a\}$; $g_0 = g$
$g_a^{(t)} := \sup\{s \leq t,\ X_s = a\}$; $g_0^{(t)} = g^{(t)}$
$\sigma_a := \sup\big\{s \geq 0\ ;\ X_s \in [-a, a]\big\}$; $\sigma_{a,b} := \sup\big\{s \geq 0\ ;\ X_s \in [a, b]\big\}$
$f_Z^{(P)}$: density of the r.v. Z under P
T : a $(\mathcal{F}_t,\ t \geq 0)$ stopping time
$P_0^{(3)}$ (resp. $\widetilde{P}_0^{(3)}$) : the law of a 3-dimensional Bessel process (resp. of the opposite of a 3-dimensional Bessel process) started at 0
$P_0^{(3,\mathrm{sym})} = \dfrac{1}{2}(P_0^{(3)} + \widetilde{P}_0^{(3)})$
$W_0^{\tau_l}$: the law of a 1-dimensional Brownian motion stopped at τ_l
$\Pi_{0,0}^{(t)}$: the law of the Brownian bridge $(b_u,\ 0 \leq u \leq t)$ of length t and s.t. $b_0 = b_t = 0$
$\omega \circ \widetilde{\omega}$: the concatenation of ω and $\widetilde{\omega}$ $(\omega, \widetilde{\omega} \in \Omega)$
$\omega = (\omega_t, \omega^t)$: decomposition of ω before and after t
$\Gamma^+ = \big\{\omega \in \Omega\ ;\ X_t \underset{t\to\infty}{\longrightarrow} \infty\big\}$, $\Gamma^- = \big\{\omega \in \Omega\ ;\ X_t(\omega) \underset{t\to\infty}{\longrightarrow} -\infty\big\}$
$\mathbf{W}^+ = 1_{\Gamma^+} \cdot \mathbf{W}, \quad \mathbf{W}^- = 1_{\Gamma^-} \cdot \mathbf{W}$
$W^F\big(F \in L_+^1(\Omega, \mathcal{F}_\infty, \mathbf{W})\big)$: the finite measure defined on $(\Omega, \mathcal{F}_\infty)$ by : $W^F(G) = \mathbf{W}(F \cdot G)$
$\mathcal{C}$: the class of "good" weight processes for which Brownian penalisation holds
$(\nu_x^{(q)},\ x \in \mathbb{R})$: a family of σ-finite measures associated with the additive functional $(A_t^{(q)},\ t \geq\ 0)$

$(Z_t,\ t \geq 0)$: a positive Brownian supermartingale

$Z_\infty := \lim\limits_{t\to\infty} Z_t \quad W$ a.s. ; $z_\infty := \lim\limits_{t\to\infty} \dfrac{Z_t}{1+|X_t|} \quad \mathbf{W}$ a.s.

$\big(\Delta_t(F),\ t \geq 0\big),\ \big(\Sigma_t(F),\ t \geq 0\big)$: two quasimartingales associated with $F \in L^1(\Omega, \mathcal{F}_\infty, \mathbf{W})$

$(\Phi_s,\ s \geq 0)$: a predictable positive process

$\big(k_s(F),\ s \geq 0\big)$ a predictable process such that $\mathbf{W}\big(F|\mathcal{F}_g\big) = k_g(F)\ \big(F \in L^1_+(\Omega, \mathcal{F}_\infty, \mathbf{W})\big)$

$(\chi_t,\ t \geq 0)$: a $\mathcal{C}(\mathbb{R}_+ \to \mathbb{R})$ valued Markov process

$(\mathbb{P}_t,\ t \geq 0)$: the semigroup associated to $(\chi_t,\ t \geq 0)$

$\mathbf{W}^{a,b}_x = a\mathbf{W}^+_x + b\mathbf{W}^-_x$

$\widetilde{\mathbf{W}}^{a,b} = \displaystyle\int dx \mathbf{W}^{a,b}_x$: is an invariant measure for $(\chi_t,\ t \geq 0)$

$\widetilde{\Omega} = \mathcal{C}(\mathbb{R} \to \mathbb{R}_+)$: the space of continuous functions from $\mathbb{R}$ to $\mathbb{R}_+$

$< q, l >:= \displaystyle\int_{\mathbb{R}} l(x)q(dx),\ q \in \mathcal{I},\ l \in \widetilde{\Omega}$

$\mathcal{L} : \Omega \to \widetilde{\Omega}$ defined by $\mathcal{L}(X_t,\ t \geq 0) = (L^y_\infty,\ y \in \mathbb{R})$

$(Q_t,\ t \geq 0)$: the semigroup associated with the Markov process $\big((X_t,\ L^\bullet_t),\ t \geq 0\big)$ which is $\mathbb{R} \times \widetilde{\Omega}$ valued

$\mathcal{G}$: the infinitesimal generator of $(Q_t,\ t \geq 0)$

$(\widetilde{\mathbf{\Lambda}}^{a,b}, a, b \geq 0)$: a family of invariant measures for $\big((X_t,\ L^\bullet_t),\ t \geq 0\big)$

$(\mathbf{\Lambda}_x,\ x \in \mathbb{R})$: a family of positive and σ-finite measures on $\widetilde{\Omega}$

$\theta\ : \mathbb{R} \times \widetilde{\Omega} \to \widetilde{\Omega}$ defined by $\theta(x,l)(y) = l(x-y) \quad (x, y \in \mathbb{R},\ l \in \widetilde{\Omega})$

$(L^{X_t - \bullet}_t,\ t \geq 0)$: a $\widetilde{\Omega}$ valued Markov process

$(\overline{Q}_t,\ t \geq 0)$: the semigroup associated with $(L^{X_t-\bullet}_t,\ t \geq 0)$

$\overline{\mathcal{G}}$: the infinitesimal generator of $(\overline{Q}_t,\ t \geq 0)$

$\mathbf{\Lambda}^{a,b} = a\ \mathbf{\Lambda}^+ + b\ \mathbf{\Lambda}^-$

Chap. 2

$\Omega = \mathcal{C}(\mathbb{R}_+ \to \mathbb{C})$: the space of continuous functions from $\mathbb{R}_+$ to $\mathbb{C}$

$(X_t,\ t \geq 0)$: the coordinate process on Ω

$(W^{(2)}_x,\ x \in \mathbb{C})$ the set of Wiener measures ; $W^{(2)}_0 = W^{(2)}$

$\mathcal{J}$: the set of positive Radon measures on $\mathbb{C}$ with compact support

$(A^{(q)}_t := \displaystyle\int_0^t q(X_s)ds,\ t \geq 0)$: the additive functional associated with $q \in \mathcal{J})$

$(W^{(2,q)}_{z,\infty},\ z \in \mathbb{C})$: the set of probabilities obtained by Feynman-Kac penalisations associated with $q \in \mathcal{I}$; $W^{(2,q)}_{0,\infty} = W^{(2,q)}_\infty$

$(M^{(2,q)}_s,\ s \geq 0)$: the martingale density of $W^{(2,q)}_{z,\infty}$ with respect to $W^{(2)}_z$

φ_q : a solution of Sturm-Liouville equation $\Delta\varphi = q\varphi$

Δ : the Laplace operator

$(\mathbf{W}^{(2)}_z,\ z \in \mathbb{C})$: a family of positive and σ-finite measures on $(\Omega, \mathcal{F}_\infty)$

$\mathbf{W}^{(2)}_0 = \mathbf{W}^{(2)}$

C : the unit circle in $\mathbb{C}$

$(L^{(C)}_t,\ t \geq 0)$: the continuous local time process on C

$(\tau^{(C)}_l,\ l \geq 0)$: the right continuous inverse of $(L^{(C)}_t,\ t \geq 0)$

$(R_t,\ t \geq 0)$: the process solution of (2.2.6)

$P^{(2,\log)}_1$: the law of process $(R_t,\ t \geq 0)$

$(\rho_u,\ u \geq 0)$: a 3-dimensional Bessel process starting from 0.
$\left(H_t := \int_0^t \frac{ds}{R_s^2},\ t \geq 0\right)$
$g_C := \sup\{s \geq 0\ ;\ X_t \in C\}$
$W_0^{\left(2,\tau_l^{(C)}\right)}$: the law of a $\mathbb{C}$-valued Brownian motion stopped at $\tau_l^{(C)}$
$\widetilde{P}_1^{(2,\log)}$: the law of $(X_{g_C+s},\ s \geq 0)$
∇ : the gradient operator
K_0 : the Bessel Mc Donald function with index 0
$T_1^{(3)} := \inf\{u\ ;\ \rho_u = 1\}$
$(R_t^{(\delta)},\ t \geq 0)$: the process solution of (2.3.19)
$\left(M_t^{(2)}(F),\ t \geq 0\right)$: the Brownian martingale associated with $F \in L^1(\Omega, \mathcal{F}_\infty, \mathbf{W}^{(2)})$

Chap. 3
$\Omega = \mathcal{C}(\mathbb{R}_+ \to \mathbb{R}_+)$: the space of continuous functions from $\mathbb{R}_+$ to $\mathbb{R}_+$
S : the scale function
m : the speed measure
$(X_t,\ t \geq 0,\ P_x,\ x \in \mathbb{R}_+)$: the canonical process associated with S and m
$(\mathcal{F}_t,\ t \geq 0)$: the natural filtration of $(X_t,\ t \geq 0)$; $\mathcal{F}_\infty = \bigvee_{t \geq 0} \mathcal{F}_t$
$L = \frac{d}{dm} \frac{d}{dS}$: the infinitesimal generator of $(X_t,\ t \geq 0)$
$p(t, x, \bullet)$: the density of X_t under P_x with respect to m
$(L_t^y,\ t \geq 0,\ y \geq 0)$: the jointly continuous family of local times of X
$(L_t,\ t \geq 0)$: the local time process at level 0
$(\tau_l,\ l \geq 0)$: the right continuous inverse of $(L_t,\ t \geq 0)$
$P_x^{\tau_l}$: the law of the process $(X_t,\ t \geq 0)$ started at x and stopped at τ_l
$g_y := \sup\{t \geq 0\ ;\ X_t = y\}$; $g := g_0$
$g_y^{(t)} := \sup\{s \leq t\ ;\ X_s = y\}$; $g^{(t)} := g_0^{(t)}$
$T_0 := \inf\{t \geq 0\ ;\ X_t = 0\}$
$(\widehat{X}_t,\ t \geq 0)$: the process $(X_t,\ t \geq 0)$ killed at T_0
$\widehat{p}(t, x, \bullet)$: the density of $\widehat{X}_t$ under P_x with respect to m
$(P_x^{\uparrow},\ x \in \mathbb{R}_+)$: the laws of X conditionned not to vanish ; $P^{\uparrow} := P_0^{\uparrow}$
$f_{y,0}(t)$ defined by : $f_{y,0}(t)dt = P_y(T_0 \in dt) = P_0^{\uparrow}(g_y \in dt)$
$\mathbf{W}^*$ a σ-finite measure on $(\Omega, \mathcal{F}_\infty)$
$\Pi_0^{(t)}$: the law of the bridge of length t
$\mathbf{W}_g^*$: the restriction of $\mathbf{W}^*$ to $\mathcal{F}_g$
$\left(M_t^{(\lambda,x)} = \frac{1 + \frac{\lambda}{2} S(X_t)}{1 + \frac{\lambda}{2} S(x)} \cdot e^{-\frac{\lambda}{2} L_t},\ t \geq 0\right)$: the martingale density of $P_{x,\infty}^{(\lambda)}$ with respect to P_x
$(M_t^*(F),\ t \geq 0)$: the positive $\big((\mathcal{F}_t,\ t \geq 0),\ P_0\big)$ martingale associated with $F \in L^1(\Omega, \mathcal{F}_\infty, \mathbf{W}^*)$
$\left(P_x^{(-\alpha)},\ x \geq 0\right)$: the family of laws of a Bessel process with dimension $d = 2(1-\alpha)$ $(0 < d < 2$, or equivalently $0 < \alpha < 1)$ started at x
$\mathbf{W}^{(-\alpha)}$: the measure $\mathbf{W}^*$ in the particular case of a Bessel process with index $(-\alpha)$ $(0 < \alpha < 1)$
$\Pi_0^{(-\alpha,t)}$: the law of the Bessel bridge with index $(-\alpha)$ and length t
$P_x^{(-\alpha,\tau_l)}$: the law of a Bessel process with index $(-\alpha)$ started at x and stopped at τ_l

φ_q : a particular solution of the Sturm-Liouville equation :

$$\frac{1}{2}\,\varphi''(r) + \frac{1-2\alpha}{2r}\,\varphi'(r) = \frac{1}{2}\,\varphi(r)\,q(r), \quad r \geq 0$$

with q a positive Radon measure with compact support
$(m_u,\ 0 \leq u \leq 1)$: the Bessel meander with dimension d
$P_0^{\left(\frac{\delta}{2}-1,\, m,\, \nearrow\nwarrow\right)}$: the law of the process obtained by putting two Bessel processes with index $\left(\frac{\delta}{2}-1\right)$ back to back; these processes start from 0 and are stopped when they first reach level m

Chap. 4
E : a countable set
$(X_n,\ n \geq 0)$: the canonical process on $E^{\mathbb{N}}$
$(\mathcal{F}_n,\ n \geq 0)$: the natural filtration, $\mathcal{F}_\infty = \bigvee_{n\geq 0} \mathcal{F}_n$
$(\mathbb{P}_x,\ x \in E)$: the family of probabilities associated to Markov process $(X_n,\ n \geq 0)$ s.t. $\mathbb{P}(X_{n+1} = z | X_n = y) = p_{y,z}$ and $\mathbb{P}_x(X_0 = x) = 1$
$\left(L_k^y = \sum_{m=0}^{k} 1_{X_m = y},\ k \geq 0\right)$: the local time of $(X_n,\ n \geq 0)$ at level y (with $L_{-1}^y = 0$)
ϕ : a positive function from E to $\mathbb{R}_+$, harmonic with respect to $\mathbb{P}$, except at the point x_0 and such that $\phi(x_0) = 0$
$\psi_r(x) := \frac{r}{1-r}\,\mathbb{E}_{x_0}\big(\phi(X_1)\big) + \phi(x) \quad \big(r \in]0,1[,\ x \in E\big)$
$\big(\mu_x^{(r)},\ x \in E,\ r \in]0,1[\big)$: a family of finite measures on $(E^{\mathbb{N}}, \mathcal{F}_\infty)$
$\mathbb{Q}_x = \left(\frac{1}{r}\right)^{L_\infty^{x_0}} \mu_x^{(r)}$, independent of $r \in]0,1[$
$\mathbb{Q}_x^{(\psi,y_0)}$: the σ-finite measure $\mathbb{Q}_x$ constructed from the point y_0 and the function ψ
q : a function from E to $[0,1]$ such that $\{q < 1\}$ is a finite set
$\big(M(F, X_0, X_1, \cdots, X_n),\ n \geq 0\big)$: the $\big((\mathcal{F}_n,\ n \geq 0),\ \mathbb{P}_x\big)$ martingale associated with $F \in L^1(\Omega, \mathcal{F}_\infty, \mathbb{Q}_x)$
$\tau_k^{(y)}$: the k-th hitting time of y
$(\tau_k^{(y)},\ k \geq 0)$: the inverse of $(L_k^y,\ k \geq 0)$
$\mathbb{Q}_y^{[y_0]}$: the restriction of $\mathbb{Q}_y$ to trajectories which do not hit y_0
$\widetilde{\mathbb{Q}}_y$: the restriction of $\mathbb{Q}_y$ to trajectories which do not return to y
$\mathbb{P}_x^{\tau_k^{(y_0)}}$: the law of the Markov chain $(X_n,\ n \geq 0)$ starting from x and stopped at $\tau_k^{(y_0)}$
$2\widetilde{\mathbb{Q}}_a^+$: the law of a Bessel random walk strictly above a
$2\widetilde{\mathbb{Q}}_a^-$: the law of a Bessel random walk strictly below a
$\widetilde{\mathbb{Q}}_a := \widetilde{\mathbb{Q}}_a^+ + \widetilde{\mathbb{Q}}_a^-$
$g_a := \sup\{n \geq 0\ ;\ X_n = a\}$
$\phi^{[y_0]}$ defined by $\phi^{[y_0]}(y) = \mathbb{Q}_y^{[y_0]}(1)$
$\simeq$: the equivalence relation defined in Subsection 4.2.4
$\mathbb{Q}_x^{[\psi]}$: the measure $Q_x^{(\psi,y_0)}$ where $[\psi]$ denotes the equivalence class of ψ