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MINIMAL UNIFORM CONVERGENCE SPACES

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ABSTRACT. In this paper the study of minimal P uniform convergence spaces is initiated by investigating minimal Hausdorff uniform convergence spaces and minimal uniformly regular uniform convergence spaces.

1. **Introduction.** Minimal P and P-closed topological spaces have been of interest for some time (see [1]). In [5] these concepts were introduced into the setting of convergence spaces and have been studied further in [6]. Our object here is to investigate minimal Hausdorff and minimal uniformly regular uniform convergence spaces and their relationships with their induced convergence structures.

Notions not explicitly mentioned here are standard and may be found in [2] or [3]. In particular, a uniform convergence structure (hereafter abbreviated u.c.s.) is taken in the sense of [2]. One other abbreviation is employed, namely "u.f." for "ultrafilter." If X is a set, a filter Φ on $X \times X$ is *Δ-symmetric* if it has a base of sets A, each of which is symmetric and contains the diagonal Δ *in* $X \times X$. Each u.c.s. has a base of Δ-symmetric filters. A u.c.s. I is a *pseudo uniformity* if a filter is in I whenever each refining u.f. is in I, and is *uniformly regular* if it is Hausdorff and $\mathrm{cl}\,\Phi \in I$ whenever $\Phi \in I$. Here, the closure is taken in the product space. If q is a convergence structure on X, $[q]$ is the collection of all u.c.s.'s on $X \times X$ which induce q. For each Hausdorff q there is a coarsest member of $[q]$, I_q, called the coarse u.c.s. relative to q. This structure is studied in [4] where the following are established.

PROPOSITION 1.1. *If q is Hausdorff, then I_q is generated by all Δ-symmetric filters Φ which have the following property*: $\Phi(\mathscr{F}) \to x$ *whenever* $\mathscr{F} \to x$.

PROPOSITION 1.2. *If q is Hausdorff, then I_q is uniformly regular if and only if q is a regular topology.*

If P is a property of u.c.s.'s, I is *minimal P* if I has property P and no strictly coarser u.c.s. has property P. In §2 we characterize the minimal Hausdorff u.c.s.'s as those which are compact, Hausdorff, pseudo uniformities, and show that the correspondence $q \to I_q$ is one-one between the class of minimal Hausdorff convergence structures and the class of minimal Hausdorff u.c.s.'s.

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Minimal uniform regularity is much more difficult. In §4 we show that a minimal uniformly regular u.c.s. I need not be coarse and need not induce a minimal regular convergence structure. However, if I is minimal uniformly regular and totally bounded, then I is coarse, and the structure induced by a coarse minimal uniformly regular u.c.s. is characterized as being a semi-minimal regular topological space.

2. **Minimal Hausdorff u.c.s.'s.** We will employ the following lemmas.

LEMMA 2.1. *Let X be a set and suppose that the composition $\Lambda_1 \circ \Lambda_2$ of two filters Λ_1 and Λ_2 on $X \times X$ exists. Also assume $\Lambda \geqq \Lambda_1 \circ \Lambda_2$, where Λ is an u.f. Then there are u.f.'s $\Sigma_1 \geqq \Lambda_1$ and $\Sigma_2 \geqq \Lambda_2$ such that $\Lambda \geqq \Sigma_1 \circ \Sigma_2$.*

PROOF. The set $\{\Sigma : \Sigma \geqq \Lambda_1, \Lambda \geqq \Sigma \circ \Lambda_2\}$, ordered by inclusion, has a maximal element, say Σ_1, by an application of Zorn's Lemma. We shall first show that Σ_1 is an u.f..

Let $A \cup B \in \Sigma_1$, and suppose $A \notin \Sigma_1$ and $B \notin \Sigma_1$. Let Σ_A be the filter generated by $\{A \cap S : S \in \Sigma_1\}$, and Σ_B the filter generated by $\{B \cap S : S \in \Sigma_1\}$. Under the assumptions $A \notin \Sigma_1$ and $B \notin \Sigma_1$, both Σ_A and Σ_B are well-defined, and $\Sigma_A > \Sigma_1 \geqq \Lambda_1$ and $\Sigma_B > \Sigma_1 \geqq \Lambda_1$ both hold. Thus, in order to contradict the maximality of Σ_1, it remains only to show that either $\Lambda \geqq \Sigma_A \circ \Lambda_2$ or $\Lambda \geqq \Sigma_B \circ \Lambda_2$.

Suppose, on the contrary, that both of the preceding inequalities are false. Then there are sets S_1, S_2 in Σ_1 and L_1, L_2 in Λ_2 such that $(A \cap S_1) \circ L_1$ and $(B \cap S_2) \circ L_2$ are not in Λ. Letting $S = S_1 \cap S_2$ and $L = L_1 \cap L_2$, and using the fact that Λ is an u.f., we have $((A \cap S) \circ L) \cup ((B \cap S) \circ L) = ((A \cup B) \cap S) \circ L \notin \Lambda$. But this is a contradiction, since $(A \cup B) \cap S \in \Sigma_1$, and it follows that Σ_1 is an u.f.

The conclusion of the lemma now follows by applying a similar argument to the set $\{\Sigma : \Sigma \geqq \Lambda_2, \Lambda \geqq \Sigma_1 \circ \Sigma\}$.

LEMMA 2.2. *Let Λ be a filter on $X \times X$, $\mathscr{F}$ a filter on X such that $\Lambda(\mathscr{F})$ exists. Let $\mathscr{G}$ be a u.f. on X such that $\mathscr{G} \geqq \Lambda(\mathscr{F})$. Then there are u.f.'s $\Sigma \geqq \Lambda$ and $\mathscr{H} \geqq \mathscr{F}$ such that $\mathscr{G} \geqq \Sigma(\mathscr{H})$.*

PROOF. The proof is similar to that of Lemma 2.1 and is omitted.

LEMMA 2.3. *Suppose $\Phi \in I$, I a Hausdorff u.c.s., and $\Phi \to (x, y)$. Then $x = y$.*

PROOF. If p_1, p_2 are the projection maps, $p_1\Phi \geqq \Psi_1(\dot{x})$, $p_2\Phi \geqq \Psi_2(\dot{y})$ for some Ψ_1, $\Psi_2 \in I$. A simple computation shows that $\dot{x} \times \dot{y} > \Psi_1^{-1} \circ \Phi \circ \Psi_2$, so $\dot{x} \times \dot{y} \in I$, and $x = y$ follows since I is Hausdorff.

LEMMA 2.4. *Let I be a u.c.s., $\mathscr{F}$ a Cauchy filter, Φ a Δ-symmetric member of I. Then $\Phi(\mathscr{F})$ is a Cauchy filter equivalent to $\mathscr{F}$.*

PROOF. $\mathscr{F} \geqq \Phi(\mathscr{F})$ and $\Phi(\mathscr{F}) \times \Phi(\mathscr{F}) \geqq \Phi \circ \mathscr{F} \times \mathscr{F} \circ \Phi \in I$. Thus $\Phi(\mathscr{F})$ is Cauchy and clearly equivalent to $\mathscr{F}$.

PROPOSITION 2.5. *If I is minimal Hausdorff and $I \in [q]$, then q is compact.*

PROOF. It is clear that $I = I_q$. Suppose $\mathscr{H}$ is a non-convergent u.f.. Let $y \in X$ and define p to be the finest pseudo topology which satisfies the following conditions:

(1) $\mathscr{F} \to x$ relative to p if and only if $\mathscr{F} \to x$ relative to q, $x \neq y$.

(2) $\mathscr{F} \to y$ relative to p if and only if $\mathscr{F} \to y$ relative to q or else $\mathscr{F} \geqq \mathscr{G}$, $\mathscr{G}$ an I_q Cauchy filter equivalent to $\mathscr{H}$.

Notice that p is Hausdorff, for if not we have $\mathscr{F} \to x$ relative to q, $x \neq y$, $\mathscr{F} \vee \mathscr{G}$ exists where $\mathscr{G}$ is an I_q Cauchy filter equivalent to $\mathscr{H}$. But then $\mathscr{F} \wedge \mathscr{G}$ is Cauchy, and hence $\mathscr{G} \to x$ relative to q. This contradicts the fact that $\mathscr{G}$ is equivalent to $\mathscr{H}$.

Next it is asserted that $I_p < I_q$. If Φ is Δ-symmetric in I_q, let $\mathscr{F} \to x$ relative to p. Now if $\mathscr{F} \to x$ relative to q, then $\Phi(\mathscr{F}) \to x$ relative to q by Proposition 1.1, and so $\Phi(\mathscr{F}) \to x$ relative to p. On the other hand suppose $\mathscr{F} \to y$ relative to p, where $\mathscr{F}$ is an I_q Cauchy filter equivalent to $\mathscr{H}$. By Lemma 2.4, $\Phi(\mathscr{F})$ is an I_q Cauchy filter which is equivalent to $\mathscr{H}$, and so $\Phi(\mathscr{F}) \to y$ relative to p. Our assertion now follows by Proposition 1.1. Thus $I_p < I_q = I$, and I_p is Hausdorff, which contradicts the minimality of I.

THEOREM 2.6. *For a Hausdorff u.c.s. (X, I) the following are equivalent:* (a) *(X, I) is minimal Hausdorff;*

(b) *I is a compact, psuedo uniformity;*

(c) *Each uniformly continuous, one-one map of (X, I) onto a Hausdorff u.c.s (Y, J) is a uniform homeomorphism.*

PROOF. (a) implies (b). Define ρI on $X \times X$ by requiring $\Phi \in \rho I$ if and only if each u.f. finer than Φ is in I. That ρI is indeed a u.c.s. is shown by Lemma 2.1. Clearly $\rho I \leqq I$ and ρI is Hausdorff because it induces the same convergence on u.f.'s as does I. Hence $\rho I = I$ by minimality, and I is a pseudo uniformity. The fact that I is compact is a consequence of Proposition 2.5. (b) implies (c). Suppose that f is a one-one, uniformly continuous map onto a Hausdorff (Y, J). By assumption it suffices to show that if $(f \times f)(\Lambda) \in J$, Λ an u.f., then $\Lambda \in I$. Now $\Lambda \to (x, y)$ for some $(x, y) \in X \times X$ by compactness, and $(f \times f)\Lambda \to (f(x), f(y))$; hence $f(x) = f(y)$ by Lemma 2.3. Thus $x = y$, and $\Lambda \geqq p_1\Lambda \times p_2\Lambda \geqq (p_1\Lambda \times \dot{x}) \circ (\dot{x} \times p_2\Lambda) \in I$.

(c) implies (a). This is obvious.

COROLLARY 2.7. *The correspondence $q \to I_q$ is one-one between the class of all minimal Hausdorff convergence structures and the class of all minimal Hausdorff u.c.s.'s.*

PROOF. If q is minimal Hausdorff, then it is a compact, pseudo topology by [5]; hence I_q is compact. To show that I_q is a pseudo uniformity suppose $\mathscr{F} \to \chi$ implies $\Lambda(\mathscr{F}) \to x$ for each u.f. Λ finer than Φ. By Lemma 2.2, if $\mathscr{U} \geqq \Phi(\mathscr{F})$, $\mathscr{U}$ an u.f., then $\mathscr{U} \geqq \Lambda(\mathscr{G})$ for u.f.'s $\Lambda \geqq \Phi$ and $\mathscr{G} \geqq \mathscr{F}$. Since q is a pseudo topology, $\Phi(\mathscr{F}) \to x$ and so $\Phi \in I_q$ by Proposition 1.1. By Theorem 2.6, I_q is minimal Hausdorff. The remainder of the Corollary is clear. The next result will be used below as well as in §4.

PROPOSITION 2.8. *Let (X, I) be a u.c.s., Φ Δ-symmetric in I, $\mathscr{F}$ a filter on X. Then $\mathscr{F}$ and $\Phi(\mathscr{F})$ have the same adherent points.*

PROOF. Let x be an adherent point of $\Phi(\mathscr{F})$. There is an u.f. $\mathscr{G} \geqq \Phi(\mathscr{F})$ such that $\mathscr{G} \to x$. A slight variation of Lemma 2.2 shows that there is an u.f. $\mathscr{H} \geqq \mathscr{F}$ such that $\mathscr{G} \geqq \Phi(\mathscr{H})$. It can be shown that $\mathscr{H} \times \mathscr{H} > \Phi \circ \mathscr{G} \times \mathscr{G} \circ \Phi \in I$; thus $\mathscr{H}$ is Cauchy. Also, $\mathscr{G} \times \mathscr{H} > \mathscr{H} \times \mathscr{H} \circ \Phi \circ \mathscr{G} \times \mathscr{G} \in I$, and so $\mathscr{G}$ and $\mathscr{H}$ are equivalent Cauchy filters. Thus $\mathscr{H} \to x$, and x is an adherent point of $\mathscr{F}$. The remainder of the proposition is clear from the relation $\mathscr{F} \geqq \Phi(\mathscr{F})$.

PROPOSITION 2.9. *If I is a Hausdorff u.c.s., there is a minimal Hausdorff u.c.s. J such that $J \leqq I$.*

PROOF. Let p be the convergence structure on X induced by I. Let $a \in X$, and define q on X as follows:

(a) If $x \neq a$, then $\mathscr{F} \to x$ relative to q if each u.f. finer than $\mathscr{F}$ p-converges to x.

(b) $\mathscr{F} \to a$ relative to q if each u.f. finer than $\mathscr{F}$ either p-converges to a or else fails to p-converge.

From this construction, it is easy to see that q is minimal Hausdorff and $q \leqq p$. To complete the proof we will show that $I_q \leqq I$. Let $\Phi \in I$ and let $\mathscr{F} \to x$ relative to q. By Proposition 1.1, it suffices to show that $\Phi(\mathscr{F}) \to x$ relative to q.

First, assume $x \neq a$. Let $\mathscr{G}$ be an u.f., $\mathscr{G} \geqq \Phi(\mathscr{F})$. By a variation of Lemma 2.2, there is an u.f. $\mathscr{H}$ such that $\mathscr{H} \geqq \mathscr{F}$ and $\mathscr{G} \geqq \Phi(\mathscr{H})$. Then, since $\mathscr{H} \to x$ in (X, p), it follows that $\Phi(\mathscr{H}) \to x$ in (X, p). Thus, $\Phi(\mathscr{H}) \to x$ in (X, q), and so $\mathscr{G} \to x$ in (X, q). From the definition of q, $\Phi(\mathscr{F}) \to x$ in (X, q).

Next, assume $x = a$, and let $\mathscr{G}$ and $\mathscr{H}$ be defined as above. If $\mathscr{H} \to a$ relative to p, then a repetition of the argument above leads to the conclusion that $\Phi(\mathscr{H}) \to a$ relative to q. If $\mathscr{H}$ does not p-converge, then it follows by Proposition 2.8 that $\Phi(\mathscr{H})$ has no p-adherent points, and so, by construction of q, $\Phi(\mathscr{H}) \to a$ relative to q. Thus, $\mathscr{G} \to a$ in (X, q) for each u.f. $\mathscr{G} \geqq \Phi(\mathscr{F})$; hence $\Phi(\mathscr{F}) \to a$ relative to q.

3. Minimal Uniformly Regular Spaces; General Results.

PROPOSITION 3.1. *A minimal uniformly regular u.c.s. is a pseudo uniformity.*

PROOF. If I is minimal uniformly regular, let ρI be as defined in Theorem 2.6. Let $\Phi \in \rho I$, and suppose $\Lambda \geqq \mathrm{cl}_{\rho I}\Phi$, Λ an u.f. Since I and ρI agree on u.f.'s $\Lambda \geqq \mathrm{cl}_I\Phi$ and we can find an u.f. $\Sigma \geqq \Phi$, such that $\Lambda \geqq \mathrm{cl}_I\Sigma$. Then $\Sigma \in I$ by definition of ρI, and $\mathrm{cl}_I\Sigma \in I$ by uniform regularity. We have shown that each u.f. finer than $\mathrm{cl}_{\rho I}\Phi$ is in I, so ρI is uniformly regular, as it is clearly Hausdorff. It follows from minimality that I is a pseudo uniformity.

PROPOSITION 3.2. *A compact, uniformly regular u.c.s. is minimal uniformly regular if and only if it is a psuedo uniformity.*

PROOF. A compact, uniformly regular, pseudo uniformity is minimal Hausdorff by Theorem 2.6, hence minimal uniformly regular. The converse is Proposition 3.1.

PROPOSITION 3.3. *Let q be a minimal regular convergence structure, I uniformly regular, $I \in [q]$. Then there exists a minimal uniformly regular u.c.s. J such that $J \leqq I$.*

PROOF. Let $\mathscr{A}$ be the collection of all uniformly regular structures which induce q. Let $\mathscr{C}$ be a chain in $\mathscr{A}$. Then $\inf\mathscr{C}$ is generated by all finite compositions $\Phi_1 \circ \cdots \circ \Phi_n$, $\Phi_i \in \dot{I}_i$, $I_i \in \mathscr{C}$; and hence is generated by $\{\Phi : \Phi \in I, I \in \mathscr{C}\}$ since $\mathscr{C}$ is a chain. It follows easily that $\inf\mathscr{C} \in [q]$ and that $\inf\mathscr{C}$ is uniformly regular. So $\mathscr{A}$ has a minimal member J. If $K \leqq J$, K uniformly regular, then $K \in [q]$ by the minimality of q. Thus $K = J$ and J is minimal unformly regular.

We now see that a minimal uniformly regular u.c.s. I need not be coarse. In fact, if q is a minimal regular convergence structure which is not a topology (see [5]), the fact that the fine u.c.s. for q is uniformly regular and Proposition 3.3 shows that there is a minimal uniformly regular $J \in [q]$. But $J \neq I_q$ by Proposition 1.2. In the next section, by investigating minimal uniformly regular u.c.s.'s which are coarse, we will also see that not every minimal uniformly regular u.c.s. induces a minimal regular convergence structure.

4. Totally Bounded Minimal Uniformly Regular Structures. In this section it will be shown that a totally bounded, minimal uniformly regular u.c.s. is coarse, and the topology induced by a minimal uniformly regular I_q will be characterized. From this we will see that a minimal uniformly regular u.c.s. need not induce a minimal regular convergence structure or a minimal regular topology. The following lemma will be needed.

LEMMA 4.1. *Let p be a Hausdorff pseudo topology, q a Hausdorff convergence structure such that $q \geqq p$ and $q = p$ on u.f.'s. Then $I_q \geqq I_p$.*

PROOF. Let Φ be Δ-symmetric in I_q, $\mathscr{F} \to x$ relative to p. If $\mathscr{U} \geqq \Phi(\mathscr{F})$, $\mathscr{U}$ an u.f., let $\mathscr{H}$ be an u.f. such that $\mathscr{H} \geqq \mathscr{F}$ and $\mathscr{U} \geqq \Phi(\mathscr{H})$. By assumption $\mathscr{H}$ q-converges to x, so $\mathscr{U}$ q-converges to x by Proposition 1.1. Since p is a pseudo topology, $\Phi(\mathscr{F}) \to x$ relative to p and $\Phi \in I_p$.

PROPOSITION 4.2. *Suppose I is totally bounded, minimal uniformly regular, $I \in [q]$. Then $I = I_q$ and q is a regular topology.*

PROOF. By Proposition 0.3 of [4], q is a regular topology which agrees, on u.f.'s, with λq, the topological modification of q. By Lemma 4.1, $I_{\lambda q} \leqq I_q \leqq I$ and, by Proposition 1.2, $I_{\lambda q}$ is uniformly regular. Thus, $I_{\lambda q} = I_q = I$.

DEFINITION. Let q be a regular, Hausdorff topology on X. Then q is *semiminimal regular* if, whenever $p < q$, p a regular Hausdorff topology, there exist u.f.'s $\mathscr{F}$, $\mathscr{G}$ and $x \in X$ such that $\mathscr{F}$, $\mathscr{G}$ are not q-convergent, $\mathscr{F}$ p-converges to x, $\mathscr{G}$ does not p-converge to x.

Note that a minimal regular topology is obviously semi-minimal regular.

THEOREM 4.3. *Let q be a Hausdorff convergence structure. Then I_q is minimal uniformly regular if and only if q is a semi-minimal regular topology.*

PROOF. If I_q is minimal uniformly regular, then q is a regular, Hausdorff topology by Proposition 4.2. Suppose $p < q$, p a regular Hausdorff topology. Then $I_p \not\geqq I_q$ so there is a Δ-symmetric $\Phi \in I_q - I_p$. This means that there is a filter $\mathscr{F}_1$ and a point $x \in X$ such that $\mathscr{F}_1$ p-converges to x and $\Phi(\mathscr{F}_1)$ does not p-converge to x. Let $\mathscr{F} \geqq \mathscr{F}_1$ be an u.f. such that $\Phi(\mathscr{F})$ does not p-converge to x.

Notice that $\mathscr{F}$ is not q-convergent, for if $\mathscr{F}$ q-converges to x (x is the only possibility.), then $\Phi(\mathscr{F})$ q-converges to x since $\Phi \in I_q$; thus $\Phi(\mathscr{F})$ p-converges to x, which is a contradiction.

Now some u.f. $\mathcal{G} \geqq \Phi(\mathcal{F})$ must fail to p-converge to x. If $\mathcal{G}$ q-converges to some z, then $\mathcal{F}$ has z as q-adherent point by Lemma 2.8. Hence $\mathcal{F}$ has z as p-adherent point so $z = x$. But then $\mathcal{G}$ p-converges to x, a contradiction. In summary, we have u.f.'s $\mathcal{F}$, $\mathcal{G}$, neither of which is q-convergent, $\mathcal{F}$ p-converges to x, $\mathcal{G}$ does not p-converge to x. So q is semi-minimal regular.

Conversely, suppose that q is a semi-minimal regular topology. Then I_q is uniformly regular. Assume $I < I_q$, I uniformly regular, $I \in [p]$. By [7], I_q is totally bounded so I is totally bounded. Hence, by results of [**4**] and Lemma 4.1, λp is a regular Hausdorff topology and $I_{\lambda p} \leqq I < I_q$. By assumption there are u.f.'s $\mathcal{F}$, $\mathcal{G}$ and a point $x \in X$, such that $\mathcal{F}$, $\mathcal{G}$ are not q-convergent, $\mathcal{F}$ λp-converges to x, $\mathcal{G}$ does not λp-converge to x. From the characterization of I_q, $\mathcal{G} \times \mathcal{F} \in I_q$ so $\mathcal{G} \times \mathcal{F} \in I_{\lambda p}$. Then $\mathcal{G} \geqq (\mathcal{G} \times \mathcal{F})(\mathcal{F}) \to x$ relative to λp, a contradiction.

Corollary 4.4. *Let (X, q) be a Hausdorff topological space. Then $[q]$ contains a minimal uniformly regular u.c.s. if and only if q is semi-minimal regular.*

Proof. This assertion follows immediately from Theorem 4.3 and Theorem 1.5 of [**4**].

The following will be used to obtain a workable characterization of semi-minimal regularity.

Definition. A regular, Hausdorff topological space (X, q) is *almost locally compact* if there exists $y \in X$ such that the neighborhood filter at x has a base of compact sets for $x \neq y$.

The straightforward proofs of the next two lemmas are omitted.

Lemma 4.5. *Let (X, q) be a regular, Hausdorff topological space which is not almost locally compact. If $p < q$, p a regular, Hausdorff topology, then there is no point $y \in X$ such that each non-q-convergent u.f. p-converges to y.*

Lemma 4.6. *Let (X, q) be a regular, Hausdorff, almost locally compact space which is not compact. Then there is a point $y \in X$ and a compact topology p, $p < q$, such that each non-q-convergent u.f. p-converges to y.*

Theorem 4.7. *A regular, Hausdorff topology is semi-minimal regular if and only if it is either compact or not almost locally compact.*

THEOREM 4.7. *A regular, Hausdorff topology is semi-minimal regular if and only if it is either compact or not almost locally compact.*

PROOF. If q is semi-minimal regular but not compact, then q is not almost locally compact as a consequence of Lemma 4.6.

Conversely, if q is not almost locally compact, let p be a regular, Hausdorff topology, $p < q$. There is an $x \in X$ and an u.f. $\mathscr{F}$ which p-converges to x but does not q-converge to x. By Lemma 4.5, there is a non-q-convergent u.f. $\mathscr{G}$ which does not p-converge to x. Hence q is semi-minimal regular. The result follows from this and the obvious fact that a compact, Hausdorff topology is semi-minimal regular.

COROLLARY 4.8. *There is a minimal uniformly regular u.c.s. whose induced convergence structure is neither minimal regular as a convergence structure nor minimal regular as a topology.*

PROOF. By Theorem 4.7, the usual topology q on the rationals is semi-minimal regular so, by Theroem 4.3, I_q is minimal uniformly regular. But it is clear from results of [1] and [5] that q is not minimal regular as a topology or as a convergence structure.

CONCLUDING REMARKS. In the study of minimal uniform regularity, the two most basic problems are the following:

(1) Characterize the u.c.s.'s I which are minimal uniformly regular;

(2) Characterize the convergence structures q such that $[q]$ contains a minimal uniformly regular member.

We have solved Problem (1) for the case when I is totally bounded and Problem (2) for the case when q is a topology. Both problems remain unsolved in the general case.

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