

Schröder Material at the Russell Archives

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Abstract. A description is given of the holdings of the Bertrand Russell Archives relating specifically to Schröder. These include Russell's copy of various works of Schröder, as well as Russell's none too numerous or detailed notes on these materials. There is no Russell-Schröder correspondence, although Schröder figures in Russell's correspondence with others, primarily with P.E.B. Jourdain and with Louis Couturat.

Russell's appraisal of Schröder's work was largely negative, and there is relatively little in the Russell Archives that relates to Schröder. What material there is, however, reenforces the conclusion drawn from published pronouncements in *The principles of mathematics* and elsewhere that Russell had little regard for Schröder's work in particular or for the entire algebraic tradition in general.

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The inscriptions on the title page of Russell's copy of the first and third volumes of Schröder's *Vorlesungen über die Algebra der Logik*, indicate that Russell acquired his copy of the work in September 1900. (Both inscriptions read "B. Russell, September 1900.") It was during the period from the beginning of 1899 to 1913, and especially during the decade that followed his acquisition of the *Algebra*, from 1901 to 1910, that Russell did his most significant work in logic. During the very earliest part of this period, that is especially from early 1899 to 1902, Russell worked particularly on the logic of relations, as a list of his publications and unpublished logical writings of that period clearly suggests: these works include the unpublished 20-page manuscript *The classification of relations* dating from January 1899, and the published papers *Sur la logique des relations avec des applications à la théorie des séries* (*Revue de Mathématiques* 7 (1901), 115–148), *On the notion of order* (*Mind* (n.s.) 10 (1901), 30–51), *On finite and infinite cardinal numbers* (*American Journal of Mathematics* 24 (1902), 378–383; which is section III of Whitehead's paper *On cardinal numbers*), and *Théorie général des séries bien-ordonnées* (*Revue de Mathématiques* 8 (1902), 12–43).

In the *Principles of mathematics*, Russell [1903, p. 10] wrote of the *Algebra* that Schröder produced a work which gave "by far the most complete account of the non-Peanesque methods" of logic ever to have been published; and in his intellectual autobiography [1959, p. 65; 1985, p. 51], he added that in the *Algebra*, Schröder "published a work in three big volumes summarising all that had previously been done" in logic. Nevertheless, Russell's published and unpublished writings of the period suggest, sometimes implicitly, sometimes explicitly, that their author did not have a high regard for the Boole–Schröder calculus.

There are numerous examples, both published and unpublished, of Russell's negative attitude towards the Boole–Schröder algebraic logic. The published ones are already familiar. In the *Principles of mathematics*, for example, Russell [1903, p. 24] wrote concerning the calculus of relations that "Peirce and Schröder have realized the great importance of the subject, but unfortunately their methods, being based, not on Peano, but on the older Symbolic Logic derived (with modifications) from Boole, are so cumbrous and difficult that most of the applications which ought to be made are practically not feasible." Similar, often harsher, judgments are

expressed in others of Russell's writings, for example in Russell's treatment of Norbert Wiener's 1913 Harvard University doctoral thesis comparing Schröder's *Algebra* with Whitehead and Russell's *Principia*. In response to Wiener's favorable treatment of Schröder's work, Russell claimed that Wiener had dealt only with "the more conventional parts of *Principia Mathematica*" ([Russell 1913; quoted by [Grattan-Guinness 1975, p. 130]). A much more detailed examination of turn-of-the-century appraisals of the work of Peirce, Schröder, and Russell, and of Russell's negative attitude towards Peirce and Schröder in particular and the non-Peanesque tradition in general, is given by [Anellis and Houser, forthcoming].

The reactions of algebraic logicians to Russell's negative appraisal of the algebraic tradition of Boole, Peirce, and Schröder in general and to Peirce's and Schröder's work in particular is perhaps best – albeit with some hyperbole – exemplified by Chrisitine Ladd-Franklin's remark (probably made for a Columbia University class lecture; quoted by [Anellis & Houser, forthcoming, p. 15]) that "It should now be clear how the logic of *Principia* is related to the logic we have presented, following the materials of Peirce and Schröder... . But Whitehead and Russell plainly 'imply' that P[eirce] and S[chröder] were absolutely non-existent!" A more accurate and perhaps also more impartial summary of Russell's view of the work of Schröder was given in Wiener's doctoral thesis (quoted by [Grattan-Guinness 1975, p. 108]), with the statement that "Russell claimed that the Peirce–Schröder approach was inferior with regard to...technical fluency... ."

We should not be overly surprised to learn, then, that there is not a significant amount of materials relevant to the study of Schröder to be found in the Russell Archives. What is available, however, deserves closer scrutiny, and I shall therefore give some indication of what is available.

The Bertrand Russell Archives (BRA) are kept at the Mills Memorial Library of McMaster University, in Hamilton, Ontario, Canada. The Archivist is Kenneth Blackwell. Blackwell is preparing BRACERS (Bertrand Russell Archives Catalogue Entry Retrieval System), a complete electronic catalog of BRA holdings. The *Catalogue of Bertrand Russell Collection*, compiled by John Slater [1988], includes sections on books, pamphlets, articles, critical works, and lists 844 items. McMaster University obtained the BRA in 1968, when Russell sold his papers and correspondence (see [Trelford 1968]). The BRA is comprised of several archives,

including the section called "Mathematical Logic Manuscripts" which contains the bulk of the mathematical and logical documents. In addition, the BRA contains a section devoted to Russell's personal library. On the shelves holding the books owned by Russell, we will find Russell's copy of the three volumes of the first edition (i.e. vols. I, II, Pt. I, III) of Schröder's *Algebra*; Russell's manuscript notes on the *Algebra* may be found in a folder labelled file #230:030460. Russell's copies of Schröder's papers *Der Operationskreis des Logikkalkul* and *Sur une extension de l'idée d'ordre* are located in Russell's offprint file. General descriptions of the contents of the logic papers of the BRA are given by [Grattan-Guinness 1974; 1975a; 1985].

Between February 1891 and March 1902, Russell maintained a notebook in which he kept a running log of his readings. The list, entitled "What Shall I Read?" (and now published as "Appendix II" of vol. 1 of Russell's *Collected papers*), is incomplete, since it does not include books which Russell studied from 1890 to 1893 to prepare for the Mathematical Tripos. Nor does it include a number of other books which Russell is known to have owned and to have read. None of the material of Schröder which Russell owned and read is entered in this catalog of readings.

That Russell acquired his copy of Schröder's *Vorlesungen über die Algebra der Logik* in September 1900 is evidenced not only by the entry of his name and the date on the title page, but also by his mention of it in a letter of 1910 to P.E.B. Jourdain, in which the September 1900 date is given. That Russell read the work is evidenced by the marginal notes which Russell entered in the text, as well as by the notes in file #230:030460.

Russell's marginal notations in his copy of the *Algebra* are not numerous, but are scattered throughout the three volumes. Many of these "notes" consist of little more than underlinings or marginal bracketings of various passages of text. Many of these underlinings involve passages in which Peirce is concerned; thus, for example, in the passage on p. 33 in vol. III which reads

Peirce bezeichnet das identische Produkt „a,b“. Ganz abgesehen davon, dass dieses *Komma* als Malzeichen für ein kommutative Knüpfung wegen seiner Unsymmetrie hinsichtlich

rechts und links als weniger geeignet erscheint, muss ich solche Verwendung eines so häufig als Interpunktionszeichen gebrauchten Trennungszeichens nach wie vor für *gänzlich unannehmbar* erklären wegen der Verwirrung die sie anzurichten nicht verfehlen kann sowohl und vor allem *im Texte*, als auch in den Formeln, wo Funktionen von mehreren Argumenten in Betracht kommen, die ja auch durch Kommata zu trennen wären,

Russell has underlined in pencil the word "Kommata" and emphasized it with an exclamation point in the margin.

Although most of Russell's notations involve little more than underlinings or similar means of emphasis, there are a few comments scattered through the text. Most of these comments are negative, and many are directly concerned with Russell's disagreement with Schröder on the nature of classes and the role which individuals play in the development of a class. There is in particular Russell's distinction between a class and a collection, the latter being a class which is defined by the enumeration of its members. This is the case for Russell's marginal note alongside the passage on p. 100 of vol. I,

Gerade indem sie die Klasse als eine möglicherweise auch ganz willkürlich zusammengesetzte – um nicht zu sagen „zusammengewürfelte“ – in's Auge fasst, wird die Logik der *Klassen*, unter denen von selbst auch die Umfänge aller Begriffe mit figurieren, eine wesentlich höhere Allgemeinheit erzielen als jede Logik, welche von vornherein nur von den Inhalten der Begriffe handeln will,

where Russell asserts that all collections are classes. These issues are dealt with at some length by Russell in the *Principles*. Along these same lines, Russell indicates in the margin alongside the passage on p. 319 of vol. II,

...Daher stellen wir uns jetzt die Aufgabe, wenn wenigstens der Begriff des Gebietes, der Klasse überhaupt als bekannt gilt und das

Verständniss der Formelsprache des Kalkuls vorausgesetzt werden darf, zur *Definition des Punkts und Individuum* herabzusteigen,

that individuals are required before there can be classes, with similar remarks on the pages that follow. The most explicit marginal note expressing Russell's criticism of Schröder's treatment of classes and collections is found on p. 321 of vol. II, regarding Schröder's definition (β) at the top of the page of *individuals*, according to which

$$(i \neq 0) \prod_x \{(ix_1 \neq 0) = 0\}, = i$$

that, unlike Peano, Schröder is unable to distinguish between a class containing only one individual and the one individual which that class contains; more precisely, Russell wrote there that

Schröder cannot dist. as Peano does, betw. a class containing only one indiv. & the one indiv wh. it contains.

Obs. also: What? Defs? is the cls. individual not any part^{al} member of this cls. This illustrates the need of P's \in : for any class is contained in the class individual; wh = universe; what we need is $x \in$ indiv. not $x \neq$ indiv.

Similarly, in connection with Schröder's treatment on p. 349 of vol. II of the number 2 as a quantity (Anzahl) of individuals of a class, Russell wrote that

Peano is better: an Individual is whatever has the relⁿ \in to some other term:

$$x \in y . \supset . x \in \text{Indiv}$$

This is not a formal def & none can be given. Any formal df involves a circle.

In general, Russell's remarks are either implicit or explicit expressions of the superiority of Peano's methods and notations over those of Schröder. In only one instance, however, does Russell specifically and peremptorily dismiss Schröder's statement (concerning Postulate ((1+)), on p. 213 of vol. I) as outright rubbish, without explanation, simply writing "Rot!" in the margin. Again, this concerns Russell's view of the distinctions between classes and collections and the treatment of collections as individuals. Russell made a similar remark about the falsity of Schröder's postulate (p. 338, vol. II) that "Jede von 0 verschiedene (nicht inhaltsleere) Klasse lässt sich darstellen als eine identische Summe von lauter (unter sichverschidenen) Individuen" in his unpublished 28-page manuscript of 1901 on *Recent Italian work on the foundations of mathematics*.

The file containing Russell's notes on Schröder's *Algebra* ([Russell 1900]) hold six pages of Russell's manuscript notes (on eight sides) and consist almost exclusively of lists of those formulae from the *Algebra*. The pages are respectively headed "Schröder III. Lecture VII", "Logic founded on diversity", "Schröder volume III [from p. 26]", "Schröder III", "Schröder III. p. 89ff" (and with "Lecture IV" on the same side, about one-third of the way down from the top of the page), "Schröder III, Lecture IV (continued)", and "Schröder III, Lecture IV (continued)" (and with "Lecture V" on the same side, about mid-page).

The BRA does not include any correspondence between Russell and Schröder, and it is believed that the two never corresponded. There are, however, several mentions of Schröder in Russell's letters. In a letter to Philip Jourdain dated 15 April 1910 (and quoted in [Grattan-Guinness 1977, p. 134]), Russell expressed the views which he reiterated in his reply to Wiener, that Schröder's methods were inferior in regard to their technical fluency; "I read Schröder on Relations in September 1900 and found his methods hopeless," he wrote. This, then, is the letter in which Russell confirms that he read the *Algebra* in September 1900. There is also mention of Schröder in an exchange between Russell and French historian and philosopher of logic Louis Couturat. In a letter to Couturat of 21 January 1901, Russell (Russell Archives item R36) compares his work extending Peano's methods to series to Schröder's work on order, and expresses the opinion once more that treatment by Peanesque methods is much more felicitous than the Schröderian

treatment. Russell was particularly concerned about making a distinction between class inclusion and material implication (see, e.g., *Sur la logique des relations avec des applications à la théorie des séries*). Couturat (Russell Archives item C35) replied on 27 January 1901 that he agreed with the necessity of distinguishing between \in and \supset , but also warned against denigrating the Boole–Schröder system or dismissing it in favor of blind adherence to Peano. A general discussion of the Russell–Couturat correspondence is given by [Schmid 1983].

The material in the archives reenforces the view, examined in [Anellis & Houser, forthcoming] that Russell was extremely critical not only of Schröder, but of the entire algebraic tradition from Boole onward, despite the fact that Peano, to whom Russell considered himself the most indebted, belonged to that tradition, and despite the fact that contemporaries of Whitehead and Russell saw the *Principia* essentially as the apogee of that tradition.

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Schüler III.

$$\prod_{u,v} \text{ or } \prod_u \cdot \prod_v (A_u \in B_v) = (\sum_u A_u \in \prod_v B_v)$$

[In words: If every prop. of one class implies every prop. of another, then any of one implies all of the other.]

$$\sum_{u,v} \text{ or } \sum_u \sum_v (A_u \in B_v) = (\prod_u A_u \in \sum_v B_v)$$

[If some prop. of one class implies some one of another, then all of one together imply some one of the other.]

$$\prod_u (A_u = 0) = \cdot \sum A_u = 0 \quad \sum_u (A_u = 0) = \cdot \prod_u A_u = 0$$

$$\sum_u \prod_v A_{uv} \in \prod_v \sum_u A_{uv} \quad \text{but not v.v.}$$

1 means approx? identical with, 0 means diff! from.

- 1) $(a \in b)(b \in c) \in (ac \in bd)(a+c \in b+d)(a;c \in b;c)(a+c \in b+d)$
- 4) $a; (b+c) = a;b + a;c$ $a \neq bc = (a \neq b)(a \neq c)$
 $(a+b); c = a;c + b;c$ $ab \neq c = (a \neq c)(b \neq c)$
- 5) $a; bc \in a;b \cdot a;c$ $a \neq b + a \neq c \in a \neq (b+c)$
 $ab; a \in a; \cdot b;c$ $a \neq c + b \neq c \in (a+b) \neq c$
- 6) $a;(b;c) = (a;b); c = a;b;c$ $a \neq (b \neq c) = (a+b) \neq c = a \neq b \neq c$
- 7) $a;(b+c) \in a;b \neq c$ $(a+b); c \in a \neq b \neq c$
- 8) $\bar{a} = a$, $\bar{\bar{a}} = \bar{a}$, $\bar{\bar{\bar{a}}} = a$
- 10) $\overline{a;b} = \bar{a} + \bar{b}$ $\overline{a \neq b} = \bar{a}; \bar{b}$
- 11) $\overline{ab} = \bar{a} \bar{b} = \bar{\bar{b}} \bar{a}$ $\overline{a+b} = \bar{a} + \bar{b}$
- 12) $\overline{a;b} = \bar{b}; \bar{a}$ $\overline{a \neq b} = \bar{b} \neq \bar{a}$

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