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ON BANACH SPACES HAVING THE PROPERTY G. L.

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A Banach space E has the property G. L. if every absolutely summing operator defined on E factors through an L_1-space. Some properties of spaces having G. L. property are investigated, using methods of Banach ideals of operators.

1. Introduction and notations. The property G. L. is known to be shared by a number of important classes of Banach spaces: in [6] it is shown that if E'' is isomorphic to a complemented subspace of a Banach lattice (in particular, if E has local unconditional structure in the sense of [4]) then E has the G. L. property. Subspaces of L_1 spaces as well as quotients of $C(K)$ spaces have G. L. property. Moreover, in [17] it is shown that if E is a subspace of a Banach space F *s.t.* $\Pi_2(\mathscr{L}_\infty, F) = \mathscr{L}(\mathscr{L}_\infty, F)$ (in particular if F has cotype 2) and F has the property G. L. then E has the property G. L. In fact, it is easy to see that it is enough for E to be finitely represented in F. In this paper, we try to investigate the property G. L. using methods of Banach ideals of operators. It is shown that this property is characterized by a perfect ideal $[\Gamma, \gamma]$. We obtain a description of the conjugate ideal $[\Gamma^*, \gamma^*]$ and deduce that $[\Gamma, \gamma]$ is a symmetric ideal hence E has G. L. iff E' has it.

It is also shown that a number of properties, known to hold for spaces having *l.u.st.* in the sense of [4] are common to all the spaces having G. L. For example, if E is a space having G. L. which does not contain l_∞^n-*s* uniformly, then either E contains l_1^n-*s* uniformly and uniformly complementably, or E does not contain l_1^n-*s* uniformly at all.

It follows that if E is a space having G. L. and F a Banach space, then there exist compact nonnuclear operators from E to F and from F to E. These are partial generalizations to results of Davis and Johnson (see [2] and [9]). We show also that for spaces having G. L. the property $\Pi_2(\mathscr{L}_\infty, E) = \mathscr{L}(\mathscr{L}_\infty, E)$ implies that E is of cotype 2; we show a dual implication as well.

The paper is divided into two parts. In §2 we describe some tools in Banach ideals of operators; in §3 we use these tools in investigating spaces having G. L. It seems to us that these tools may be useful in other contexts.

The notations are of two kinds:

(1) *General notations.* We use standard notations of Banach

space theory. If E is a Banach space its dual space is E' and for $x \in E$, $x' \in E'$ we denote by $\langle x, x' \rangle$ the scalar product of x and x'.

We deal with Banach spaces over the field of real numbers. Modification to the complex numbers case is straightforward. For a positive measure space (Ω, Σ, μ) and $1 \leqq p \leqq \infty$ we denote by $L_p(\mu)$ the Banach space of scalar, μ-measurable functions f with $|f|^p$ integrable (with classical modification for $p = \infty$) with the usual norm.

We denote by $L_p(E) = L_p(\mu, E)$ the space of Bochner measurable E-valued functions with $\|f(\cdot)\| \in L_p(\mu)$ equipped with the norm $\|f\| = \| \ \|f(\cdot)\| \ \|_{L_p(\mu)}$.

The term "operator" means "bounded linear operator between Banach spaces". If E, F are Banach spaces, $\mathscr{L}(E, F)$ is the Banach space of operators from E into F equipped with the norm of operators.

Let E, F be Banach spaces; we say that E is finitely represented in F (abbreviation: $Ef.rF$) if for every finite dimensional subspace E_1 of E and $\varepsilon > 0$ there exists a subspace F_1 of F and an isomorphism $u: E_1 \to F_1$ with $\|u\| \|u^{-1}\| \leqq 1 + \varepsilon$. If P is a property which makes sense for Banach spaces we say that E has super-P if every space F with $Ff.rE$ has the property P.

(2) *Definitions and notations concerning Banach ideals of operators and tensor products of Banach spaces.* A standard reference in Banach ideals of operators is [8] (see also, [15] and [14]); as a reference concerning tensor products one can use [20]. If $[A, a]$ is a Banach ideal of operators we denote by $[A^*, a^*]$ the conjugate ideal and say that $[A, a]$ is perfect if $[A, a] = [A^{**}, a^{**}]$. $[A', a']$ is the adjoint ideal ($T \in A'(E, F)$ iff $T' \in A(F', E')$).

Let $[A, a]$ be a normed ideal of operators and E, F Banach spaces, a norm (called "an ideal norm") is naturally induced on the tensor product $E \otimes F$ by considering it as algebraically contained in $\mathscr{L}(E', F)$. We denote $E \otimes F$ with this norm by $E \otimes_a F$ and its completion by $E \hat{\otimes}_a F$. Let E, F be Banach spaces and $u \in E \otimes F$. Let E_1, F_1 be subspaces of E and F respectively *s.t.* there is a representation of u as $u = \sum_{i=1}^n x_i \otimes y_i$ with $x_i \in E_i$, $y_i \in F_1$ for all i. We denote by $a(u, E_1, F_1)$ the norm of u as an element of $E_1 \otimes_a F_1$. If E and F are not considered as subspaces of some other spaces we denote $a(u, E, F) = a(u)$.

We say that an ideal norm a is semi-tensorial norm if for every pair of Banach spaces E, F, one which is finite dimensional, and every $u \in E \otimes F$ hold: $a(u) = \inf \{a(u, E_1, F_1);\ E_1 \subset E,\ F_1 \subset F,\ E_1$ and F_1 finite dimensional and $u \in E_1 \otimes F_1\}$.

We list here a number of ideals that we shall use in the sequel.

(a) $[\mathscr{L}, \|\cdot\|]$ the ideal of all bounded operators.

(b) $[\Pi_p, \pi_p]$ $(1 \leqq p \leqq \infty)$ the ideal of p-summing operators.

(c) $[I_p, i_p]$ the ideal of p-integral operators. $U \in I_p[E, F]$ if there exists a probability space (Ω, Σ, μ) and operators $V \in \mathscr{L}(E, L_\infty(\mu))$, $W \in \mathscr{L}(L_p(\mu), F'')$ $s.t. WiV = j_F U$ where i is the formal "inclusion" map of $L_\infty(\mu)$ into $L_p(\mu)$ and j_F the canonnical inclusion of E into E''.

We define $i_p(U) = \inf\{\|V\|\|W\|; V, W, (\Omega, \Sigma, \mu)$ as in the definition$\}$. We say that U is strongly p-integral if the preceeding factorization is for U instead of $j_F U$.

(d) $[N_p, \nu_p]$ $1 \leqq p < \infty$ the ideal of p-nuclear operators.

(e) $[\Gamma_p, \gamma_p]$ the ideal of operators factorizable through L_p. $U \in \Gamma_p(E, F)$ if there exists an $L_p(\mu)$ space and operators $A \in \mathscr{L}(E, L_p(\mu))$, $B \in \mathscr{L}(L_p(\mu), F'')$ $s.t. j_F U = BA$. We define $\gamma_p(U) = \inf \|B\|\|A\|$.

(f) (A new definition). $[M, \mu]$ the ideal of operators factorizable through a Banach lattice. $U \in M(E, F)$ iff there exists a Banach lattice L and $A \in \mathscr{L}(E, L)$, $B \in \mathscr{L}(L, F'') s.t. j_F U = BA$. $\mu(U) = \inf \|B\|\|A\|$. Using ultraproducts of Banach spaces ([1]) or the methods of [5] one can show that $[M, \mu] = [H^{**}, \eta^{**}]$ where $[H, \eta]$ is the ideal of weakly nuclear operators introduced in [7]. Therefore a Banach space E has $l.u.st$ in the sense of [6] iff E'' is isomorphic to a complemented subspace of a Banach lattice ([5]).

It is known that the ideals in (a), (b), (c) and (e) are perfect and the same is true for the ideal in (f). It is also not hard to check that all the ideal norms on tensor products induced by the above ideals are semi-tensorial.

Let E, F be Banach spaces, *the greatest tensor-norm*, π, is defined on $E \otimes F$ by $\pi(u) = \inf\{\sum_{i=1}^n \|x_i\|\|y_i\|; u = \sum_{i=1}^n x_i \otimes y_i\}$ for $u \in E \otimes F$. There is an identification $(E \hat{\otimes}_\pi F)' = \mathscr{L}(F, E')$ defined by

$$\langle u, T\rangle = \text{trace } Tu = \sum_{i=1}^n \langle x_i, Ty_i\rangle$$

for

$$u = \sum_{i=1}^n x_i \otimes y_i \in E \otimes F .$$

2. Let I be an index set and $\{[A_i, a_i]\}_{i \in I}$ a family of normed ideals of operators.

DEFINITION 2.1. (a) The greatest lower bound $[\bigwedge_i A_i, \bigwedge_i a_i]$ of the family is defined by:

$$\Big(\bigwedge_i A_i\Big)(E, F) = \{T \in \mathscr{L}(E, F);\ \forall i,\ T \in A_i(E, F) \text{ and } \sup_i a_i(T) < \infty\}$$

$$\Big(\bigwedge_i a_i\Big)T = \sup_i a_i(T) \quad \text{for} \quad T \in \Big(\bigwedge_i A_i\Big)(E, F)\,.$$

(b) The least upper bound $[\bigvee_i A_i, \bigvee_i a_i]$ of the family is defined by:

$$\Big(\bigvee_i A_i\Big)(E, F) = \{T \in \mathscr{L}(E, F);\ T = \sum_{j \in J} T_j;\ J \subset I,\ J \text{ finite and for all } j \in J\ T_j \in A_j(E, F)\}$$

$$\Big(\bigvee_i a_i\Big)(T) = \inf\Big[\sum_{j \in J} a_j(T_j)\Big] \quad \text{for} \quad T \in \Big(\bigvee_i A_i\Big)(E, F)\,,$$

the inf being taken over all finite subsets $J \subset I$ *s.t.* there is a representation $T = \sum_{j \in J} T_j$ with $T_j \in A_j(E, F)$.

PROPOSITION 2.2. (a) *$[\bigwedge_i A_i, \bigwedge_i a_i]$ and $[\bigvee_i A_i, \bigvee_i a_i]$ are normed ideals of operators.*

(b) *If for all i $[A_i, a_i]$ are Banach ideals then so is $[\bigwedge_i A_i, \bigwedge_i a_i]$ and if, in addition, I is finite, then $[\bigvee_i a_i, \bigvee_i a_i]$ is also a Banach ideal.*

(c) *If for all i $[A_i, a_i]$ are perfect then so is $[\bigwedge_i A_i, \bigwedge_i a_i]$.*

The proof is routine.

PROPOSITION 2.3. $[\bigwedge_i A_i^*, \bigwedge_i a_i^*] = [(\bigvee_i A_i)^*, (\bigvee_i a_i)]$.

Proof. Consider the following diagram, in which E, F are Banach spaces, E_1, F_1 finite dimensional Banach spaces and T, U, S, V operators.

$$\begin{array}{ccc} E & \xrightarrow{T} & F \\ {\scriptstyle V}\big\downarrow & & \big\downarrow{\scriptstyle U} \\ E_1 & \xrightarrow{S} & F_1 \end{array}$$

(a) Suppose $T \in (\bigvee_i A_i)^*(E, F)$ then

$$|\text{trace } TVSU| \leqq \Big(\bigvee_i a_i\Big)^*(T)\|V\|\|U\|\Big(\bigvee_i a_i\Big)(S)\,,$$

hence, for all $i \in I$

$$|\text{trace } TVSU| \leqq \Big(\bigvee_i a_i\Big)^*(T)\|V\|\|U\|a_i(S)\,,$$

therefore $\forall i \in I\ a_i^*(T) \leqq (\bigvee_i a_i)^*(T)$ and it follows that

$$T \in \Big(\bigwedge_i A_i^*\Big)(E, F) \quad \text{and} \quad \Big(\bigwedge_i a_i^*\Big)(T) \leqq \Big(\bigwedge_i a_i\Big)^*(T)\,.$$

(b) Suppose $T \in \bigwedge_i A_i^*(E, F)$. Let $J \subset I$ be finite and $S = \sum_{j \in J} S_j$ be a representation of S *s.t.*

$$\sum_{j \in J} a_j(S_j) \leqq \Big(\bigvee_i a_i\Big)(S) + \varepsilon\,.$$

We have:

$$\begin{aligned}
|\text{trace } TVSU| &\leqq \sum_{j \in J} |\text{trace } TVS_jU| \\
&\leqq \sum_{j=1}^{n} a_j^*(T)\,\|V\|\,\|U\|\,a_j(S_j) \\
&\leqq \sup_i a_i^*(T)\,\|V\|\,\|U\|\Big(\sum_{j=1}^{n} a_j(S_j)\Big) \\
&\leqq \Big(\bigwedge_i a_i^*\Big)(T)\,\|V\|\,\|U\|\Big[\Big(\bigvee_i a_i\Big)(S) + \varepsilon\Big]\,,
\end{aligned}$$

therefore $T \in (\bigvee_i A_i)^*(E, F)$ and $(\bigvee_i a_i)^*(T) \leqq (\bigwedge_i a_i^*)(T)$.

COROLLARY 2.4. *If $[A_i, a_i]$ are perfect, then*

$$\Big[\Big(\bigwedge_i A_i\Big)^*, \Big(\bigwedge_i a_i\Big)^*\Big] = \Big[\Big(\bigvee_i A_i^*\Big)^{**}, \Big(\bigvee_i a_i^*\Big)^{**}\Big]\,,$$

in particular, if E and F are finite dimensional then (without assuming perfectness of $[A_i, a_i]$) for every $T \in \mathscr{L}(E, F)$ $(\bigwedge_i a_i)^*(T) = (\bigvee_i a_i^*)(T)$.

Proof. Since for all i $[A_i, a_i] = [A_i^{**}, a_i^{**}]$ we get

$$\Big(\bigwedge_i A_i\Big)^* = \Big(\bigwedge_i A_i^{**}\Big)^* = \Big[\Big(\bigvee_i A_i^*\Big)^*\Big]^* = \Big(\bigvee_i A_i^*\Big)^{**}$$

with equality of the norms. The second assertion is an obvious consequence of the first.

DEFINITION 2.5. (a) Let $[A, a]$ and $[B, b]$ be normed ideals of operators and G a fixed Banach space. We define for Banach spaces E, F:

$$\Big(\frac{A}{B}\Big)_G(E, F) = \{T \in \mathscr{L}(E, F);\ \forall U \in B(F, G) \quad UT \in A(E, G)\}\,.$$

From the closed-graph theorem it follows that for every $T \in (A/B)_G(E. F)$ there exists a $k > 0$ *s.t.* for all $U \in B(F, G) a\ (UT) \leqq$

$kb(U)$. We define $(a/b)_G(T) = \inf\{k;\ k \text{ as above}\}$.

(b) Let $[A, a]$ and $[B, b]$ be normed ideals of operators, E and F Banach spaces. We define

$$\frac{A}{B}(E, F) = \{T \in \mathscr{L}(E, F);\ \text{for every Banach space } G \text{ and } U \in B(F, G)\ \ UT \in A(E, G)\}\ .$$

It can be shown in a standard way that for every $T \in A/B(E, F)$ there exists a $k > 0$ *s.t.* for every Banach space G and $U \in B(F, G)$ $a(UT) \leqq kb(U)$. We define $a/b(T) = \inf\{k;\ k \text{ as above}\}$.

(c) Let $[A, a]$, $[B, b]$, E and F be as in (b). We define

$$\frac{A}{B}f(E, F) = \{T \in \mathscr{L}(E, F);\ \exists k > 0\ s.t. \text{ for every Banach space } G \text{ of finite dimension and } U \in \mathscr{L}(F, G)\ \ a(UT) \leqq kb(U)\}$$

$$\frac{a}{b}f(T) = \inf\{k,\ k \text{ as above}\} \text{ for } T \in \frac{A}{B}f(E, F)\ .$$

PROPOSITION 2.6. *$[(A/B)_G, (a/b)_G]$, $[A/B, a/b]$ and $[A/Bf, a/b\,f]$ are normed ideals of operators.*

If $[A, a]$ is a Banach ideal then these ideals are Banach ideals. If $[A, a]$ is perfect then $[A/B, a/b] = [A/Bf, a/b\,f]$.

Proof. The verification of the first and third assertions is routine. We prove the second assertion for A/B.

Let $\{T_n\}_{n \in N}$ be a Cauchy sequence in $A/B\ (E, F)$. It is easy to check the following facts:

(1) There exists an operator $T \in A/B(E, F)$ *s.t.* for every Banach space G and $U \in B(F, G)$ $a(UT_n - UT) \xrightarrow[n\to\infty]{} 0$.

(2) The numerical sequence $a/b(T_n - T)$ is Cauchy, hence $a/b(T_n - T) \xrightarrow[n\to\infty]{} l \geqq 0$.

It is left to show that $l = 0$. Suppose $l > 0$. By (2) there is an integer n_0 *s.t.* for any $n \geqq n_0$ there exists a Banach space G_n and an operator $U_n \in B(F, G_n)$ with $b(U_n) \leqq 1$ *s.t.* $a(U_n(T_n - T)) > l/2$. We get for $m > n \geqq n_0$.

(3) $l/2 < a(U_n(T_n - T)) \leqq a(U_n(T_n - T_m)) + a(U_n(T_m - T))$.

Choose $n_1 > n_0$ *s.t.* for all U with $b(U) \leqq 1$ and $n, m \geqq n_1$ we have $a(U(T_n - T_m)) < l/8$ (which is possible since $\{T_n\}$ is Cauchy in $A/B(E, F)$). Fix $n > n_1$ and let $m_1 > n_1$ be *s.t.* for $m > m_1$ we have $a(U_n(T_m - T)) < l/8$ (such m_1 exists by 1).

Applying (3) to the fixed n and some $m > m_1$ we get $l/2 < l/4$ which is a contradiction that completes the proof.

PROPOSITION 2.7. *Let $[A, a]$ and $[B, b]$ be normed ideals of operators such that $[A, a]$ is perfect and b is a semi-tensorial norm. Then $[A/B, a/b]$ is perfect.*

Proof. By Proposition 2.6 it is enough to show that $[A/Bf, a/b\, f]$ is perfect. Let $T \in (A/B\, f)^{**}(E, F)$, then for every finite dimensional subspace M of E and finite codimensional subspace N of F $a/b\, f(q_N T i_M) \leqq (a/b\, f)^{**}(T)$ where $i_M: M \to E$ is the inclusion map and $q_N: N \to F/N$ the canonical surjection. Let G be a finite dimensional Banach space and $U \in B(F, G)$, since b is semi-tensorial we have:

$$\begin{aligned} b(U) &= \inf\{b(U, F^1, G);\ F^1 \text{ finite dimensional subspace of } F'\} \\ &= \inf b(U_1) \end{aligned}$$

the last infinum is taken over all operators U_1 and finite codimensional subspaces N of F such that U has a factorization of the form:

$$\begin{tikzcd} F \arrow[rr, "U"] \arrow[dr, "q_N"'] & & G \\ & F/N \arrow[ur, "U_1"'] & \end{tikzcd} \tag{1}$$

For given $\varepsilon > 0$ let N and U_1 be as in (1) with $b(U_1) \leqq b(U) + \varepsilon$. We have $a(UTi_M) = a(U_1 q_N T i_M) \leqq b(U_1) a/b\, f(q_N T i_M) \leqq (b(U) + \varepsilon)(a/b\, f)^{**}(T)$. Since ε is arbitrary and $[A, a]$ is perfect it follows that $a(UT) \leqq b(U)(a/b)^{**}(T)$, therefore $T \in A/Bf(E, F)$ and $a/b\, f(T) = (a/b\, f)^{**}(T)$.

PROPOSITION 2.8. *Let $[A, a]$ and $[B, b]$ be normed ideals of operators, E and F Banach spaces of finite dimension and $T \in \mathscr{L}(E, F)$. Then $(a/b\, f)^*(T) = \inf \sum_{i=1}^n a^*(U_i) b(V_i)$, the infinum being taken over all representations of T of the form $T = \sum_{i=1}^n U_i V_i$ with $V_i \in \mathscr{L}(E, G_i)$; $U_i \in \mathscr{L}(G_i, F)$ and G_i finite dimensional Banach spaces.*

Proof. For fixed finite dimensional G and $S \in \mathscr{L}(F, E)$ we have

$$\left(\frac{a}{b}\right)_G (S) = \sup\{a(US);\ U \in \mathscr{L}(E, G),\ b(U) \leqq 1\}\ .$$

Define the operator $\hat{S}: B(E, G) \to A(F, G)$

by $$\hat{S}(U) = US\ . \quad \text{Then}$$

$$\left(\frac{a}{b}\right)_G (S) = \|\hat{S}\|\ .$$

The correspondence $S \leftrightarrow \hat{S}$ enable us to identify $(A/B)_G(F, E)$ with a subspace of $\mathscr{L}(B(E, G), A(F, G))$. Therefore $(A/B)_G^*(E, F) =$

$[(A/B)_G(F, E)]'$ is a quotient space of $A^*(G, F) \otimes_\pi B(E, G)$ with the following identification: for $\phi = \sum_{i=1}^n U_i \otimes V_i \in A^*(G, F) \otimes_\pi B(E, G)$ and $S \in (A/B)_G(F, E)$ we define

$$\begin{aligned}\langle S, \phi\rangle = \langle \phi, \hat{S}\rangle &= \sum_{i=1}^n \langle U_i, \hat{S}(V_i)\rangle \\ &= \sum_{i=1}^n \langle U_i, V_i S\rangle = \sum_{i=1}^n \text{trace } U_i V_i S = \text{trace } TS\end{aligned}$$

where

$$T = \sum_{i=1}^n U_i V_i .$$

From the last discussion it follows that for $T \in \mathscr{L}(E, F)$

$$\left(\frac{a}{b}\right)_G^*(T) = \inf \left\{\sum_{i=1}^n a^*(U_i)b(V_i);\ T = \sum_{i=1}^n U_i V_i;\ V_i \in \mathscr{L}(E, G) U_i \in \mathscr{L}(G, F)\right\} .$$

We complete the proof by noting that

$$\left[\frac{A}{B} f, \frac{a}{b} f\right] = \left[\bigwedge_{\dim G<\infty} \left(\frac{A}{B}\right)_G, \bigwedge_{\dim G<\infty} \left(\frac{a}{b}\right)_G\right]$$

and by using Corollary 2.4 which shows that for finite dimensional E and F

$$\left[\left(\frac{A}{B} f\right)^*, \left(\frac{a}{b} f\right)^*\right] = \left[\bigvee_{\dim G<\infty} \left(\frac{A}{B}\right)_G^*, \bigvee_{\dim G<\infty} \left(\frac{a}{b}\right)_G^*\right].$$

3.

DEFINITION 3.1. We define the ideal $[\Gamma, \gamma]$ by:

$$[\Gamma, \gamma] = \left[\frac{\Gamma_1}{\Pi_1}, \frac{\gamma_1}{\pi_1}\right]. \quad \text{Explicitly:}$$

$T \in \Gamma(E, F)$ iff for every Banach space G and $U \in \Pi_1(F, G)$ $UT \in \Gamma_1(E, G)$. For such an operator T $\gamma(T) = \sup \gamma_1(UT)$, the supremum being taken over all Banach spaces G and $U \in \Pi_1(F, G)$ with $\pi_1(U) = 1$.

DEFINITION 3.2. We say that a Banach space E has the property G. L. (Gordon-Lewis) if for every Banach space G $\Pi_1(E, G) \subset \Gamma_1(E, G)$. Of course, E has property G. L. iff the identity operator on E is in $\Gamma(E, E)$.

PROPOSITION 3.3. *A Banach space E has the property G. L. if* *and only if there exist $k > 0$ s.t. for every finite dimensional Banach space G and $U \in \mathscr{L}(E, G)$ $\gamma_1(U) \leqq k\pi_1(U)$.*

Proof. This is a result of the equality

$$\left[\frac{\Gamma_1}{\Pi_1}, \frac{\gamma_1}{\pi_1}\right] = \left[\frac{\Gamma_1}{\Pi_1}f, \frac{\gamma_1}{\pi_1}f\right]$$

which is, in turn, a consequence of Proposition 2.6 and the fact that $[\Gamma_1, \gamma_1]$ is perfect.

PROPOSITION 3.4. *Let E and F be finite dimensional Banach spaces and $T \in \mathscr{L}(E, F)$. Then* (a) $\gamma^*(T) = \inf[\sum_{i=1}^n \pi_1'(U_i)\pi_1(V_i)]$, *the infinum being taken over all representations of the form $T = \sum_{i=1}^n U_i V_i$ with $V_i \in \Pi_1(E, G_i)$, $U_i \in \Pi_1'(G_i, F)$ and G_i finite dimensional Banach spaces.*

(b) $\gamma^*(T) = \inf[\sum_{i=1}^n \|\mu_i\|\|\nu_i\|]$, *the infimum being taken over all representations of the form $T = \sum_{i=1}^n T_i$ s.t for all i there exist positive Radon measures, μ_i on the unit ball $B(E')$ of E' and ν_i on the unit ball $B(F)$ of F s.t. for all $x \in E$, $y' \in F'$ and $1 \leqq i \leqq n$ hold:*

$$|\langle T_i x, y'\rangle| \leqq \int_{B(E')} |\langle x, x'\rangle| d\mu_i(x') \int_{B(F)} |\langle y, y'\rangle| d\nu_i(y) .$$

Proof. (a) Follows from Propositions 2.8 and 3.3 combined with the fact ([10]) that $[\Gamma_1^*, \gamma_1^*] = [\Pi_1', \pi_1']$.

(b) Is a consequence of (a) and the following lemma which is proved by methods of [10].

LEMMA 3.5. (c) *Let $T \in \mathscr{L}(E, F)$ (E, F not necessarily finite dimensional) then*

$$(1) \qquad \inf \pi_1'(U)\pi_1(V) = \inf \|\nu\|\|\mu\|$$

where the infinum on the left is taken over all Banach spaces G and representations $jT = UV$ with j the canonical inclusion of F into F'', $U \in \Pi_1'(G, F'')$ and $V \in \Pi_1(E, G)$. The infimum on the right is taken over all positive Radon measures μ on $B(E')$ and ν on $B(F'')$ (with the relative ω^*-topologies) s.t. for all $x \in E$, $y' \in F'$ hold

$$|\langle Tx, y'\rangle| \leqq \int_{B(E')} |\langle x, x'\rangle| d\mu(x') \int_{B(F'')} |\langle y', y''\rangle| d\nu(y'') .$$

(d) *If in* (c) *E and F are finite dimensional then the infinum on the left hand side of* (1) *can be taken over all finite dimensional Banach spaces G.*

Proof. (d) follows from (c) since π_1 and π_1' are semi-tensorial (in fact, tensorial) norms. We prove (c).

Let $jT = UV$ be a factorization of jT with $U \in \Pi_1'(G, F'')$ and $V \in \Pi_1(E, G)$. By the Pietsch factorization theorem there exist positive Radon measures, μ on $B(E')$ and ν on $B(F'')$ *s.t.* for $x \in E$, $y' \in F'$ $\| Vx \| \leqq \int_{B(E')} |\langle x, x'\rangle| d\mu(x')$, $\| U'|_{F'} y' \| \leqq \int_{B(F'')} |\langle y', y''\rangle| d\nu(y'')$ and $\|\mu\| \leqq \pi_1(V) + \varepsilon$, $\|\nu\| \leqq \pi_1(U') + \varepsilon$. Therefore $\|\nu\|\|\mu\| \leqq (\pi_1'(U) + \varepsilon)(\pi_1(V) + \varepsilon)$ and

$$(2) \qquad |\langle Tx, y'\rangle| = |\langle Vx, U'y'\rangle| \leqq \int_{B(E')} |\langle x, x'\rangle| d\mu \int_{B(F'')} |\langle y', y''\rangle| d\nu .$$

On the other hand, suppose μ and ν are Radon measures on $B(E')$ and $B(F'')$ respectively *s.t.* (2) hold for every $x \in E$, $y' \in F'$ then we define operators:

$$U_0 \colon F' \longrightarrow L_1(\nu) ; \quad U_0(y') = \langle \cdot, y'\rangle$$

and

$$V_0 \colon E \longrightarrow L_1(\mu) ; \quad V_0(x) = \langle x, \cdot\rangle .$$

Let $H = \overline{U_0(F')}$, $G = \overline{V_0(E)}$ and let $\langle\!\langle \cdot \rangle\!\rangle$ be the bilinear form on $V_0(E) \times U_0(F')$ defined by $\langle\!\langle V_0x, U_0y'\rangle\!\rangle = \langle Tx, y'\rangle$, from (2) it follows that this form is well defined and bounded with norm $\leqq 1$, hence it defines an operator $W \in \mathscr{L}(G, H')$ with $\|W\| \leqq 1$ and $\langle\!\langle V_0x, U_0y'\rangle\!\rangle = \langle WV_0x, U_0y'\rangle$. We have then the following commutative diagram:

$$(3) \qquad \begin{array}{ccccc} E & \xrightarrow{T} & F & \xrightarrow{j} & F'' \\ & \searrow^{U_1} & & \nearrow_{V_1'} & \\ & & G \xrightarrow{W} H' & & \end{array}$$

where U_1 and V_1 are U_0 and V_0 considered as operators into G and H respectively. Of course $\pi_1(U_1) \leqq \|\mu\|$ and $\pi_1(V_1') \leqq \|\nu\|$ which completes the proof of Lemma 3.5 and Proposition 3.4.

REMARK 3.6. In [7] Gordon and Lewis show that for all E, F and $T \in \mathscr{L}(E, F)$

$$(1) \qquad \mu^*(T) = \inf \|\mu\| ,$$

the infinum being taken over all positive Radon measures on $B(E') \times B(F'')$ (with the product of the ω^*-topologies) which satisfy for all x, y':

$$(2) \qquad |\langle Tx, y'\rangle| \leqq \int_{B(E')\times B(F'')} |\langle x, x'\rangle\langle y', y''\rangle| d\mu(x', y'') .$$

In fact, using compactness of the unit balls it is not hard to check that for finite dimensional E and F we can replace "inf $||\mu||$" by "inf $\sum_{i=1}^{n} ||\mu_i|| ||\nu_i||$" in (1); μ_i, ν_i positive Radon measures on $B(E')$ and $B(F)$ respectively *s.t.* for all x, y'

$$(3) \qquad |\langle Tx, y'\rangle| \leqq \sum_{i=1}^{n} \int_{B(E')} |\langle x, x'\rangle| d\mu_i(x') \int_{B(F)} |\langle y, y'\rangle| d\nu_i(y)$$

(all the $\mu_i \otimes \nu_i$ but one may be taken as scalar multiples of $\delta(x_i') \otimes \delta(y_i)$—the products of valuations at points $x_i' \in B(E')$, $y_i \in B(F)$, the one $\mu_i \otimes \nu_i$ left may be a scalar multiple of the product of Lebesgue measures on $B(E')$ and $B(F)$). The difference between μ^* and γ^* is therefore the possibility to represent T as a sum $\sum_{i=1}^{n} T_i$ where each T_i is "majorized" by the product $\mu_i \otimes \nu_i$. It follows of course that $\mu^* \leqq \gamma^*$, hence $\mu \geqq \gamma$ and we get the result of [6]: if E'' is isomorphic to a complemented subspace of a Banach lattice then E has property G. L.

COROLLARY 3.7. $[\Gamma, \gamma] = [\Gamma', \gamma']$, *therefore E has the property G. L. if and only if E' has it.*

Proof. $[\Gamma^*, \gamma^*] = [\Gamma^{*\prime}, \gamma^{*\prime}]$; this is obvious for pairs of finite dimensional Banach spaces from (a) or (b) of Proposition 3.4 and passes over to all pairs of Banach spaces since $[\Gamma^*, \gamma^*]$ is perfect. Now perfectness of $[\Gamma, \gamma]$ gives $[\Gamma, \gamma] = [\Gamma^{**}, \gamma^{**}] = [\Gamma^{*\prime *}, \gamma^{*\prime *}] = [\Gamma^{**\prime}, \gamma^{**\prime}] = [\Gamma', \gamma']$.

The last corollary enables us to prove that a number of properties known to hold for spaces having *l.u.st.* are true also for spaces having the property G. L.

We use the next lemma of Pisier ([16] and [17]) which was originally proved for spaces E with E'' isomorphic to a complemented subspace of a Banoch lattice. However, Pisier's proof uses only the fact that such an E, and also E', has the property G. L.

LEMMA 3.8. *Let E have the property G. L.*

(a) *If E does not contain l_∞^n's uniformly, then there exist q, $2 \leqq q < \infty$ and $C > 0$ s.t.*

(1) *For any E valued operator $A\pi_q(A) \leqq C\pi_1'(A)$.*

(b) *If neither E nor E' contain l_∞^n's uniformly, then there exist q, $2 \leqq q < \infty$, p, $1 < p \leqq 2$ and $C > 0$ s.t.:*

(2) *For any E-valued operator $A\pi_q(A) \leqq C\pi_p'(A)$.*

The next theorem and its corollary is in a certain way a generalization of results of Johnson and Davis ([9] and [2]).

THEOREM 3.9. *Let E be finitely represented in a Banach space F such that F has the property G. L. and F does not contain l_∞^n-'s uniformly. Then either E contains l_1^n-'s uniformly and uniformly complementably or E does not contain l_1^n-'s uniformly.*

We need two lemmas.

LEMMA 3.10. *Let $[A, a]$ and $[B, b]$ be normed ideals of operators s.t. a is a semi-tensorial norm and $[B, b]$ is perfect and right injective (which means: if E, F, G are Banach spaces, $F \subset G$ and $T \in \mathscr{L}(E, F)$ then the b-norms of T considered as operator from E to F or from E to G are the same).*

Let F be a Banach space *s.t.* the following holds:

(1) There exists a $k > 0$ *s.t.* for every Banach space G and $T \in A(G, F)$ $b(T) \leqq ka(T)$.

Let E be a Banach space *s.t.* $Ef.r.F$ then (1) is true for E as well.

Proof. Let G be a Banach space and $T \in A(G, E)$. Let G_1 be a finite dimensional subspace of G and $T_1 = T|_{G_1}: G_1 \to E$. Then $a(T_1) \leqq a(T)$. Since a is semi-tensional and G_1 finite dimensional then $a(T_1) = \inf\{a(T_1: G_1 \to N)$; N a finite dimensional subspace of E with $T_1(G_1) \subset N\}$. Given $\varepsilon > 0$ there exists therefore a finite dimensional subspace $N \subset E$ with $T_1(G_1) \subset N$ *s.t.* $\bar{T}_1: G_1 \to N$ — the astriction of T_1, satisfies $a(\bar{T}_1) \leqq (1 + \varepsilon)a(T_1)$. We can find a $N_1 \subset F$ and an isomorphism $i: N \to N_1$ with $\|i\| \leqq 1$; $\|i^{-1}\| \leqq 1 + \varepsilon$. Let $j: N_1 \to F$ be the inclusion map from N_1 into F, then $a(ji\bar{T}_1) \leqq (1 + \varepsilon)a(T)$ and (1) gives:

$$b(ji\bar{T}_1) \leqq k(1 + \varepsilon)a(T)\,, \quad \text{injectivity of } [B, b]$$

implies now that $b(i\bar{T}_1) \leqq k(1 + \varepsilon)a(T)$. Therefore $b(\bar{T}_1) \leqq k(1 + \varepsilon)^2 a(T)$ which implies $b(T_1) \leqq k(1 + \varepsilon)^2 a(T)$. Since ε is arbitrary and $[B, b]$ perfect we conclude that $b(T) \leqq ka(T)$.

We say that a Banach space E has *property $I - K$* (respectively $I - N_r$) if for every Banach space G and strongly integral operator $T: G \to E$ T is compact (respectively — T is r-nuclear). It is known (combining results of Diestel [3] and Pisier [18]) that the property super $(I - N_1)$ is super reflexivity.

LEMMA 3.11. *The following are equivalent:*

(a) *E has the property super $(I - K)$.*

(b) *E does not contain l_1^n's uniformly.*

Proof. It is known that if E contains l_1^n-s uniformly than l_1, as well as $L_1[0, 1]$ are finitely represented in E. The formal "inclusion" map $L_\infty[0, 1] \to L_1[0, 1]$ is strongly integral, noncompact operator, therefore in this case E fails to have super $(I - K)$. Suppose, on the other hand, that E does not contain l_1^n-s uniformly but there exists an integral noncompact operator into E. The adjoint of this operator is a strongly integral noncompact operator T defined on E', hence it is a Dunford-Pettis operator (which means that it takes ω-Cauchy sequences into norm convergent sequences). Since E does not contain l_1^n-s uniformly — E' does not contain an isomorph of l_1, it follows from a result of Rosenthal [19] that every bounded sequence in E' contains a ω-Cauchy subsequence, but then T must be compact — a contradiction. Therefore E has $(I - K)$. Since "not containing l_1^n-'s uniformly" is a super-property it turns out that E has in fact super $(I - K)$.

Proof of Theorem 3.9. From Lemma 3.8 follows the existence of $c > 0$ and $2 \leqq q < \infty$ *s.t* for every Banach space G and $A: G \to F$

$$\pi_q(A) \leqq c\pi_1'(A) . \tag{1}$$

From Lemma 3.10 we deduce that (1) holds for E as well. If E does not contain l_1^n-s uniformly and uniformly complementably E' does not contain l_∞^n-s uniformly and follows as in [16] the existence of $d > 0$ and $1 < p \leqq 2$ *s.t.* for every G and $A: G \to E$ $\pi_1'(A) \leqq d\pi_p'(A)$. Therefore there exists $k > 0$ $2 \leqq q < \infty$, $1 < p \leqq 2$ *s.t* for every G and A as above

$$\pi_q(A) \leqq k\pi_p'(A) . \tag{2}$$

By Lemma 3.10 (2) is true for every Banach space which is finitely represented in E. Now, let G be a Banach space and $T: G \to E$ a strongly integral operator. Then T has a factorization

$$\begin{tikzcd} G \arrow[r, "T"] \arrow[d, "B"'] & E \\ L_\infty(\Omega, \mu) \arrow[r, "j"] & L_1(\Omega, \mu) \arrow[u, "A"'] \end{tikzcd} \tag{3}$$

with (Ω, μ) a probability space and j the formal "inclusion" map.

We look at the factorization

$$\begin{tikzcd} L_\infty(\Omega, \mu) \arrow[rr, "j"] \arrow[dr, "i_1"'] & & L_1(\Omega, \mu) \\ & L_{p'}(\Omega, \mu) \arrow[ur, "i_2"'] & \end{tikzcd} \tag{4}$$

where $1/p + 1/p' = 1$ and i_1, i_2 are the formal "inclusion" maps. Then $Ai_2 \in \pi'_p(L_{p'}(\mu), E)$ and from (2) follows $Ai_2 \in \pi_q(L_{p'}(\mu), E)$, a known result of Persson and Pietsch [14] combined with the fact that i_1B is strongly p' integral then shows that

$$T = Ai_2i_1B \in N_r(G, E) \quad \text{with} \quad \frac{1}{r} = \frac{1}{p'} + \frac{1}{q}\,.$$

Since the same is true for every Banach space finitely represented in E, E has super $(I - N_r)$ and of course it has super $(I - K)$. Lemma 3.11 then shows that E does not contain l_1^n-s uniformly.

REMARK. We do not know if the property super $(I - N_r)$ is in fact strictly stronger than "not containing l_1^n-'s uniformly".

COROLLARY 3.12. *Let E be a Banach space which either has the property G. L. or is finitely represented in a Banach space F s.t. F has property G. L. and does not contain l_∞^n-'s uniformly. Then for any Banach space G there exist compact nonnuclear operators from E into G and from G into E.*

Proof. From Theorem 3.9 it follows that in both cases one of the three possibilities hold: (a) E contains l_∞^n-s uniformly.

(b) E contains l_1^n-s uniformly and uniformly completably.

(c) E does not contain l_1^n-s uniformly.

In each of these cases the result follows, in (a) or (b) from results of [9] and in (c) from the result of [2].

Let E be a Banach space. We say that E has Grothendieck property ($G.\ P.$) if $\Pi_2(\mathscr{L}_\infty, E) = \mathscr{L}(\mathscr{L}_\infty, E)$ (see [4] for discussion of this property). Maurey [12] showed that if E has cotype-2 then E has $G.\ P.$, Pelczynski [13] shows that the inverse implication is true if E has *l.u.st.* We can generalize:

THEOREM 3.13. *Let E be a Banach space having the property G. L. Then*

(a) *E has G. P. if and only if E is of cotype-*2.

(b) *E' has G. P. and E' does not contain l_1^n-s uniformly if and only if E is of type* 2.

Proof. In both assertions only the "only-if" parts are new and will be proved.

By Corollary 3.7 we know that E' also has the G. L. property.

(a) Suppose E has $G.\ P.$ As in [16] the fact that $\mathscr{L}(\mathscr{L}_\infty, E) = \Pi_2(\mathscr{L}_\infty, E)$ combined with the G. L. property of E' shows that there exists $c > 0$ *s.t.* Any E-valued operator A satisfies

$$(1) \qquad \pi_2(A) \leqq c\pi_1'(A) .$$

By [16] (1) is equivalent to the following condition:

(2) Let S be a subspace of an $L_1(\mu)$ space and $\omega: S \to L_2(\nu)$ a bounded operator. Then $\omega \otimes I_E$ (I_E — the identity operator of E) can be extended to a bounded operator $S \hat{\otimes}_{\Delta_1} E \to L_2(F)$ (for a subspace S of $L_p(\mu)$, Δ_p denotes the norm on $S \otimes E$ as a subspace of $L_p(\mu, E)$: of course $L_p(\mu) \hat{\otimes}_{\Delta_p} E = L_p(\mu, E)$).

We choose S to be the closed linear span in $L_1[0, 1]$ of the Rademacher functions $\{r_n\}$. ($r_n(t) = \operatorname{sign} 2^n \pi t$; $n = 0, 1, \cdots$.) It is known that S is isomorphic to l_2. Let ω be the isomorphism from S to l_2:

$$\omega(\sum b_n r_n) = (b_n)_{n \in N} .$$

From (2) it follows that

$$\omega \otimes I_E: S \hat{\otimes}_1 E \longrightarrow l_2$$

is bounded. Therefore, for $x_1, \cdots, x_n \in E$ we have:

$$\begin{aligned} \Big(\sum_{j=1}^n \|x_j\|^2\Big)^{1/2} &= \Big\|(\omega \otimes I_E)\Big(\sum_{j=1}^n r_j \otimes x_j\Big)\Big\|_{l_2(E)} \\ &\leqq \|\omega \otimes I_E\| \Big\|\sum_{j=1}^n r_j \otimes x_j\Big\|_{L_1([0,1],E)} \\ &= \|\omega \otimes I_E\| \int_0^1 \Big\|\sum_{j=1}^n r_j \otimes x_j\Big\| dt \end{aligned}$$

therefore E is of cotype 2.

(b) Let E' have G. P. and suppose E' does not contain l_1^n-'s uniformly. Then E does not contain l_∞^n-'s uniformly and Pisier's method ([16]) yields the existence of $C > 0$ and $1 < p \leqq 2$ *s.t.* Any E'-valued operator A satisfies

$$(3) \qquad \pi_2(A) \leqq C\pi_p'(A) .$$

(3) is equivalent to

(4) Let ω be a bounded operator $\omega: L_p(\mu) \to L_2(\nu)$, then $\omega \otimes I_{E'}$ is extendable to a bounded operator $\omega \otimes I_{E'}: L_p(\mu, E') \to L_2(\nu, E')$. For such a ω we get therefore that

$$(\omega \otimes I_{E'})': [L_2(\nu, E')]' \longrightarrow [L_p(\mu, E')]'$$

is bounded.

It is easy to check (identifying $L_2(\nu, E'')$ and $L_{p'}(\mu, E'')$ with subspace of $[L_2(\nu, E')]'$ and $[L_p(\mu, E')]'$) that

$$(\omega \otimes I_{E'})'(L_2(\nu, E'')) \subset L_{p'}(\mu, E'')$$

and

$$(\omega \otimes I_{E'})' = \omega' \otimes I_{E''}$$

considered as operators $L_2(\nu, E'') \to L_{p'}(\mu, E'')$.

Therefore $\omega' \otimes I_{E''}$ is well defined and bounded. Now, take $L_2(\nu) = l_2$, $L_p(\mu) = L_p[0, 1]$ and $\omega: L_p[0, 1] \to l_2$ defined by

$$\omega(f) = (\langle f, r_n \rangle)_{n \in N} .$$

ω is bounded and $\omega': l_2 \to L_{p'}[0, 1]$ is the embedding of l_2 in $L_{p'}[0, 1]$:

$$\omega'(g) = \sum g_j r_j \quad \text{for} \quad g = (g_j)_{j \in N} \in l_2 .$$

We get for $x_1, \cdots, x_n \in E$:

$$\begin{aligned}\left(\int_0^1 \left\|\sum_{j=1}^n r_j(t) x_j\right\|^{p'} dt\right)^{1/p'} &= \left\|\sum_{j=1}^n r_j \otimes x_j\right\|_{L_{p'}(E'')} \\ &= \left\|(\omega' \otimes I_{E''})\left(\sum_{j=1}^n e_j \otimes x_j\right)\right\| \leqq \|\omega' \otimes I_{E''}\| \left\|\sum_{j=1}^n e_j \otimes x_j\right\|_{l_2(E'')} \\ &= \|\omega' \otimes I_{E''}\| \left(\sum_{j=1}^n \|x_j\|^2\right)^{1/2}\end{aligned}$$

(e_j being the unit vectors in l_2). Therefore E is of type 2.

Some concluding remarks. The property G. L. as it is defined is in some sense an "external" property. It is interesting to find some "internal" geometric characterization of this property. Up to now we know of no example of Banach space having the G. L. property for which E'' is not isomorphic to a complemented subspace of a Banach lattice, though Remark 3.6 hints that the existence of such example is probable (a result of Lewis [11, Cor. 4.2], together with the fact that each subspace of l_1 has G. L. constant 1, shows that the two norms are not equal).

Another course of problems may arise with respect to properties of spaces having the G. L. property, e.g., how far properties of spaces having *l.u.st* or isomorphic to complemented subspaces of Banach lattices pass over to spaces having G. L. property. Also one can ask how one can use such properties to the solution of problems concerning general Banach spaces. For example with respect to the problem of compact-nonnuclear operators arises the problem: suppose E satisfies $\mathscr{L}(E, l_2) = \Pi_1(E, l_2)$, does this imply that E can be embedded in a space having G. L. property which does not contain l_n^∞-s uniformly?

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