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SHALIKA'S GERMS FOR p-ADIC GL(n), II: THE SUBREGULAR TERM

JOE REPKA

For an elliptic torus in GL(n) over a p-adic field an explicit formula is established for the germ associated to the "subregular" unipotent class, i.e. the class whose Jordan canonical form contains a 1×1 block and an $(n-1) \times (n-1)$ block. In particular this, together with previously known information, gives all the germs for GL(3).

0. In [**1**], an ad hoc method was described for calculating the germ associated to the regular unipotent class. Here that approach is refined to deal with the subregular class. Neither the technique nor the final result is particularly clean; it would be desirable to express the germ in terms more suggestive of generalizations.

The results obtained here are consistent with conjectures made by J. Rogawski in his thesis ([**3**]).

The idea is to construct a function f whose orbital integrals vanish for all unipotent classes except the regular and subregular classes. The germ can easily be calculated from the unipotent orbital integrals and the orbital integrals of f over the classes of regular elements of an elliptic torus.

§1 establishes notations and defines the function f. §§2–7 contain the calculation of the elliptic orbital integrals of f, which are given by Proposition 4. The unipotent orbital integrals are given in §8, and the Theorem in §9 contains the main result, a formula for the subregular germ, preceded by a brief summary of the notation. Finally, §10 describes the result for GL(3) more explicitly.

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1. Let F be a p-adic field, $\mathfrak{o} = \mathfrak{o}_F$ and $\mathfrak{p} = \mathfrak{p}_F$ its ring of integers and prime ideal, respectively, and $q = |\mathfrak{o}/\mathfrak{p}|$. Let $G = \mathrm{GL}(n, F)$, $K = \mathrm{GL}(n, \mathfrak{o})$, and $K_1 = \{k \in K\colon k \equiv \mathrm{id}, \bmod \mathfrak{p}\}$, the congruence subgroup.

Let

$$u_1 = \begin{pmatrix} 1 & 0 & 0 & & & 0 \\ 0 & 1 & 1 & 0 & & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ & & & \ddots & \ddots & \ddots \quad 0 \\ & & & & \ddots & \ddots \quad 1 \\ & & & & & \ddots \quad 1 \end{pmatrix}$$

be the element with diagonal entries all equal to 1, all the entries on the superdiagonal except for the topmost equal to 1, and the top entry in the superdiagonal and all remaining entries equal to 0.

Let $S_1 = \{k \in K: k \equiv u_1, \bmod \mathfrak{p}\} = u_1 K_1$.

Following [**1**], let u_0 be the element with all diagonal and superdiagonal entries equal to 1 and all other entries equal to 0. We refer to the conjugacy classes of u_0 and u_1 as the "regular" and "subregular" unipotent conjugacy classes, respectively.

PROPOSITION 1. *The only unipotent conjugacy classes of G which meet S_1 are those of u_0 and u_1.*

Proof. The proof is similar to that of Proposition 1 of [**1**]. If u is in any other unipotent conjugacy class, then $(u - \mathrm{id})^{n-2} = 0$, but for any $s \in S_1$, $(s - \mathrm{id})^{n-2} \neq 0$. □

Define f to be the characteristic function of S_1. We shall apply Shalika's theory (cf. [**1**], §3). Let T be an elliptic torus of G so that $T = T_F$ is isomorphic to $E^\times$, where E/F is an extension field of degree n. Because of Proposition 1, we have that, for $t \in T'$ close to the identity,

$$(1.1) \qquad \int_{T\backslash G} f(g^{-1}tg)\, d\dot{g} = \Gamma_0(t) \int_{Z(u_0)\backslash G} f(g^{-1}u_0 g)\, d\dot{g} + \Gamma_1(t) \int_{Z(u_1)\backslash G} f(g^{-1}u_1 g)\, d\dot{g}.$$

The function Γ_0 was calculated in [**1**]. By computing the three integrals in (1.1), we shall find Γ_1, the germ associated to the subregular class.

2. As in [**1**], for $g \in G$ we write $\chi(g) \in F^n$ for the n-tuple consisting of the coefficients of the characteristic polynomial of $g - \mathrm{id}$. For each $t \in T'$, write C_t: $T\backslash G \to G$ for the map C_t: $g \mapsto g^{-1}tg = t^g$. Let $\overline{G}(t) = C_t^{-1}(S_1)$. The measure of $\overline{G}(t)$ is the orbital integral of f over the conjugacy class of t, which we need to compute.

For each $s \in S_1$, define $P(s)$ to be the $(n-1) \times n$ matrix obtained by deleting the last row of $s - u_1$. This gives a mapping $P\colon S_1 \to M_{n-1,n}(\mathfrak{p}) \cong \mathfrak{p}^{(n-1)n}$ (as in [**1**], superscripts on $\mathfrak{p}$ will always refer to Cartesian products of copies of $\mathfrak{p}$, and not to powers of the ideal). Our intention is to parametrize the elements of S_1 which are conjugate to t by their first $n-1$ rows. We have the composite map $P \circ C_t\colon \overline{G}(t) \to S_1 \to \mathfrak{p}^{(n-1)n}$, and we shall characterize its image and find its Jacobian. From this it will be easy to find the measure of $\overline{G}(t)$.

For fixed $t \in T' \cap K_1$, let U be a neighbourhood of t in $T' \cap K_1$ chosen so that no two elements of U are conjugate. Let $A \subset T' \times T\backslash G$ be the set $A = \{(t, g)\colon t \in U,\ t^g \in S_1\}$. An easy calculation with determinants shows that if $s \in S_1$ then $\chi(s) \in \mathfrak{p}^n$.

Consider the commuting diagram in Figure 1.

$$\begin{array}{ccccc}
T' \times T\backslash G \supset A & \overset{C}{\to} & S_1 & \overset{W}{\to} & \mathfrak{p}^n \times \mathfrak{p}^2 \times \mathfrak{p}^{(n-1)n-2} \\
\chi \times \mathrm{id}\downarrow & & \chi \times P\downarrow & & \downarrow \\
\mathfrak{p}^n \times T\backslash G \supset B & \dashrightarrow & \mathfrak{p}^n \times \mathfrak{p}^{(n-1)n} & \overset{W'}{\to} & \mathfrak{p}^n \times \mathfrak{p}^2 \times \mathfrak{p}^{(n-1)n-2}
\end{array}$$

FIGURE 1

The map labelled C in the diagram is the conjugation map taking (t, g) to t^g. The broken arrow at the bottom left is the identity on the first factor, and, for fixed $\chi(t)$ in the first factor, acts as $P \circ C_t$ on the second factor. The left-hand and centre vertical arrows are as labelled, and the set B is just the image of A, i.e.

$$B = \{(\chi(t), g)\colon t \in U,\ t^g \in S_1\}.$$

The map labelled W is more complicated. For $s \in S_1$, take $s - u_1$ and delete the last row and the first two entries in the first row. The remaining entries give an element of $\mathfrak{p}^{(n-1)n-2}$, and this is the third factor of the map W. To describe the first two factors we must find a particular conjugate of $s - \mathrm{id}$ of the special form

$$(2.1)\qquad \begin{pmatrix} a & b & 0 & 0 & & 0 \\ 0 & 0 & 1 & 0 & & 0 \\ 0 & 0 & 0 & 1 & \ddots & 0 \\ & & & & \ddots & 1 \\ \alpha_0 & \alpha_1 & \alpha_2 & \alpha_3 & & \alpha_{n-1} \end{pmatrix}.$$

The entries $\alpha_0, \alpha_1, \ldots, \alpha_{n-1}$; a, b will give an element of $\mathfrak{p}^n \times \mathfrak{p}^2$ which will be the first factors of that map.

To describe this conjugate more carefully, note that $s - \mathrm{id}$ has units on the superdiagonal except for the topmost entry; that entry and all others are in $\mathfrak{p}$. We shall conjugate by a succession of matrices of the form $\mathrm{id} + \lambda E_{ij}$, where $\lambda \in \mathfrak{p}$ and E_{ij} has a 1 in the ijth position and 0's elsewhere. This operation subtracts λ times the jth row from the ith and adds λ times the ith column to the jth.

By adding multiples of the $(n-1)$th row to the rows above it, we can clear the last column above the superdiagonal. This will also add multiples of earlier columns to the $(n-1)$th column. Then we can clear the $(n-1)$th column above the superdiagonal (adding things to the $(n-2)$th column), and so on, continuing until all entires above the superdiagonal are 0. The resulting matrix is uniquely determined by the above description. Then we clear the entries to the left of the superdiagonal in the second row, then in the third row, and so on through the $(n-1)$th row. It is easy to see that this process does not disturb the zeros above the superdiagonal. Finally, make the units on the superdiagonal into 1's by conjugating by a diagonal matrix whose diagonal entries are units and whose first two diagonal entries are equal.

The result is a uniquely determined matrix of the form (2.1), and using its entries we have now defined the map W in Figure 1.

It is not hard to see how the entries a, b are obtained from the corresponding entries of $s - u_1$. Indeed

$$a = (s_{11} - 1) + p_1, \qquad b = s_{12} + p_2, \tag{2.2}$$

where p_1 and p_2 are polynomials in $(s_{11} - 1)$ and s_{12}. The coefficients of these polynomials are determined by the remaining entries of the first $n - 1$ rows of $s - \mathrm{id}$ (they are rational functions of those entries) and are all in $\mathfrak{p}$. Furthermore, the transformation which takes the last row of $s - u_1$ into $(\alpha_0, \alpha_1, \ldots, \alpha_{n-1})$ has Jacobian equal to a unit, because it is a composition of translations, unipotent transformations, and multiplying each entry by a unit. From all this we conclude that the Jacobian of W has absolute value 1, relative to the obvious co-ordinates.

The right-hand vertical map in Figure 1 replaces the last row $(\alpha_0, \alpha_1, \ldots, \alpha_{n-1})$ of the matrix (2.1) with the coefficients of that matrix's characteristic polynomial, and acts as the identity on the other factors. The map W' is now easy to describe. It is the identity on the first factor. The second factor, which amounts to the first $n - 1$ rows of a matrix $s - u_1$, goes to an element of $\mathfrak{p}^2 \times \mathfrak{p}^{(n-1)n-2}$ as follows: the first two coefficients are just the numbers a, b defined by (2.2) in terms of the first $n - 1$ rows of $s - u_1$, and the remaining coefficients are just the remaining entries of $s - u_1$. It is easily seen that W' has Jacobian equal to 1.

3. Next we find the Jacobian of the right-hand vertical map.

It is easy to calculate the characteristic polynomial of the matrix (2.1). Expanding by cofactors along the first row, we find that the characteristic polynomial is

$$\begin{aligned}(3.1)\quad &(X-a)(X^{n-1}-\alpha_{n-1}X^{n-2}-\alpha_{n-2}X^{n-3}-\cdots-\alpha_1)-b\alpha_0\\ &\quad = X^n-(\alpha_{n-1}+a)X^{n-1}-(\alpha_{n-2}-a\alpha_{n-1})X^{n-2}\\ &\quad -\cdots-(\alpha_1-a\alpha_2)X-(b\alpha_0-a\alpha_1).\end{aligned}$$

So the Jacobian matrix of the right-hand vertical arrow is triangular, with one diagonal entry equal to $-b$ and the remaining diagonal entries all ± 1. Consequently the absolute value of the Jacobian determinant is $|b|$.

In §5 of **[1]**, the absolute value of the Jacobian of the mapping χ: $T \cap K_1 \to \mathfrak{p}^n$ was found to be $|D_{E/F}|^{1/2} \cdot |D(t)|^{1/2}$ (here $D_{E/F}$ is the discriminant of E over F). Since the Jacobian of the map C: $T' \times T\backslash G \to S_1$ is known to be $D(t)$, it is now easy to write the modulus of the Jacobian of each map in Figure 1.

4. For $t \in T' \cap K_1$, we write $\Phi(X) = \Phi_t(X)$ for the characteristic polynomial of $t - \mathrm{id}$. We wish to find all conjugates of t in S_1, but we'll start with matrices of the special form (2.1).

PROPOSITION 2. *Let a, $b \in \mathfrak{p}$. There is a matrix of the form* (2.1) *with $\alpha_0, \alpha_1, \ldots, \alpha_{n-1} \in \mathfrak{p}$ which is conjugate to t −* id *if and only if* $|\Phi_t(a)| < |b|$.

Proof. Write $\Phi(X) = X^n - \beta_{n-1}X^{n-1} - \beta_{n-2}X^{n-2} - \cdots - \beta_0$. Note each $\beta_i \in \mathfrak{p}$. From (3.1) we see that we need to find $\alpha_i \in \mathfrak{p}$ so that $\alpha_{n-1} + a = \beta_{n-1}$, $\alpha_{n-2} - a\alpha_{n-1} = \beta_{n-2}, \ldots, \alpha_k - a\alpha_{k+1} = \beta_k, \ldots, b\alpha_0 - a\alpha_1 = \beta_0$.

From this we get

$$\begin{aligned}\alpha_{n-1} &= \beta_{n-1} - a,\\ \alpha_{n-2} &= \beta_{n-2} + a\alpha_{n-1} = \beta_{n-2} + a\beta_{n-1} - a^2\\ &\ \ \vdots\\ \alpha_k &= \beta_k + a\beta_{k+1} + a^2\beta_{k+2} + \cdots + a^{n-k-1}\beta_{n-1} - a^{n-k}\\ &\ \ \vdots\end{aligned}$$

and finally

$$b\alpha_0 = \beta_0 + a\alpha_1 = \beta_0 + a\beta_1 + \cdots + a^{n-1}\beta_{n-1} - a^n = -\Phi(a).$$

It is clear that this system will have a (unique) solution with $\alpha_i \in \mathfrak{p}$ if and only if $|\Phi(a)|<|b|$. □

The version we want is a simple reformulation of the proposition.

COROLLARY. *Referring to Figure* 1, *the image in the lower right corner of* $S_1 \cap t^G$, *the set of all elements in* S_1 *which are conjugate to* t, *is*

$$\left\{\left(\chi(t);\, a,\, b;\, y_1,\, y_2, \ldots, y_{(n-1)n-2}\right) : |\Phi_t(a)|<|b|,\, y_i \in \mathfrak{p} \text{ arbitrary}\right\}.$$

Proof. The only thing needing to be checked is the surjectivity, i.e. that every value of $(a, b;\, y_1, \ldots) \in \mathfrak{p}^2 \times \mathfrak{p}^{(n-1)n-2}$ is the image of some $s \in S_1 \cap t^G$. This follows from (2.2) by Hensel's Lemma. □

5. We now concentrate on the bottom row of Figure 1. Since both maps act as the identity on the first factor, for each fixed $t \in T' \cap K_1$ we have a map $\bar{G}(t) \to \mathfrak{p}^2 \times \mathfrak{p}^{(n-1)n-2}$, whose Jacobian and whose image we know. From this we shall find the measure of $\bar{G}(t)$, which is the orbital integral of f over the conjugacy class of t.

The map which goes along the top row of Figure 1 and down the right-hand side has Jacobian with modulus equal to $|D(t)| \cdot 1 \cdot |b|$ (here b is the co-ordinate of the matrix (2.1) which occurs in the image space, but that happily turns out to be what we need). The map going down the left side has Jacobian of modulus $|D_{E/F}|^{1/2} \cdot |D(t)|^{1/2}$, so the map along the bottom row must have Jacobian with modulus $|D_{E/F}|^{-1/2} \cdot |D(t)|^{1/2} \cdot |b|$. To find the measure of the set $\bar{G}(t)$ we must integrate the reciprocal of this number over its image, which is described by the Corollary to Proposition 2.

6. We need to describe $\{(a, b) \in \mathfrak{p} \times \mathfrak{p} : |\Phi_t(a)|<|b|\}$. To do this, identify $t \in T'$ with an element of $E^\times$, also called t.

Define $d(t, F) = \min\{|t - y| : y \in F\}$, the "distance from t to F". Here the absolute value on E is the one which extends the normalized absolute value on F.

It is easy to prove that, provided $(q, n) = 1$, $d(t, F) = |t - (1/n)\mathrm{Tr}_{E/F}(t)|$, with the absolute value as above, and from this that $d(t, F) = |\Phi_t((1/n)\mathrm{Tr}(t) - 1)|^{1/n}$, which is perhaps easier to work with in terms of matrices (recall Φ_t is the characteristic polynomial of $t - \mathrm{id}$ rather than t, which accounts for the "-1"). However, both these formulae may fail if $(q, n) \neq 1$.

Now let $x_0 \in F$ be such that $|(t-1) - x_0| = d(t, F)$ (the above discussion shows that x_0 could be taken to be $(1/n)\mathrm{Tr}(t) - 1$ if $(q, n) = 1$). Note that for $t \in T' \cap K_1$, we have $x_0 \in \mathfrak{p}$.

LEMMA 3. *Let $t \in T' \cap K_1$, let $\Phi(X)$ be the characteristic polynomial of t — id, and choose x_0 as above. Then*

$$|\Phi(a)| = \begin{cases} |a - x_0|^n, & \text{if } |a - x_0| > d(t, F), \\ d(t, F)^n, & \text{if } |a - x_0| \leq d(t, F). \end{cases}$$

Proof. Note that $d(t, F) = d(t-1, F)$. Now $\Phi(a) = \prod(a - (t_i - 1))$, where $t_1, \ldots, t_n$ are the conjugates of t over F. And $|a - (t_i - 1)| = |a - (t - 1)|$, for all i. Moreover, $|a - (t-1)| = |(a - x_0) + (x_0 - (t-1))|$. Since $|x_0 - (t-1)| = d(t, F)$, the result is obvious except possibly in the case where $|a - x_0| = d(t, F) = |x_0 - (t-1)|$, when it is at least clear that $|a - (t-1)| \leq d(t, F)$. But if the strict inequality held it would contradict the definition of $d(t, F)$. □

7. We are now ready to evaluate the orbital integral of f over the conjugacy class of t. For convenience we fix $t \in T' \cap K_1$ and define the positive integer r by $d(t, F) = q^{-r/n}$.

Normalize measures as in [1]: the Haar measure on G whose restriction to K is just the product of the normalized F^+ measure on each co-ordinate; the measure on $T \cong E^\times$ is $|t|_E^{-1}\, d_E t$, where $|\cdot|_E$ is the normalized absolute value on E, d_E the normalized E^+ measure (for which $\mathfrak{o}_E$ has mass equal to 1).

PROPOSITION 4.

$$\int_{T\backslash G} f(g^{-1}tg)\, d\dot{g} = |D_{E/F}|^{1/2} \cdot |D(t)|^{-1/2} q^{-(n-1)n+1} \times \left(n - 1 + \frac{1}{q} - \left(1 - \frac{1}{q}\right) \sum_{k=r}^{\infty} q^{-[k/n]}\right).$$

Proof. As described in §2, the orbital integral equals the measure of the set $\overline{G}(t)$. The bottom row of Figure 1 gives a map from $\overline{G}(t)$ onto a set described in §4, and the Jacobian was calculated in §5. So we see that

$$\int_{T\backslash G} f(g^{-1}tg)\, d\dot{g} = \text{measure } \overline{G}(t)$$
$$= \iint \cdots \int |D_{E/F}|^{1/2} \cdot |D(t)|^{-1/2} \cdot |b|^{-1}\, da\, db\, dy_1\, dy_2 \cdots dy_{(n-1)n-2},$$

where the integral is over

$$\{(a, b, y_1, \ldots, y_{(n-1)n-2}) \in \mathfrak{p}^{(n-1)n} \colon |\Phi_t(a)| < |b|\}.$$

So this equals

$$|D_{E/F}|^{1/2} \cdot |D(t)|^{-1/2} \cdot q^{-(n-1)n+2} \iint |b|^{-1}\, da\, db.$$

Letting $|b| = q^{-k}$, the double integral becomes

$$\left(1 - \frac{1}{q}\right) \sum_{k=1}^{\infty} \int_{A_k} da,$$

where $A_k = \{a \in \mathfrak{p} \colon |\Phi_t(a)| < q^{-k}\}$. By Lemma 3, $A_k = \varnothing$ if $q^{-r} \geq q^{-k}$. If $r > k$, then

$$\begin{aligned} A_k &= \{a \in \mathfrak{p} \colon |a - x_0|^n < q^{-k}\} = \{a \in \mathfrak{p} \colon |a - x_0| < q^{-k/n}\} \\ &= \{a \in \mathfrak{p} \colon |a - x_0| \leq q^{-[k/n]-1}\}, \end{aligned}$$

and

$$\int_{A_k} da = q^{-[k/n]-1},$$

so the sum equals

$$\begin{aligned} \sum_{k=1}^{r-1} q^{-[k/n]-1} &= \sum_{k=1}^{\infty} q^{-[k/n]-1} - \sum_{k=r}^{\infty} q^{-[k/n]-1} \\ &= \left(\frac{n}{q-1} - \frac{1}{q}\right) - \frac{1}{q} \sum_{k=r}^{\infty} q^{-[k/n]}. \end{aligned}$$

The result follows. □

8. Proposition 4 tells us the first integral in (1.1). To get the other two we need to normalize measures on $Z(u_0)$ and $Z(u_1)$. On $Z(u_0)$ we use the normalization described in [**1**]. For $Z(u_1)$ we proceed analogously. For each $z \in Z(u_1)$, write $z = au$, where a is diagonal and u is unipotent. Then let $dz = da\, du$, where da is the product of the standard $F^\times$ measure on each of the (two) parameters of a, and du is the product of the standard F^+ measure on each parameter of u.

The orbital integrals of f over the classes of u_0 and u_1 are then calculated as in §7 of [**1**]. (The calculation for u_1 is perhaps simplified if you first conjugate everything by the matrix obtained from the identity matrix by interchanging the first two rows. This makes $Z(u_1)$ entirely upper triangular.) We find that

$$(8.1)\qquad \int_{Z(u_0)\backslash G} f(g^{-1}u_0 g)\,d\dot{g} = q^{-(n-1)n+1}(n-1+1/q),$$

$$\int_{Z(u_1)\backslash G} f(g^{-1}u_1 g)\,d\dot{g} = q^{-(n-1)n+2}.$$

9. We are now able to describe the germ associated to the subregular unipotent conjugacy class. We recall the notation: $T \cong E^{\times}$ is an elliptic torus, $t \in T'$ is identified with an element of E. The integer r is defined by $q^{-r/n} = d(t, F) = \min\{|t-y| : y \in F\}$, where the absolute value on E extends the normalized absolute value on F. The normalizations of the measures are given in §§7 and 8; the germ $\Gamma_1(t)$ is defined by equation (1.1). We assume $r > 0$.

THEOREM. *The germ associated to the elliptic torus $T \cong E^{\times}$ and the subregular unipotent conjugacy class is*

$$\Gamma_1(t) = -q^{-1}\left(1-\frac{1}{q}\right)|D_{E/F}|^{1/2}\cdot|D(t)|^{-1/2}\sum_{k=r}^{\infty} q^{-[k/n]}.$$

Proof. Proposition 4 gives the left side of (1.1), $\Gamma_0(t)$ was calculated in [**1**], and the other two integrals in (1.1) are given by (8.1). □

Note that $\Gamma_1(t)$ is always negative.

10. In the particular case of GL(3), this can be said more clearly. If T is an unramified torus (i.e. E/F is unramified) then $r/3 \in \mathbf{Z}$, and the germ is

$$(10.1)\qquad \begin{aligned}\Gamma_1(t) &= -3q^{-1}|D_{E/F}|^{1/2}\cdot|D(t)|^{-1/2}q^{-r/3}\\ &= -3q^{-1}|D_{E/F}|^{1/2}\cdot|D(t)|^{-1/2}d(t,F).\end{aligned}$$

If T is ramified, then it is easy to see that $r/3 \notin \mathbf{Z}$, and it follows that the germ is

$$(10.2)\quad \Gamma_1(t) = \begin{cases} -(2+1/q)q^{-2/3}|D_{E/F}|^{1/2}\cdot|D(t)|^{-1/2}d(t,F), & \text{if } r \equiv 1, \bmod(3),\\ -(1+2/q)q^{-1/3}|D_{E/F}|^{1/2}\cdot|D(t)|^{-1/2}d(t,F), & \text{if } r \equiv 2, \bmod(3).\end{cases}$$

The other germs for GL(3) were found in [**2**] and [**1**]. It is perhaps worthwhile to discuss the non-central singularities of GL(3); they are necessarily of the form diag(a, a, b), with $a \neq b$. First we note that elements of an elliptic torus cannot approach this singularity (if an element of a cubic extension of F approaches $a \in F$, then its conjugates also approach a, so it is impossible for one of them to approach b). So any torus whose elements approach diag(a, a, b) can be contained in the Levi component of a standard maximal parabolic subgroup and the whole question — including the calculation of germs — reduces to one on GL(2), which is straightforward.

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University of Toronto
Toronto, Ontario M5S 1A1, Canada