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GROUPS OF KNOTS IN HOMOLOGY 3-SPHERES THAT ARE NOT CLASSICAL KNOT GROUPS

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In this paper we attempt to enlarge classical knot groups K by adding a root to a meridian of K. Thus if K is a classical knot group with a meridian μ, then the groups we study are of the form $G = K \underset{\mu=t^q}{*} \langle t \rangle$. This group can always be realized as the group of a knotted 3-sphere in S^5. By using explicit geometric constructions we also show that such a group G is a 2-knot group and the group of a knot in a homology 3-sphere. Finally, we show that G is not realizable by any knot in S^3.

1. Introduction. In [$\mathbf{R}_1$] J. Ratcliffe gave an example of a group Γ that is the group of a fibered knot in a homology 3-sphere which cannot be realized as the group of a classical knot. Let K be the group of the trefoil knot and let $\mu \in K$ represent a meridian. As seen in [$\mathbf{R}_1$] Γ can be expressed as a free product with amalgamation $K \underset{\mu=t^2}{*} \langle t \rangle$.

In this paper we generalize the result in [$\mathbf{R}_1$]. We study groups G obtained from classical knot groups K by forming an amalgamated free product of K with Z. More specifically if K is a classical knot group with meridian μ, then $G = K \underset{\mu=t^q}{*} \langle t \rangle$. Note that if $q = 1$ then $G = K$. Hence we assume $q > 1$. One natural question to ask about G is if G is the group of a knot in any dimension. We show that G is the group of a knotted 2-sphere in S^4. Furthermore we show that G can be realized as the group of a knot in a homology 3-sphere. However, G is not a classical knot group.

I would like to thank the referee for his suggestion of how to construct the 2-sphere in §3. This made the third section a lot simpler than it was in the first version of this paper.

2. Preliminaries. In this paper we work in the smooth category. S^n and B^n denote the standard n-sphere and n-ball. If N and M are manifolds and $f\colon M \to N$ is a map then both of the induced homomorphisms $\pi_1(M) \to \pi_1(N)$ or $H_1(M) \to H_1(N)$ will be denoted by f_*. Homeomorphism between spaces and isomorphism between groups are denoted by $\cong$. An n-dimensional knot is the image of a smooth embedding Σ^n of S^n into S^{n+2} or R^{n+2}. By the knot group we mean $\pi_1(S^{n+2} - \Sigma^n)$. For $n = 1$, we call these groups classical knot groups.

We define the deficiency of a group presentation with n generators and m relators to be the integer $n - m$. The following well known proposition is due to Kervaire [**K**].

PROPOSITION 2.1. *If a group K has a deficiency one presentation and $K/K' \cong Z$ then $H_2(K) = 0$.*

Consider an oriented knot Σ in S^3. Remove an open neighbourhood N of Σ in S^3 to produce the knot manifold $X = S^3 - N$. The preferred meridian, longitude pair (μ, λ) of Σ are two nontrivial simple closed curves on Bd(X) such that μ bounds a disk in N and λ is homologically trivial in X.

DEFINITION 2.2. A (p, q)-curve is a simple closed curve J on Bd(X) that is homotopic to $\mu^p\lambda^q$ where p and q are relatively prime. We also call J a (p, q)-cable about Σ.

3. A 2-knot with group $K \underset{\mu=t^q}{*} \langle t \rangle$. Let K be a classical knot group with meridian μ. We construct a new group G by adding a qth root (via amalgamted free product) to the meridian μ of K, i.e., $G = K \underset{\mu=t^q}{*} \langle t \rangle$. The following proposition is easy to verify using Kervaire's characterization of high dimensional knot groups ([**K**], Theorem 1).

PROPOSITION 3.1. *The group G is a high dimensional knot group.*

Proof. Since K is a classical knot group it has a deficiency one presentation. We can thus obtain a deficiency one presentation of G from a presentation of K by adding one more generator (t) and one more relation $(\mu = t^q)$. Moreover since K is a knot group and hence satisfies the conditions of Kervaire's characterization it is straightforward to check that $G/G' \cong Z$ and that $G/\langle\langle t \rangle\rangle = 1$. By Proposition 2.1 we obtain $H_2(G) = 0$, and it follows that G can be realized as the group of a knotted 3-sphere in S^5. □

Let Σ be a knot in S^3 with group K. We shall construct a knotted 2-sphere Σ^4 with $\pi_1(S^4 - \Sigma^2) = K \underset{\mu=t^q}{*} \langle t \rangle$. The equatorial cross-section of this 2-sphere will be a $(1, q)$-cable about the composite knot $\Sigma \# - \Sigma^*$, where $-\Sigma^*$ is the mirror image of Σ with its orientation reversed.

DEFINITION 3.2. A knot Σ in S^3 is a slice knot if there exists a smooth disk D in B^4 such that $\mathrm{Bd}(D) = \Sigma$.

THEOREM 3.3. *Let Σ be a knot in S^3 and let L be the $(1, q)$-cable about $\Sigma \# -\Sigma^*$. Then L is a slice knot.*

Proof. For any knot Σ, the knot $\Sigma \# -\Sigma^*$ is a slice knot **[F, M]**. To construct a slice disk D in B^4 with $\mathrm{Bd}(D) = \Sigma \# -\Sigma^*$, we do as follows. First note that $(S^3, \Sigma) = (B^3, \beta) \cup_\partial (B^3, B^1)$ where β is a knotted arc and (B^3, B^1) is a standard ball pair. Remove the ball pair (B^3, B^1) from (S^3, Σ) and cross (B^3, β) with the interval to obtain a disk $D = \beta \times I$ contained in B^4. Then $\mathrm{Bd}(D) = \Sigma \# -\Sigma^*$. Thus D is the desired disk. Let $N = D \times \operatorname{int} B^2$ be an open neighbourhood of the slice disk and let $M = B^4 - N$. Then $D \times \mathrm{Bd}(B^2)$ is in $\mathrm{Bd}(M)$. We shall attach a 2-handle $B^2 \times B^2$ which contains a slice disk for the trivial $(1, q)$-torus knot to M along $D \times \mathrm{Bd}(B^2)$. If $B^2 \times B^2$ is attached along $S^1 \times B^2$ then let the torus knot be the $(1, q)$-cable about the core of the solid torus $B^2 \times S^1$. Note that the attaching sphere $S^1 \times \{0\}$ represents the qth power of the meridian of the torus knot. Since the image of $\{*\} \times S^1$ under the attaching map is $\mathrm{Bd}(D) = \Sigma \# -\Sigma^*$ it follows that the image of the boundary of the slice disk for the $(1, q)$-torus knot is L. Thus there exists a disk in B^4 with boundary L. □

Since L is a slice knot, we can use L as the equatorial section of a knotted 2-sphere in S^4 by joining together smooth disks in B^4_+ and B^4_- bounded by L. We denote the 2-sphere obtained this way by $S(\Sigma, q)$.

THEOREM 3.4. *Let Σ be a knot in S^3 with group K and meridian μ, $q > 1$ and let $S(\Sigma, q)$ be the 2-knot described above. Then*

$$\pi_1\big(S^4 - S(\Sigma, q)\big) = K \underset{\mu = t^q}{*} \langle q \rangle.$$

Proof. If $\tilde{D}$ is the slice disk for the $(1, q)$-cable about $\Sigma \# -\Sigma^*$ it suffices to show that $\pi_1(B^4 - \tilde{D}) = K \underset{\mu = t^q}{*} \langle t \rangle$. Using the notation from the proof of Theorem 3.3 we have that

$$B^4 - \tilde{D} = M \cup_{S^1 \times B^2 - \mathrm{Bd}(\tilde{D})} (B^2 \times B^2) - \tilde{D}.$$

Since M is homotopic to $B^4 - D$ which equals $(B^3 - \beta) \times I$ it follows that $\pi_1(M) = \pi_1(B^3 - \beta) = K$. Moreover, since $\mathrm{Bd}(\tilde{D}_1)$ is the (trivial) $(1, q)$-torus knot we get that the fundamental group of $B^2 \times B^2 - \tilde{D}$ is infinite cyclic generated by the meridian of the torus knot. Thus the fundamental group of $B^4 - \tilde{D}$, is obtained from $\pi_1(M)$ by attaching a qth root of the original meridian μ, i.e. $\pi_1(B^4 - \tilde{D}) = K \underset{\mu = t^q}{*} \langle t \rangle$. □

4. 3-manifolds that can realize the group $K \underset{\mu=t^q}{*} \langle t \rangle$. We now consider how close G is to being a classical knot group. As the following shows the Alexander polynomial $\Delta_G(t)$ for G is symmetric and it satisfies $\Delta_G(1) = \pm 1$.

THEOREM 4.1. *Let K be a classical knot group with Alexander polynomial $\Delta_K(t)$ and let $G = K \underset{\mu=t^q}{*} \langle t \rangle$. Then the Alexander polynomial $\Delta_G(t)$ for G satisfies $\Delta_G(t) = \Delta_K(t^q)$.*

Proof. Let $K = \langle x_0, x_1, \ldots, x_n;\ R_1, \ldots, R_n \rangle$ be a standard Wirtinger presentation for K and let $A = [\partial R_i / \partial x_j]$ be the $n \times (n+1)$ Alexander matrix with respect to this presentation.

$$A \text{ is equivalent to } B = \begin{bmatrix} \dfrac{\partial R_1}{\partial X_0} & \cdots & \dfrac{\partial R_1}{\partial X_{n-1}} & 0 \\ & & & \vdots \\ \dfrac{\partial R_n}{\partial X_0} & \cdots & \dfrac{\partial R_n}{\partial X_{n-1}} & 0 \end{bmatrix}$$

which is obtained from A by adding the first n columns to the last column. The Alexander polynomial for K, $\Delta_K(t)$ is the generator of the principal ideal generated by the determinants of all the $n \times n$ submatrices of B. Thus $\Delta_K(t) =$ determinant of the $n \times n$ submatrix obtained from B by deleting the last column. The group G has a presentation $\langle t, x_0, \ldots, x_n;$ $R_1, \ldots, R_n, x_0 t^{-q} \rangle$ and its Alexander matrix is

$$\begin{bmatrix} & A(t^q) & & 0 \\ & & & 0 \\ 1 & 0 & \cdots\ 0 & k(t) \end{bmatrix}$$

where $k(t) = \partial(x_0 t^{-q})/\partial t$. This matrix is equivalent to

$$C = \begin{bmatrix} \dfrac{\partial R_1}{\partial X_0}(t^q) & & \cdots & \dfrac{\partial R_1}{\partial X_{n-1}}(t^q) & 0 & 0 \\ \dfrac{\partial R^n}{\partial X_0}(t^q) & & & \dfrac{\partial R_n}{\partial X_{n-1}}(t^q) & 0 & 0 \\ 1 & 0 & \cdots & 0 & 1 & k(t) \end{bmatrix}.$$

The ideal generated by all $(n+1) \times (n+1)$ minors of C is easily seen to be principal and its generator is $\Delta_K(t^q)$. Hence G has an Alexander polynomial $\Delta_G(t)$ and moreover $\Delta_G(t) = \Delta_K(t^q)$. □

Since K is a classical knot group there is a knot Σ in S^3 with knot manifold X such that $\pi_1(X) = K$. Let (μ, λ) be the preferred meridian, longitude pair for $\mathrm{Bd}(X)$, and let $A = \mu \times I$ be an annulus on $\mathrm{Bd}(X)$. By T^3 we mean the standard solid torus $S^1 \times D^2$ in R^3. Furthermore, let J be a $(1, q)$-curve on the boundary of T^3 and let $B = J \times I$ be an annulus on $\mathrm{Bd}(T^3)$. We construct a cabled 3-manifold M **[J, M]** by glueing together X and T^3 along the two annuli A and B, i.e. $M = X \cup_{A=B} T^3$.

PROPOSITION 4.2. $\pi_1(M) = K \underset{\mu=t^q}{*} \langle t \rangle$.

Proof. Let $\pi_1(T^3) = \langle t \rangle$, $\pi_1(A) = \langle \mu \rangle$ and $\pi_1(B) = \langle j \rangle$. The image of μ in $\pi_1(X)$ under the homomorphism induced by the inclusion map $A \to M$ is μ, and the image of j in $\pi_1(T^3)$ under the homomorphism induced by the inclusion map $B \to T^3$ is t^q. Thus by the Van-Kampen Theorem we conclude that $\pi_1(M) = K \underset{\mu=t^q}{*} \langle t \rangle$. □

The boundary of M is homeomorphic to $S^1 \times S^1$, and if (μ, λ) is a standard meridian, longitude pair for $\mathrm{Bd}(X)$, then a basis for $\mathrm{Bd}(M)$ is $\mu, \lambda t^{-1}$.

THEOREM 4.3. *If K is a classical knot group with meridian μ, then $G = K \underset{\mu=t^q}{*} \langle t \rangle$ is the group of a knot in a homology 3-sphere.*

Proof. We construct a 3-manifold M' by sewing a solid torus to M in such a way that a meridian of the solid torus is identified with the curve λt^{-1}. Then $\pi_1(M')$ is obtained from $\pi_1(M)$ by adding the relation $t = \lambda$. Since $\pi_1(M) = K \underset{\mu=t^q}{*} \langle t \rangle$ we get that $\pi_1(M') = K/\langle\langle \mu\lambda^{-q} \rangle\rangle$. It is now easy to see that $H_1(M') \cong 0$ and we conclude, using the usual Poincaré duality argument that M' is a homology 3-sphere.

In some recent work done by Culler, Gordon, Luecke, and Shalen, it is shown that if $(1, q)$-surgery on a nontrivial knot yields a simply connected manifold then $|q| = 0$ or 1 **[C, G, L, S]**. If a knot Σ has group K and $m, l \in K$ is the meridian and the longitude, then the fundamental group of the surgery manifold $\Sigma(1, q)$ is $K/\langle\langle ml^q \rangle\rangle$. We thus have the following proposition.

PROPOSITION 4.3. *If K is a classical knot group and $q > 1$ then $K/\langle\langle ml^q \rangle\rangle \neq 1$.*

THEOREM 4.4. *Let K be a classical knot group with meridian μ and let $G = K \underset{\mu = t^q}{*} \langle t \rangle$, $q > 1$. Then G is not a classical knot group.*

Proof. Let $M = X \cup_A T^3$ be the cabled manifold we constructed in the paragraph preceding Proposition 4.2, then $\pi_1(M) \cong G$. Suppose there exists a knot L in S^3 with $\pi_1(S^3 - L) = G$. We shall eventually show that L must be a cable knot about some core L' and that the surgery manifold $L'(1, q)$ is simply connected which contradicts Proposition 4.3.

The knot manifold X is aspherical **[P]**. Moreover, M as the union of aspherical spaces sewn together along an incompressible subspace is aspherical **[W₂]**. Let N be the knot manifold for L. Since $\pi_1(M)$ is isomorphic to $\pi_1(N)$ and M and N are aspherical spaces, there exists a homotopy equivalence $f\colon N \to M$ that induces the group isomorphism.

The annulus A is bicollared in M. Furthermore, $\pi_1(A) \to \pi_1(M)$ is injective since $\pi_1(A) \to \pi_1(X)$ and $\pi_1(A) \to \pi_1(T^3)$ are injective; also $\pi_2(A) = \pi_2(M) = \pi_3(M) = \pi_2(M - A) = 0$ since M, X and T^3 are aspherical spaces. Hence $\ker(\pi_j(A) \to \pi_j(M)) = 0$, $j = 1, 2$. By Lemma 1.1 in **[W₁]** there exists a map g that is homotopic to f such that:

1. g is transverse with respect to A, i.e. there exists a neighbourhood $g^{-1}(A) \times I$ of $g^{-1}(A)$ so that $g(x, y) = (g(x), y)$ for every $x \in g^{-1}(A)$ and $y \in I$.
2. $g^{-1}(A)$ is an orientable compact 2-manifold and $g^{-1}(A) \cap \mathrm{Bd}(M) = \mathrm{Bd}(g^{-1}(A))$.
3. If F is a component of $g^{-1}(A)$ then $\ker(\pi_j(F) \to \pi_j(N)) = 0$, $j = 1, 2$.

Choose a map g that minimizes the number of components of $g^{-1}(A)$. We shall show that for such a map g, $g^{-1}(A)$ is just one annulus F and that F separates N into a solid torus V and a knot manifold Y having the same group as X. We use techniques similar to those used in **[F, W]** and **[S]** to prove the following assertions. (Some proofs are omitted since they are the same as proofs in **[F, W]** and **[S]**.)

Claim 4.4.1. $g^{-1}(A)$ is nonempty.

Claim 4.4.2. Let F be a component of $g^{-1}(A)$. Then F is an essential annulus.

We thus have $g^{-1}(A) = F_1 \cup \cdots \cup F_K$, $k \geq 1$. Since each component of $g^{-1}(A)$ is an essential annulus we get that the core L of $S^3 - \mathrm{Int}(N)$ is either a composite knot or a cable knot ([**W**$_3$], Lemma 1.1).

Claim 4.4.3. L is a cable knot.

Since $g^{-1}(A) = F_1 \cup \cdots \cup F_K$ and L is a (p', q')-cable about a knot L' we have for each $i, 1 \leq i \leq k$, $N = Y_i \cup_{F_i} V_i$ where V_i is a solid torus and Y_i is a knot manifold. Moreover a boundary component of $\mathrm{Bd}(F_i)$ is a (p', q')-curve on $S^3 - \mathrm{Int}(Y_i)$.

Claim 4.4.4. $\pi_1(N)$ has trivial center.

Proof of 4.4.4. Since $\pi_1(N) = K \underset{\mu = t^q}{*} \langle t \rangle$, the center of $\pi_1(N) = C(K) \cap \langle \mu \rangle$ ([**M, K, S**], Cor. 4.5). If K is not a torus knot group then $C(K) = 1$ and consequently $\pi_1(N)$ has no center. On the other hand if K is a torus knot group then $C(K) = \langle \mu^{pq}\lambda \rangle$, but $\langle \mu^{pq}\lambda \rangle \cap \langle \mu \rangle = 1$ in K and consequently in $K \underset{\mu = t^q}{*} \langle t \rangle$. □

Claim 4.4.5. The annuli $F_1, \ldots, F_K$ are parallel in N.

Proof of 4.4.5. N is prime since it is a cable knot manifold. Moreover N is irreducible. Since $\pi_1(N)$ has trivial center, N cannot be a Siefert fiber space with decomposition surface a disk with 3 singular fibers as such a space has fundamental group with nontrivial center [**J**]. By Lemma 2.4 in [**J, M**] there exists a unique (up to ambient isotopy) essential annulus embedded in N. Hence each annulus F_i is parallel to F_1. □

Claim 4.4.6. For each i, $1 \leq i \leq k$, $g|_{F_i}$ is homotopic to a homeomorphism.

Proof of 4.4.6. It suffices to show that g_*: $H_1(F_i) \to H_1(A)$ is an isomorphism. Since $H_1(F_i) \cong \pi_1(F_i)$ and $H_1(A) \cong \pi_1(A)$, this implies that g_*: $\pi_1(F_i) \to \pi_1(A)$ is an isomorphism. Hence $g|_{F_i}$ is a homotopy equivalence and is therefore homotopic to a homeomorphism.

Let f_i be a generator for $H_1(F_i)$ and let a be a generator for $H_1(A)$. Then $g_*(f_i) = a^r$. We wish to show that $r = \pm 1$. We use a homotopy inverse h of g (h: $M \to N$). As done earlier in the proof we can modify h such that $h^{-1}(F_i)$ is a collection of essential annuli, $B_1, \ldots, B_n$. Since M is irreducible ([**J**], Lemma 3.1) and M is not a twisted I-bundle over the Klein bottle (if M is a twisted I-bundle over the Klein bottle then $\pi_1(M)$

abelianizes to $Z \oplus Z_2$) there exists an isotopy h_t $(0 \le t \le 1)$ such that $h_1(\mathrm{Bd}(B_i)) \cap \mathrm{Bd}(A) \neq \varnothing$. Hence $\mathrm{Bd}(B_i)$ is homologous to $\mathrm{Bd}(A)$ in $H_1(M)$, i.e. the generator a for $H_1(A)$ is homologous to a generator b_i for $H_1(B_i)$. Furthermore, $h_*(b_i) = f_i^s$. Since h is a homotopy inverse of g we get $g_*(h_*(b_i)) = b_i$, i.e. $a^{sr} = b_i$. Since a is homologous to b_i in $H_1(M)$ we obtain $a^{sr} = a$ in $H_1(M)$. Hence $sr = 1$, and it follows that $r = \pm 1$. □

Claim 4.4.7. $p'q' = q$.

Proof of 4.4.7. $z_{p'q'} \cong H_1(N)/H_1(F_i) \cong H_1(M)/H_1(A) \cong Z_q$. □

Recall that $g^{-1}(A) = F_1 \cup \cdots \cup F_K$.

Claim 4.4.8. k is odd.

Proof of 4.4.8. Since the annuli F_i, $1 \le i \le k$ are all parallel V_k contains a core v_1 of V_1. Let α be a path in V_1 from F_1 to v_1, then $\pi_1(Y_1)$ and $\alpha v_1 \alpha^{-1}$ generate $\pi_1(N)$. If k is even then g maps $\mathrm{Int}(Y_1)$ and v_1 into the same component of $M - A$. If Y_1 and v_1 are both mapped into $X - A$, then $\pi_1(M)/\pi_1(X) \cong 1$ which contradicts the fact that $H_1(M)/H_1(X) \cong Z_q$. On the other hand Y_1 cannot be mapped into T^3, because that would imply that $g_*\colon \pi_1(Y_1) \to \pi_1(T^3)$ is 1-1 which contradicts the fact that Y_1 is a knot manifold. □

Claim 4.4.9. $k = 1$.

The proof of 4.4.9 is the same as the proof of Claim 7 in **[S]**.

Since $k = 1$, we have that $N = Y \cup_F V$, where Y is the knot manifold for the knot L', V is a solid torus, and a boundary component of $\mathrm{Bd}(F)$ is a (p', q')-curve of the boundary of $S^3 - \mathrm{Int}(Y)$. Moreover we have a homotopy equivalence $g\colon Y \cup_F V \to X \cup_A T^3$ such that $g_{|F}$ is a homeomorphism. We saw in the proof of Claim 4.4.8 that $g(Y)$ is not contained in T^3. Hence $g(Y) \subseteq X$ and $g(V) \subseteq T^3$ and $g_*(\pi_1(Y)) \subset \pi_1(X)$ and $g_*(\pi_1(V)) \subseteq \pi_1(T^3)$. Let s be a generator for $\pi_1(V)$ and let m, l be a meridian, longitude pair for $\pi_1(Y)$, then $\pi_1(N) = \pi_1(Y) *_{\pi_1(F)} \pi_1(V) = \pi_1(Y) *_{m^{p'}l^{q'} = s^{q'}} \langle s \rangle$. Recall that $\pi_1(M) = \pi_1(x) *_{\pi_1(A)} \pi_1(T^3)$ where $\pi_1(x) = K$ and $\pi_1(T^3) = \langle t \rangle$. Since g_* is an isomorphism and $g_*(\pi_1(F)) = \pi_1(A)$ we have the following $g_*(\pi_1(N)) = g_*(\pi_1(Y)) *_{\pi_1(A)} g_*(\pi_1(V)) = K *_{\pi_1(A)} \langle t \rangle$. By Proposition 2.5 in **[B]** we conclude that $g_*|_{\pi_1(Y)}\colon \pi_1(Y) \to K$ and $g_*|_{\pi_1(V)}\colon \pi_1(V) \to \pi_1(T^3)$ is an isomorphism. Therefore, since

$q = p'q'$ we have the following: $g_*((m^{p'}l^{q'})^{p'}) = g_*(s^{q'p'}) = g_*(s^q) = t^{\pm q} = \mu^{\pm 1}$. Hence $\pi_1(Y)/\langle\langle (m^{p'}l^{q'})^{p'}\rangle\rangle \cong K/\langle\langle \mu^{\pm 1}\rangle\rangle = 1$. Since $\pi_1(Y)/\langle\langle (m^{p'}l^{q'})^{p'}\rangle\rangle$ abelianizes to $Z_{(p')^2}$ this implies that $p' = \pm 1$ which in turn implies that $q' = \pm q$. Hence $\pi_1(Y)/\langle\langle ml^q\rangle\rangle = 1$ which contradicts Proposition 4.3. □

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