Suzuki, S.
Osaka J. Math.
6 (1969), 475-483

ON THE KNOT ASSOCIATED WITH THE SOLID TORUS

SHIN'ICHI SUZUKI

(Received March 26, 1969)

We have some methods to compose a knot from some knots, for instance composition (or product, sum) [7], union [5], linked sum [11], etc.. Recently, F. Hosokawa defined in [4] a fusion of a link as an extension of composition. And further, we have some kinds of knots associated with the torus already proved of their non-triviality, for example torus knots, parallel knots, doubled knots [12], Schlingknoten [8], etc..

The purpose of the present note is to define a complete-fusion of a link and an associated knot with a solid torus T_μ, and to prove the non-trivialities of them.

In §1, the concept of complete-fusion is described as extensions of composition and fusion, and in §2 we give a standard model of the solid torus T_μ of genus μ and investigate the relation between complete-fusion and solid torus T_μ. In §3, we consider a generalization of knots associated with the socalled torus T_1.

By the Dehn's Lemma [2], [3], [6], we have the following well-known unknotting theorem: *a knot k is of trivial type if and only if* $\pi_1(S^3-k)\cong Z$, *infinite cyclic group.* Implicit use of this theorem will be made throughout this work.

The following considerations are based upon the semi-linear point of view.

The author wishes to express his hearty thanks to Professor H. Terasaka for leading him to this work.

1. A complete-fusion of a link

Let $L=(k_1, k_2,\cdots, k_\mu)$ be an oriented link with μ components ($\mu\geq 2$) in a 3-sphere S^3, and let $B_1,\cdots, B_{\mu-1}$ be narrow bands that span L and that are oriented coherently with L. Suppose that $L+\partial B_1+\cdots+\partial B_{\mu-1}$ is a knot[1], denote $k(L; B_1,\cdots, B_{\mu-1})$ or simply $k(L)$, where $+$ means homological addition. We say that the knot $k(L)$ is a *complete-fusion* of L, see Fig. 1. Note that if the link L is split, then this definition is a special case of fusion in [4, §2], while this definition can be meant to extend the concept of fusion to of general links. It

1) ∂=boundary, $\mathcal{J}$=interior.

will be noticed that a complete-fusion is not determined uniquely, i.e. the knot type of $k(L)$ depends upon the link type of L and joined bands $B_1,\cdots,B_{\mu-1}$.

It is easily shown that the composition (or product, sum) [7] of $k_1, k_2,\cdots, k_\mu$ is the special case of complete-fusion, and we denote this by $k^\#(L)=k_1\#k_2\#\cdots\#k_\mu$. The composition $k^\#(L)$ of L is uniquely determined.

Theorem 1.1. *Let $L=(k_1, k_2,\cdots, k_\mu)$ be a split link with μ components $(\mu\geq 2)$ in the 3-sphere S^3. If at least one of components $k_1, k_2,\cdots, k_\mu$ of L, say k_1, is of non-trivial knot type, then any complete-fusion $k(L)$ of L is of non-trivial knot type.*

Moreover, let $g(k)$ and $\Delta_k(t)$ be genus and Alexander polynomial of k respectively, then by Terasaka-Yonebayashi's arguments [10], [13], we have:

(1.2) $g(k(L))\geq g(k_1)+\cdots+g(k_\mu)$.

(1.3) $\Delta_{k(L)}(t)\equiv F(t)F(t^{-1})\ \Delta_{k_1}(t)\cdots\Delta_{k_\mu}(t)\ (\mathrm{mod}\pm t)$,

where $F(t)$ is an integral polynomial determined by the crossings of B_i with k_j.

Proof. By the definition at least one of $B_1,\cdots,B_{\mu-1}$ attachs to k_μ. So, sliding the ends of $B_1,\cdots,B_{\mu-1}$ along $k_1,\cdots,k_\mu$ and sides of other bands we may assume that each band B_i connects k_i and k_μ, $i=1,\cdots,\mu-1$, as illustrated in Fig. 1.

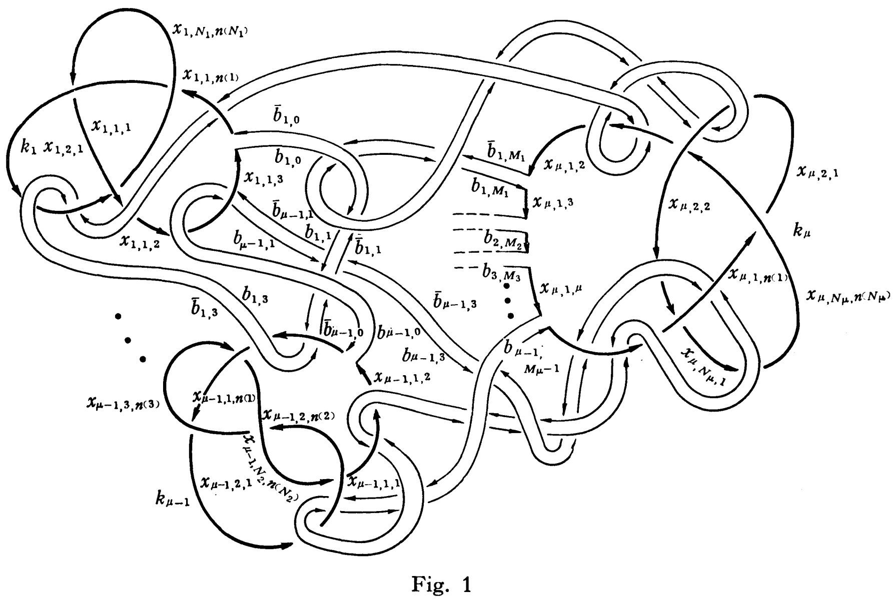


Fig. 1

Let $x_{i,1,1}, x_{i,1,2},\cdots, x_{i,1,n_1}, x_{i,2,1},\cdots, x_{i,N_i,n(N_i)}$ be the generators of the knot group $G(k(L))=\pi_1(S^3-k(L))$ corresponding to the arcs of k_i, $i=1,\cdots,\mu$, with the same letter in this order. Let $b_{i,0}, b_{i,1},\cdots, b_{i,M_i}$ and $\bar{b}_{i,0}, \bar{b}_{i,1},\cdots, \bar{b}_{i,M_i}$ be the generators of the knot group corresponding to the boundary of B_i, $i=1,\cdots,\mu-1$.

Then we have a presentation[2] of $G(k(L))$ as follows:

2) See [1], Chapter VI.

$$\left(\begin{array}{l}
\textit{generators}: \\
\qquad x_{i,1,1},\, x_{i,1,2},\cdots,\, x_{i,1,n_1},\, x_{i,2,1},\cdots,\, x_{i,N_i,n(N_i)},\ (i=1,\cdots,\mu)\,, \\
\qquad b_{i,0},\, b_{i,1},\cdots,\, b_{i,M_i},\, \bar{b}_{i,0},\, \bar{b}_{i,1},\cdots,\, \bar{b}_{i,M_i},\ (i=1,\cdots,\mu-1)\,. \\
\textit{relations}: \\
\qquad r_{i,1}=\cdots=r_{i,N_i}=1,\ (i=1,\cdots,\mu)\,, \\
\qquad s_{i,j}=1,\ t_{*,*,j}=1,\ u_{i,j}=1,\ \bar{u}_{i,j}=1\,, \\
\qquad b_{i,0}=x_{i,1,*},\ \bar{b}_{i,\mu}=x_{i,1,*},\ (i=1,\cdots,\mu-1)\,, \\
\qquad b_{i,M_i}=x_{\mu,1,*},\ \bar{b}_{i,M_i}=x_{\mu,1,*},\ (i=1,\cdots,\mu-1)\,.
\end{array}\right),$$

where $r_{i,j}=x_{i,j,n_j}\, x^{\varepsilon}_{i,k,l}\, x^{-1}_{i,j+1,1}\, x^{-\varepsilon}_{i,k,l}$, $\varepsilon=\pm 1$, which corresponds to a crossing point as shown in Fig. 2,

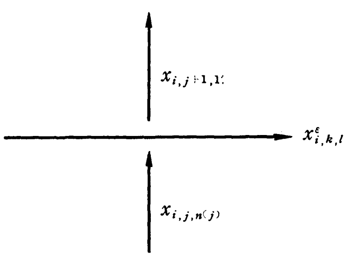


Fig. 2

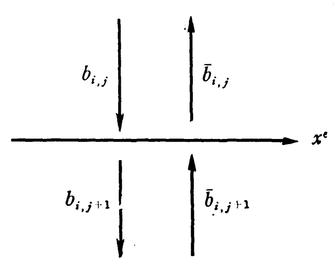


Fig. 3

$$\begin{aligned}
&s_{i,j} = x^{\varepsilon} b_{i,j}\, \bar{b}^{-1}_{i,j}\, x^{-\varepsilon}\, \bar{b}_{i,j+1}\, b^{-1}_{i,j+1},\ \varepsilon = \pm 1\,, \\
&x \in \{x_{1,1,1},\cdots,\, x_{\mu,N_\mu,n(N_\mu)}\}\,,
\end{aligned}$$

which corresponds to crossing points as shown in Fig. 3,

$$\begin{aligned}
&t_{*,*,j} = x^{\varepsilon}_{*,*,j}\, b\bar{b}^{-1}\, x^{-\varepsilon}_{*,*,j+1}\, \bar{b}b^{-1},\ \varepsilon = \pm 1\,, \\
&(b,\bar{b}) \in \{(b_{0,1},\bar{b}_{0,1}),\cdots,\, (b_{\mu-1,M_{\mu-1}},\, \bar{b}_{\mu-1,M_{\mu-1}})\}\,,
\end{aligned}$$

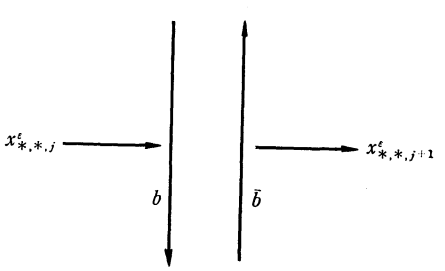


Fig. 4

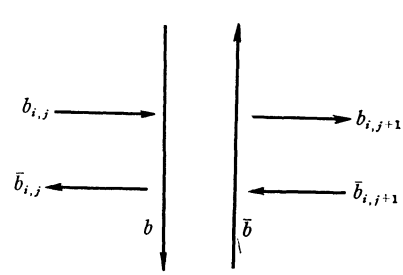


Fig. 5

which corresponds to crossing points as shown in Fig. 4,

$$u_{i,j} = b_{i,j}\, b\, \bar{b}^{-1}\, b_{i,j+1}^{-1}\, \bar{b} b^{-1}\,,$$
$$\bar{u}_{i,j} = b_{i,j+1}\, \bar{b}\, b^{-1}\, b_{i,j}^{-1}\, b\, \bar{b}^{-1}$$

which corresponds to crossing points as shown in Fig. 5.

On the other hand, we have a presentation of the knot group $G(k^{\#}(L))=\pi_1(S^3-k^{\#}(L))$ as follows:

$$\left(\begin{array}{l} \textit{generators}: \\ \qquad x_{i,1},\cdots, x_{i,N_i}, \ (i=1,\cdots,\mu)\,. \\ \textit{relations}: \\ \qquad r_{i,1}=\cdots=r_{i,N_i}=1,\ (i=1,\cdots,\mu)\,, \\ \qquad x_{i,1}=x_{\mu,1},\ (i=1,\cdots,\mu-1)\,. \end{array}\right).$$

Let $h:G(k(L))\rightarrow G(k^{\#}(L))$ be a correspondence defined by

$$h(x_{i,j,k}) = x_{i,j},\ i=1,\cdots,\mu\,,\ j=1,\cdots,N_i\,,$$
$$h(b_{i,j}) = h(\bar{b}_{i,j}) = x_{\mu,1},\ i=1,\cdots,\mu-1,\ j=0,1,\cdots,M_i\,.$$

Then h is a homomorphism onto. Because $G(k^{\#}(L))$ is not isomorphic to Z, $G(k(L))$ is not isomorphic to Z and then $k(L)$ is not of trivial type.

(1.2) and (1.3) are immediately from [10] and [13].

This completes the proof.

The following is a direct consequence of Theorem 1.1.

Corollary 1.4. *Let $L=(k_1,\cdots,k_\mu)$ be a split link with μ components. If a complete-fusion $k(L)$ is of trivial knot type, then L is of trivial link type,* i.e. *$k_1,\cdots,k_\mu$ are of trivial knot type.*

REMARK 1.5. The knot group $G(k_1)=\pi_1(S^3-k_1)$ of k_1 has the following presentation:

$$\left(\begin{array}{l|l} \textit{generators}: & \textit{relations}: \\ \qquad x_{1,1},\cdots, x_{1,N_1}\,. & \qquad r_{1,1}=\cdots=r_{1,N_1}=1\,. \end{array}\right).$$

Let h': $G(k(L))\rightarrow G(k_1)$ be a correspondence defined by

$$h'(x_{1,j,k}) = x_{1,j}\,, \qquad j=1,\cdots,N_1\,,$$
$$h'(x_{i,j,k}) = x_{1,1}\,, \qquad i=2,\cdots,\mu,\ j=1,\cdots,N_i\,,$$
$$h'(b_{i,j}) = h'(\bar{b}_{i,j}) = x_{1,1}\,, \qquad i=1,\cdots,\mu-1,\ j=0,1,\cdots,M_i\,.$$

Then h' is a homomorphism onto. So we can prove the non-triviality of Theorem 1.1 by using this homomorphism h'.

From this Remark 1.5, we can get the following theorem:

Theorem 1.6. *Let* $L=(k_1, k_2,\cdots, k_\mu)$ *be a link such that* k_1 *is of non-trivial knot type, and* k_1 *and* $(k_2,\cdots, k_\mu)$ *are split. Then any complete-fusion* $k(L)$ *of* L *is of non-trivial type.*

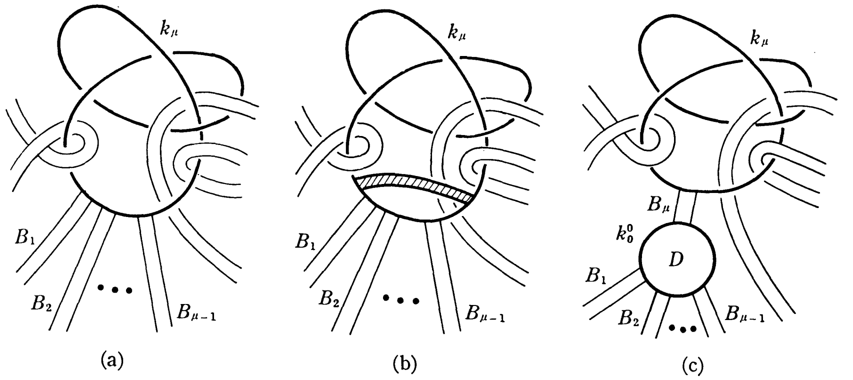
Moreover,

$$g(k(L)) \geq g(k_1)$$

and $\Delta_{k(L)}(t)$ *contains* $\Delta_{k_1}(t)$ *as its factor.*

In preparation for next section, we will give an useful fashion of $k(L)=k(L; B_1,\cdots, B_{\mu-1})$ of $L=(k_1,\cdots, k_\mu)$.

In the proof of Theorem 1.1, we assumed that each band B_i connects k_i with k_μ, $i=1,\cdots, \mu-1$, at only one arc $x_{\mu,1}$ of a projection of k_μ, (see Fig. 6(a)). Then we can take a band B_μ such that $B_\mu \cap L=\partial B_\mu \cap k_\mu$ and $k_1+\cdots+k_\mu+\partial B_\mu$ is a link, denote $L'=(k_0^0 | k_1,\cdots, k_\mu)$, with $\mu+1$ components, (see Fig. 6(b)). Especially, we may assume that $L'-k_0^0$ is the same link type of L and k_0^0 bounds a 2-cell D in S^3-L such that $D\cap(B_1\cup\cdots\cup B_\mu)=\partial D\cap(\partial B_1\cup\cdots\cup\partial B_\mu)$, (see Fig. 6(c)).



(a) (b) (c)

Fig. 6

We summarize the above:

Proposition 1.7. *As a complete-fusion* $k(L)$ *of* $L=(k_1,\cdots, k_\mu)$, *we consider always the complete-fusion* $k(L')=k(L'; B_1,\cdots, B_\mu)$ *of* $L'=(k_0^0 | k_1,\cdots, k_\mu)$ *such that*

(i) k_0^0 *is the boundary of a 2-cell* $D\subset S^3-L$,

(ii) B_i *connects* k_i *with* k_0^0, $i = 1,\cdots, \mu$,

(iii) $B_i \cap \mathcal{I}D = \phi$, $i = 1,\cdots, \mu$.

2. A standard model of the solid torus T_μ in S^3

The solid torus T_μ of genus μ (Henkelkörper vom Geschlechte μ [9, p. 219]) is a 3–cell with μ solid handles. We now construct a standard model of T_μ in S^3.

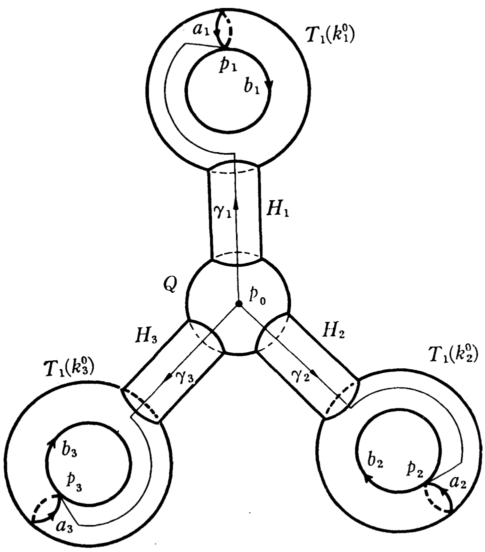


Fig. 7

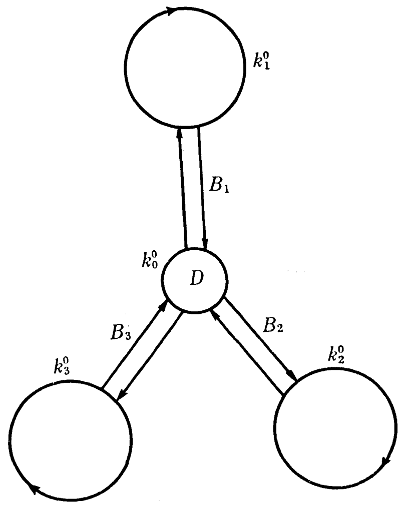


Fig. 8

It is an analoque of that sketched in Fig. 7 for $\mu=3$, and a description of T_μ is as follows.

Let $D, D_1, \cdots, D_\mu$ be $\mu+1$ mutually disjoint 2-cells in a hyperplane $S_0^2 = R_0^2 \cup \{\infty\}$ of S^3 and let $B_1, \cdots, B_\mu$ be mutually disjoint narrow bands (=2–cells) in S_0^2 such that B_i connects D with D_i as indicated in Fig. 8.

Let $\partial D_i = k_i^0$, $i=1, \cdots, \mu$. Then we have a representative $L^0=(k_1^0, \cdots, k_\mu^0)$ of the trivial link with μ components in S^3 and the composition $k^\#(L^0)=k^\#(L^0; B_1, \cdots, B_\mu)$. We will call a regular neighborhood $N(K; S^3)$ of the complex $K=D\cup B_1 \cup \cdots \cup B_\mu \cup k_1^0 \cup \cdots \cup k_\mu^0$ in S^3 a standard model of the solid torus of genus μ and denote this by T_μ. T_μ consists of a 3–cell $Q=N(D; S^3)$, μ solid cylinders $H_i=N(B_i; S^3)$ and μ solid tori (of genus 1) $T_1(k_i^0)=N(k_i^0; S^3)$, $i=1, \cdots, \mu$, united as indicated.

We take simple oriented loops $\boldsymbol{a}_i$, $\boldsymbol{b}_i$ and simple oriented arcs γ_i on ∂T_μ as Fig. 7. Let $p_0=\bigcap_{i=1}^{\mu} \gamma_i$ and $p_i=\boldsymbol{a}_i \cap \boldsymbol{b}_i$, $i=1, \cdots, \mu$. We regard $\boldsymbol{a}_i$ and $\boldsymbol{b}_i$ as p_0-based loops $\gamma_i \boldsymbol{a}_i \gamma_i^{-1}$ and $\gamma_i \boldsymbol{b}_i \gamma_i^{-1}$, respectively, in $\pi_1(T_\mu; p_0)$ and $\pi_1(\partial T_\mu; p_0)$.

It is obvious that

(2.1) $\boldsymbol{b}_1,\cdots,\boldsymbol{b}_\mu$ *form a free basis for* $\pi_1(T_\mu; p_0)$.

(2.2) $\boldsymbol{a}_1,\cdots,\boldsymbol{a}_\mu$, $\boldsymbol{b}_1,\cdots,\boldsymbol{b}_\mu$ *form a basis for* $\pi_1(\partial T_\mu; p_0)$, *and subject to the single relation*[3)]

$$\prod_{i=1}^{\mu}\boldsymbol{a}_i\boldsymbol{b}_j\boldsymbol{a}_i^{-1}\boldsymbol{b}_i^{-1}\simeq 1 \text{ on } \partial T_\mu .$$

(2.3) $\boldsymbol{a}_i\simeq 1$ *in* T_μ, $i=1,\cdots,\mu$.

By abelianizing the group we therefore get:

(2.4) $\boldsymbol{b}_1,\cdots,\boldsymbol{b}_\mu$ *form a free abelian basis for* $H_1(T_\mu)$; *and* $\boldsymbol{a}_1,\cdots,\boldsymbol{a}_\mu$, $\boldsymbol{b}_1,\cdots,\boldsymbol{b}_\mu$ *form a free abelian basis for* $H_1(\partial T_\mu)$.

By the definition of the standard solid torus T_μ, we can think that the composition $k^\sharp(L^0)$ of the trivial link $L^0=(k_0^0\,|\,k_1^0,\cdots,k_\mu^0)$ is contained in T_μ as the core such that k_i^0 is the core of $T_1(k_i^0)$, $i=1,\cdots,\mu$.

Let $L=(k_0^0\,|\,k_1,\cdots,k_\mu)$ be a link. Then for any complete-fusion $k(L)$ of L, there exists an embedding φ of T_μ in S^3 such that

(i) $\varphi(T_\mu)=N(K;S^3)$, a regular neighborhood of the complex $K=D\cup B_1\cup\cdots\cup B_\mu\cup k_1\cup\cdots\cup k_\mu$ in S^3. We denote $\varphi(T_\mu)$ by $T_\mu(k(L))$.

(ii) $\varphi(Q)=N(D;S^3)$, $\varphi(H_i)=N(B_i;S^3)$, $\varphi(T_1(k_i^0))=N(k_i;S^3)$, $i=1,\cdots,\mu$.

(iii) $\varphi(k_i^0)=k_i$, $i=1,\cdots,\mu$.

(iv) $\varphi(\boldsymbol{a}_i)=\boldsymbol{m}_i$, $\varphi(\boldsymbol{b}_i)=l_i$, $i=1,\cdots,\mu$, where $\boldsymbol{m}_i$ and l_i are the meridian and longitude respectively, of $\partial N(k_i;S^3)$ as usual meaning.

Note that an embedding φ of T_μ is determined by a complete-fusion $k(L)$ of L and is unique to within twistings of $B_1,\cdots,B_\mu$.

3. An associated knot with a torus $T_\mu(\mathbf{k}(\mathbf{L}))$

In this section we will consider some knots by using the knotted tori $T_\mu(k(L))$ in §2.

DEFINITION 3.1. Let λ be a simple oriented loop (i.e. knot) in $T_\mu\subset S^3$, and let $\varphi:T_\mu\to T_\mu(k(L))\subset S^3$ be as before. Then the knot $\varphi(\lambda)\subset S^3$ is called an *associated knot with* $T_\mu(k(L))$.

Theorem 3.2. *Let* λ *be a knot in* $T_\mu\subset S^3$ *such that*

$$\lambda\sim\beta_1\boldsymbol{b}_1+\beta_2\boldsymbol{b}_2+\cdots+\beta_\mu\boldsymbol{b}_\mu \text{ in } T_\mu .$$

Let $L=(k_1,\cdots,k_\mu)$ *be a split link. If at least one of components* $k_1,\cdots,k_\mu$ *of* L, *say* k_1, *is of non-trivial knot type, then for any complete-fusion* $k(L)$ *of* L *the knot* $\varphi(\lambda)$

3) $\simeq$ means homotopic to, $\sim$ means homologous to.

$\subset S^3$ *associated with* $T_\mu(k(L))$ *is of non-trivial type provided that* $\beta_1 \neq 0$.

Sketch of the Proof. The proof is the same as that of Theorems 1.1 and 1.6 except for obvious modifications. Let $\mathcal{P}k(L)$ be a regular projection of $k(L)$ into a hyperplane $S_0^2 = R_0^2 \cup \{\infty\}$ that is chosen suitably in the sense of knot theory, (see [1] Chapter VI). In particular, we may assume that $\mathcal{P}T_\mu(k(L))$ is a regular neighborhood of $\mathcal{P}k(L)$ in R_0^2, and that $\mathcal{P}T_\mu(k(L))$ consists of $\sum_{i=1}^{\mu} n(N_i) + \sum_{i=1}^{\mu} M_i$ domains $U_{i,j,*}$ and $V_{i,j}$ such that $U_{i,j,*} \supset x_{i,j,*}$ and $V_{i,j} \supset b_{i,j}$ where $x_{i,j,*}$ and $b_{i,j}$ are connected arcs of $\mathcal{P}k(L)$ corresponding to the generators of $G(k(L))$.

Let $y_{i,j,*,k}$ and $c_{i,j,k}$ be generators of $G(\varphi(\lambda))$ corresponding to arcs of $\mathcal{P}\varphi(\lambda)$ such that $y_{i,j,*,k} \subset U_{i,j,*}$ and $c_{i,j,k} \subset V_{i,j}$.

Let $h' : G(\varphi(\lambda)) \to G(k_1)$ be a correspondence defined by

$$\begin{aligned} h'(y_{1,j,*,k}) &= x_{1,j}, \\ h'(y_{i,j,*,k}) &= x_{1,1},\ i=2,\cdots,\mu, \\ h'(c_{i,j,k}) &= x_{1,1}. \end{aligned}$$

It is easily checked that for every relation s_i having in $U_{i,j,*}$ and $V_{i,j}$, $h'(s_i)$ becomes a trivial relation in $G(k_1)$; hence that h' is a homomorphism onto.

The homomorphism h' of $G(\varphi(\lambda))$ onto $G(k_1)$ enables us to give the following theorem that is a generalization of Theorem 3.2.

Theorem 3.3. *With* λ *as in* 3.2, *let* $L=(k_1,\cdots,k_\mu)$ *be a link such that* k_1 *is of non-trivial knot type, and* k_1 *and* $(k_2,\cdots,k_\mu)$ *are split. Then for any complete-fusion* $k(L)$ *of* L, *the knot* $\varphi(\lambda) \subset S^3$ *associated with* $T_\mu(k(L))$ *is of non-trivial type provided that* $\beta_1 \neq 0$.

REMARK 3.4. Theorem 1.1 and Theorem 1.6 are special cases of Theorems 3.2 and 3.3, respectively.

Corollary 3.5. (Parallel knots, Cables). *With* L *and* $k(L)$ *as in* 3.2 *or* 3.3, *and let* λ *be a knot on* $\partial T_\mu \subset S^3$ *such that*

$$\lambda \sim \alpha_1 \boldsymbol{a}_1 + \beta_1 \boldsymbol{b}_1 + \cdots + \alpha_\mu \boldsymbol{a}_\mu + \beta_\mu \boldsymbol{b}_\mu \text{ in } \partial T_\mu .$$

Then $\varphi(\lambda)$ *associated with* $T_\mu(k(L))$ *is of non-trivial type provided that* $\beta_1 \neq 0$.

REMARK 3.6. Let λ be a knot in T_μ, which may be homotopically trivial in T_μ. Let $\sigma_1 \cup \cdots \cup \sigma_n = \lambda \cap T_1(k_1^0)$ and $\tau_1 \cup \cdots \cup \tau_n = \lambda \cap (T_\mu - T_1(k_1^0))$, where σ_i and τ_i are simple sub-arcs of λ. We take simple arcs τ_i' in $N(H_1 \cap T_1(k_1^0); T_1(k_1^0))$, a regular neighborhood of $H_1 \cap T_1(k_1^0)$ in $T_1(k_1^0)$, corresponding to τ_i, $i=1,\cdots,n$, so that $\lambda_1=\sigma_1\cup\cdots\cup\sigma_n\cup\tau'_1\cup\cdots\cup\tau'_n$ is a knot in $T_1(k_1^0)$. Then, with L and $\varphi: T_\mu\to T_\mu(k(L))$ as in 3.2 or 3.3, we have an onto homomorphism $f: G(\varphi(\lambda))\to G(\varphi(\lambda_1))$ defined by

$$f(x_j)=\begin{cases} x_j \text{ if } x_j\subset(\sigma_1\cup\cdots\cup\sigma_n)\,, \\ \tau'_i \text{ if } x_j\subset\tau_i,\ i=1,\cdots,n\,, \end{cases}$$

where x_j and τ'_i are generators of $G(\varphi(\lambda))$ and $G(\varphi(\lambda_1))$ corresponding to arcs of $\mathcal{P}\varphi(\lambda)$ and $\mathcal{P}\varphi(\lambda_1)$. As we have the non-triviality of some kinds of knots associated with a solid torus $T_1(k)$, for example, in [5], [11], [12], we can get the non-triviality of knots $\varphi(\lambda)$ by using this homomorphism f.

KOBE UNIVERSITY

References

[1] R.H. Crowell and R.H. Fox: Introduction to Knot Theory, New York, Ginn and Co., 1963.
[2] M. Dehn: *Über die Topologie des drei-dimensionalen Raumes*, Math. Ann. **69** (1910), 137–168.
[3] T. Homma: *On Dehn's Lemma for S^3*, Yokohama Math. J. **5** (1957), 223–244.
[4] F. Hosokawa: *A concept of cobordism between links*, Ann. of Math. **86** (1967), 362–373.
[5] S. Kinoshita and H. Terasaka: *On unions of knots*, Osaka Math. J. **9** (1957), 131–153.
[6] C.D. Papakyriakopoulos: *On Dehn's Lemma and the asphericity of knots*, Ann. of Math. **66** (1957), 1–26.
[7] H. Schubert: *Die eindeutige Zerlegbarkeit eines Knotes in Primknoten*, Sitzungsber. Akad. Wiss. Heidelberg, Math. -nat. Kl. No. 3 (1949), 57–104.
[8] H. Seifert: *Schlingknoten*, Math. Z. **52** (1949), 62–80.
[9] H. Seifert and W. Threlfall: Lehrbuch der Topologie, Leipzig und Berlin, Teubner, 1934.
[10] H. Terasaka: *On null-equivalent knots*, Osaka Math. J. **11** (1959), 95–113.
[11] ——— : *On the non-triviality of some kinds of knots*, Osaka Math. J. **12** (1960), 113–144.
[12] J.H.C. Whitehead: *On doubled knots*, J. London Math. Soc. **12** (1937), 63–71.
[13] K. Yonebayashi: *On the Alexander polynomial of ribbon 2-knots*, Master's thesis, Kobe University, 1969.