have order $p^a q$) and the orders of those nonsupersolvable groups with all proper divisors the orders of only supersolvable groups. The latter proof is not selfcontained, since results of Pazderski are assumed.

Chapter 4, *Miscellaneous classes*, 27 pages by John F. Humphreys and David Johnson, first states but does not prove Suprunenko's description of primitive solvable linear groups. Then there are brief discussions of groups whose homomorphic images are all CLT-groups (QCLT-groups), groups which are the products of normal supersolvable subgroups, groups whose lattice of subgroups is lower semimodular (LM-groups), and seminilpotent groups (those whose nonnormal nilpotent subgroups have nilpotent normalizers). Incidentally, the third G in the statement of Theorem 6.1, p. 137, should be a $\mathfrak{D}$; that was the most annoying of the several misprints I noticed in the book.

The title of Chapter 5, *Classes of finite solvable groups*, by Gary L. Walls, could title the entire book. This 44-page chapter actually is a well-written presentation of the standard results on formations and $\mathfrak{F}$-normalizers due to Gaschütz, Lubeseder, Carter, and Hawkes, and the dual notion of Fitting class as developed by B. Fischer, Blessenohl, and Gaschütz. The last section, on the homomorph, a localized concept of formation developed by Wielandt, presents some results by J. A. Troccolo.

The last chapter, 19 pages by the editor, neatly summarizes much of the book by briefly stating or restating and proving known characterizations of certain classes of groups, as well as whether the classes are closed under the taking of subgroups, homomorphic images, or direct products. The classes are: CLT-groups, QCLT-groups, nilpotent-by-abelian groups, groups with the Sylow tower property, supersolvable groups, $\mathfrak{Y}$-groups ($G \in \mathfrak{Y}$ iff for all proper subgroups H, if prime p divides the index of H in G then there is a subgroup W in which H has index p), $\mathfrak{S}\mathfrak{Y}$-groups ($G \in \mathfrak{S}\mathfrak{Y}$ iff all subgroups of G are in $\mathfrak{Y}$), and LM-groups. The omnibus Theorem 5.1 presents 23 equivalent conditions for supersolvability.

I would like to caution the reader that the list of References does not attempt to include all recent research papers on the topics of this book; it does, of course, include the results actually stated or referred to in this book. The index is also briefer than I would like. Since there does not seem to exist any other recent English language survey of special classes of solvable groups, this book should be a valuable addition to the libraries of those interested in the subject.

LARRY DORNHOFF

BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 8, Number 2, March 1983

0273-0979/82/0000-0981/$02.75

Coding the universe, by A. Beller, R. B. Jensen, and P. Welch, London Mathematical Society Lecture Note Series, vol. 47, Cambridge Univ. Press, Cambridge, London, New York, New Rochelle, Melbourne, and Sydney, 1982, 353 pp., $34.50.

As the title says, this book is concerned primarily with Jensen's theorem on coding the universe with a real. Suppose M is a model of the axioms ZFC + GCH of set theory. Then Jensen's theorem says that, working almost in M, we can find a real number a which codes the entire model M in the sense that $L[a]$, the class of sets constructible from a, contains the model M. Furthermore the extension $M[a] = L[a]$ has the same cardinals and cofinalities as M and (so far as it is consistent with $V = L[a]$) the same large cardinals as M. Thus all of M is coded by the real a, with relatively little violence to the structure of M. This result has been known for several years, but it has been available only in handwritten notes, and then only with a strong extra hypothesis the elimination of which has been generally known only via rumor. This book will be exceedingly welcome to those who are interested in this difficult and important theorem.

The basic question addressed in this book is an important one for understanding the relationship between models of set theory. Suppose that M is a transitive model of ZFC containing all the ordinals. Then what extensions of M are there, that is, what models N are there which contain M and have the same ordinals as M?

Two well behaved classes of such extensions are known. The first, due to Cohen, is given by forcing extensions. Given any partial order P in M one takes a generic subset G of P; that is a subset of P such that the intersection of G with any dense subset of P in M is nonempty. Then $M[G]$, the least model of set theory containing G and all sets in M, is the desired extension of M. Such generic extensions have the useful property of being "almost in" M; the formal statement is that for any sentence σ of set theory true in $M[G]$ there is a member p of G which forces σ to be true, so that if G' is any other generic set which also contains p then σ is true in $M[G']$ as well. Thus truth in $M[G]$ is determined not by all of G but by the single condition p, which is in M. If P is a set then the extension is even closer to M, since G can do relatively little to change the structure of M very far above the cardinality of P.

The model $L[a]$ of sets constructed from the real a given by Jensen's covering lemma is a class generic extension of M; this is what was meant by the phrase "almost in" M in the first paragraph. The term "class generic" means that the partial ordering P is a proper class in M, rather than a set.

The second class of extensions are provided by large cardinals. The simplest example is a measurable cardinal: let U be a normal, κ complete ultrafilter on a cardinal κ. The $L[U]$ is an extension of L which is well behaved in the sense that the properties of $L[U]$ are entirely determined by the fact that U is a measure and not by the particular measure chosen. A more important example for the present book is given by the existence of $0^{\#}$, which is the smallest large cardinal axiom inconsistent with L. Suppose that there is a class C of indiscernibles for L; then $0^{\#}$ is the class of Gödel numbers of formulas which are true in L when the variables are interpreted as an arbitrary increasing string of members of C. Then $0^{\#}$ is a subset of ω and the class C of indiscernibles can be reconstructed from the set $0^{\#}$. Jensen's original proof of the coding theorem required the added assumption that $0^{\#}$ doesn't exist. The elimination of this added hypothesis is generally available for the first time in this book.

These two methods of extension are distinct. It has long been known that it is impossible to add $0^{\#}$ by set forcing and a result of Beller reported in this book goes a long way towards proving that it is also impossible with class forcing; at least it is hard to imagine a reasonable notion of forcing which would escape his result. On the other hand there are many partial orders for which adding $0^{\#}$ does add generic sets. This is extensively addressed in other results of Beller in the book.

So far we have generic extensions, which are close to the ground model M, and extensions by large cardinals which are essentially larger than M. Both extensions are well controlled in the sense that they do not depend critically on the fine details of how the extension was made. Are these the only possible extensions? Solovay conjectured that (at least below $0^{\#}$) they are. Specifically, he conjectured that for all sets a of ordinals either a is set generic over L or else $0^{\#} \in L[a]$. A weaker conjecture is that this is true for all sets a in $L[0^{\#}]$. Jensen, in the work reported in this book, proves that both of these conjectures are false. Let M be a model of set theory. We can assume M satisfies the GCH. (This assumption is made in the book; however if M does not, then as Jensen has shown there is a generic extension of M which does satisfy the GCH, which satisfies the same large cardinal axioms as L, and which collapses cardinals only as necessary to satisfy the GCH.) The real a given by Jensen's coding theorem cannot be in any set generic extension of L because $L[a]$ is a class generic extension which adds Cohen subsets of every regular cardinal. On the other hand $0^{\#}$ is not in $L[a]$ unless it is in M. Furthermore, if $0^{\#}$ exists then we can falsify the weaker form of Solovay's conjecture by finding in $L[0^{\#}]$ a subset a of ω which is "pseudo generic" over the same partial order. This again ensures that a codes up subsets of every cardinal which are Cohen generic over L, so that a is not in any set generic extension of L; at the same time $L[a]$ has the same indiscernibles as L does, so that $0^{\#}$ is not in $L[a]$.

The proof of Jensen's theorem starts from a technique of Solovay for adding a real to code a subset of $\aleph_1$. Suppose that $\aleph_1$ is equal to $\aleph_1$ as defined in L. We want to find a generic extension N of M containing a real a which codes A, so that $A \in L[a]$. We start with a sequence $\langle b_\nu : \nu \in \aleph_1 \rangle$ of almost disjoint subsets of $\aleph_1$, defined in L. Conditions are pairs (p, x) such that p is a function from a finite subset of ω into 2 and x is a finite subset of $\aleph_1$, where (p, x) extends (p', x') if p extends p', x contains x', and for all

$$n \in \operatorname{dom}(p) \setminus \operatorname{dom}(p')$$

and all $\nu \in x'$, $p(n) = 0$ if $\nu \notin A$. If G is generic then $a = \{n: \exists (p, x) \in G, p(n) = 1\}$ is a new subset of ω which codes the subset A of $\aleph_1$: if $\nu \in \aleph_1$ then $\nu \in A$ if and only if $a \cap b_\nu$ is infinite. Since the forcing has the countable chain condition, all cardinals are preserved.

The assumption that $\aleph_1$ is equal to $\aleph_1$ of L can be weakened; it is enough to assume that there is a sufficiently nice subset B of $\aleph_1$ so that $\aleph_1$ in $L[B]$ is the real $\aleph_1$, and such a set can be added by a preliminary stage of forcing. Jensen actually combines these two stages. He considers two sets of conditions. Conditions s in the first set, **S**, are called the "reshaping conditions" but in fact are the analogue of the functions p in the conditions defined above for Solovay's forcing, modified so as to also add the set B. The second, which is the forcing **P** actually used, combines a condition s from **S** with an analogue of the set x from the Solovay conditions.

Obviously Solovay's trick can be iterated to code subsets of $\aleph_n$ by subsets of ω for any fixed n in ω. It is less obvious that this trick can be iterated infinitely many times in order to code, for example, a subset A of $\aleph_\omega$ as a subset of ω. In order to do this we must add, for each n in ω, a subset b_n of $\aleph_n$ which codes not only $A \cap \aleph_{n+1}$ but also the new subset b_{n+1} of $\aleph_{n+1}$ which we are simultaneously adding to code the segment of A above $\aleph_{n+1}$. This apparent infinite regress is avoided by the simple idea of allowing conditions for adding b_n to code up only that part of b_{n+1} which has already been forced.

If all cardinals were regular then this would essentially finish the argument: an Easton style class extension could be used to code a class A of ordinals. Unfortunately the basic Solovay forcing breaks down at singular cardinals: if κ is singular then trying to code a subset of κ^+ by a subset of κ will simply collapse κ. The reason for this is that the Solovay trick to code a subset of κ adds its subset of κ via conditions with domain of size less than κ. If κ is regular this is enough to give the conditions the κ chain condition and hence keep κ from being collapsed; with κ singular this fails. Jensen solves this difficulty in the coding problem by a use of the fine structure of L, and it is this use of fine structure which accounts for almost all the difficulty of the proof.

Since adding a subset b_ω of $\aleph_\omega$ to code $A \cap \aleph_{\omega+1}$ would collapse $\aleph_\omega$, Jensen makes the sequence $\langle b_n: n \in \omega \rangle$ code this up at the same time as each b_n is individually coding up $A \cap \aleph_{n+1}$. This alone seems difficult enough, but it must be recalled that we do not only have $\aleph_\omega$ to deal with; we must deal with all singular cardinals at once. Thus in constructing the new subset b_κ of κ we must keep track in some coherent way of all the singular cardinals λ larger than κ such that b_κ might be helping to code subsets of λ^+. This sort of organization is precisely what Jensen's principal $\square$ was designed for, and it is this use of the fine structure of L which leads to the complications of the proof.

This book gives a detailed proof which is relatively readable to anyone with the necessary prerequisites. Needless to say, these prerequisites include a firm grounding in the basic theory of fine structure as well as familiarity with set theory in general. There are numerous misprints, mainly in the most technical parts of the exposition, but these should not be too much of a barrier to the qualified reader. The exposition could also be improved by more explanation of where the proof is and where it is going, but the real difficulty of reading this book comes simply and directly from the difficulty of the mathematics.

WILLIAM J. MITCHELL

BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 8, Number 2, March 1983

0273-0979/82/0000-0985/$01.75

Random Fourier series with applications to harmonic analysis, by Michael B. Marcus and Gilles Pisier, Annals of Mathematics Studies, No. 101, Princeton Univ. Press, Princeton, N.J., 1981, v + 150 pp., $17.50 (cloth), $7.00 (paperback).