B-sufficiency and B-ancillarity corresponding to the usual concepts. Several reprinted talks with accompanying discussions on these subjects have appeared lately. One, by the author, is in J. Roy. Statist. Soc. Ser. B **38**(1976) no. 2, 103–131 and the others in which he participated can be found by consulting the Math. Rev. Indices which list discussants as well as authors.

A short middle section gives some special mathematical results and leads to the final part on exponential families. There the regular theory of exponential families is developed and a number of examples are given. The existence and uniqueness of maximum likelihood and maximum plausibility estimates is discussed as is prediction by both methods for these families. The final chapter then deals with the existence and character of sufficient and ancillary statistics for exponential families.

The mathematical and statistical prerequisites for the book are modest and most of the proofs are pretty straightforward. A little measure theory and a standard senior statistics course should suffice. This does not mean, unfortunately, that the book is easy to read. The ratio of definitions to theorems is very high as is common in this subject. This makes for a lot of cases in which it is easier to follow the proof than it is to figure out what the theorem really says. This is mainly a problem of attention span (the reviewer's is somewhat below average) but is a problem.

On the positive side, the author provides many good examples, i.e. examples which are relevant to current statistical practice while also illustrating the points involved. Each chapter contains a complements section and a notes section which between them tell a great deal about who did what, how it could be generalized, and who did it differently. All in all, an interesting book for the specialist but a little heavy going for the general mathematical public.

T. S. PITCHER

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The collected works of Harold Davenport, edited by B. J. Birch, H. Halberstam, and C. A. Rogers, Volumes I, II, III, and IV, Academic Press, London, New York, San Francisco, 1977, xxxiii + 1910 pp., $30.35, $29.35, $34.25, $20.35.

The Collected Works of Harold Davenport fill four handsome volumes. Three photographs and the facsimile of a handwritten page form the frontispieces.

Almost a decade has passed since Davenport died, but few people realize that, in part, perhaps, because some of his work continued to appear as late as 1975.

As one starts browsing through the almost 2000 pages, one of the first impressions is that of an unusual fluency of the exposition not only in English, but also in German and French. When one then looks at the frontispieces one remembers the man (most of us will remember him like the photos of volumes 3 and 4) and recognizes his handwriting (at least this reviewer never received a typed letter from Davenport).

Then perhaps, one asks oneself the question; What role will history assign to the contributions of Harold Davenport to mathematics? The answer is not obvious. Perhaps everyone who asks the question, will have to find his own answer. Throughout his mature lifetime he was a towering figure. On many topics (e.g., on Diophantine approximations) he was the undisputed supreme authority. On the other hand, his interests were rather sharply focused on a few important and very difficult problems and he may not have had the breath of the very greatest mathematicians. But still–at least number theory is not quite the same after his work, as it had been before. Perhaps–at least in this reviewer's views–his role in reshaping number theory may be compared to that of his friend E. Landau. But then again–it may be best to leave the question open and to look instead at Davenport's work.

The volume of Davenport's publications is large because he considered it the duty of a mathematician to publish everything (nontrivial!) he knew, not only his best work. And not everything Davenport published can be found in these Collected Works. Obviously, his books and lecture notes are not reprinted here; nor are the nonresearch papers, such as survey articles, obituary notices, the (previously published) texts of some of his lectures, etc. Among the 25 or so papers listed, but not republished here are three out of four notes in the Comptes Rendus (Paris) in French, one survey article from the Jahresbericht der Deutschen Mathematiker Vereinigung in German, three other papers in French and 18 in English. All among these papers that have substantial mathematical content are either survey articles, or abstracts, or partial duplications of papers republished in these Collected Works.

As indicated in the Preface to Volume 1, the papers are grouped by subject matter. Within a given category, they are listed in roughly chronological order. The first two volumes contain Davenport's main work on Diophantine approximation (but see also vol. 4, e.g., paper No. 182) and the geometry of numbers (edited by C. A. Rogers, with assistance from J. W. S. Cassels, W. M. Schmidt and G. L. Watson), while vol. 3 contains papers on the Hardy-Littlewood method (edited by B. J. Birch, with assistance from D. J. Lewis). Volume 4 (edited by H. Halberstam, with assistance from Armitage, Bombieri, Burgess, Erdös, Lewis, Montgomery, and Watson) consists of four sections, as follows: (I) character sums and exponential sums; (II) polynomials and Diophantine equations; (III) Dirichlet and other series; and (IV) miscellaneous results. After each group of related papers one finds a section of, generally rather short, commentaries, on such topics as the evolution of Davenport's thought on the problem, his successive ever sharper results on it, the extent to which the solutions given by Davenport are definitive and, whenever it has been the case, the progress made by others on the problem.

There is, of course, no point to list here the individual papers (the list of all 198 titles can be found at the end of each volume), even less to analyse them individually, or even in clusters of related papers. As already stated, short, but eminently competent comments can be found in the volumes themselves. However, as pointed out by C. A. Rogers in his bibliographical notes, in vol. 1, "... the main bulk of [Davenport's] work was centred round a few key problems that he regarded as of outstanding importance ...". Perhaps one could attempt to assess the relevance of Davenport's work on some of these problems. This has indeed been done already and by much more competent hands (C. A. Rogers, B. J. Birch, H. Halberstam, and D. A. Burgess; see **[10]** and **[10a]**). For this reason, the present reviewer limits himself to a simple mention of some of the "key problems" and a statement of a few of the results obtained by Davenport.

Even this modest goal is not easily attained, because the number of those "key problems" is not all that small. With all their ramifications considered by Davenport, they do, in fact, cover quite some ground and it will be necessary to make a selection of problems and contributions to be mentioned–and, like all such selections, also the present one will inevitably reflect the reviewers own biases.

Before any discussion of Davenport's work, however, it is necessary to recall that, especially in his later years, Davenport published many joint papers. It would be cumbersome to mention for each result the name of all his collaborators and of those who influenced his thought. On the other hand, it also would be unfair to ignore their contributions and Davenport would not have liked to take alone credit for joint work. In fact, even in papers published under his name alone, he gives generously credit to people who had influenced his work–be they mature mathematicians, like Mordell, or young students. For this reason we list here in alphabetic order the names of joint authors of papers republished in the present four volumes: A. Baker, R. P. Bambah, B. J. Birch, E. Bombieri, H. Chatland, S. Chowla, J. G. van der Corput, P. Erdös, H. Halberstam, M. Hall, H. Hasse, H. Heilbronn, E. Landau, W. J. LeVeque, D. J. Lewis, K. Mahler, G. Pólya, D. Ridout, C. A. Rogers, K. F. Roth, A. Schinzel, W. M. Schmidt, H. P. F. Swinnerton-Dyer, and G. L. Watson.

Returning now to the "key problems", it is not really the case that these constitute discrete entities. Consider, for instance, Waring's problem, one of Davenport's lasting interest. It deals with the representation of integers as sums of a fixed number s of kth powers. This question is immediately generalized to the representation by diagonal forms of degree k in s integral variables and, more generally, by aribtrary forms of degree k. In particular, one may study the representation of zero by indefinite forms, and this leads to the general problem of Diophantine equations and systems. The same questions can be asked about forms, equations, systems, etc., over finite fields, rather than over the rationals. Next, if zero itself cannot be represented, how small can the forms be made? This question leads to Diophantine inequalities and Diophantine approximation. In many of these questions the main tool used is the Hardy-Littlewood "circle method", in the Vinogradov version. This, in turn, requires the study of certain exponential sums. These sums are, of course, interesting in their own right and lead to important results, e.g., on the distribution of quadratic residues and nonresidues, and, more generally, of kth power residues and nonresidues, the size of the smallest primitive root, etc. These last questions are closely related to the distribution of primes, and lead, on the one hand, to the study of Dirichlet (and also other) series; on the other hand to the consideration of the large sieve. While these topics have all been studied by Davenport, who made significant contributions to many of them, the preceding titles do not represent by any means an exhaustive list of his work.

Let us recall now (selectively) some of the results obtained by Davenport.

1. Waring's problem. Let $N_s^{(k)}(n)$ be the number of integers less than n, that are sums of s positive kth powers and let $G(k)$ be the least s, so that every sufficiently large integer should be a sum of s (or fewer) kth powers. Here are some of Davenport's results.

$$N_s^{(k)} > n^{\alpha_{s,k}-\varepsilon},$$

with

$$\alpha_{s,k} = 1 - \frac{1-k^{-1}}{1-k^{-1}(1-k^{-1})^{s-2}}(1-2k^{-1})(1-k^{-1})^{s-2}.$$

The particular case $N_3^{(3)} > n^{47/54-\varepsilon}$ is somewhat stronger. Every sufficiently large integer m is the sum of 14 fourth powers, unless $m \equiv 15$ or 16 (mod 16); it is the sum of 15 fourth powers, unless $m = 16^h \cdot k$, where k has only a finite number of values; finally, every sufficiently large integer is the sum of 16 fourth powers. There are infinitely many integers (e.g., $m = 16^h \cdot 31$), that are not sums of fewer than 16 fourth powers, hence $G(4) = 16$. Similarly, $G(5) \leqslant 23$ and $G(6) \leqslant 36$. The best previous results were $G(5) \leqslant 28$ (Hua, see **[6]**) and $G(6) \leqslant 42$ (Estermann see **[5]**). It seems that most of these results have not been superseded until now.

2. Cubics. Let C, Q, L, N stand for forms (i.e., for homogeneous polynomials) with integral coefficients, of degrees 3, 2, 1, 0 respectively. Also, define h, as the smallest integer, that permits a decomposition of the form $C(x_1, \ldots, x_n) = \sum_{j=1}^{h} L_j(x_1, \ldots, x_n)Q_j(x_1, \ldots, x_n)$. Here are some of Davenport's results: $C(x_1, \ldots, x_n) = 0$ has nontrivial solutions if $n \geqslant 16$. If $\phi(x_1, \ldots, x_n) = C + Q + L + N$ and if ϕ satisfies the (obviously necessary) "congruence condition" of solvability of $\phi \equiv 0 \pmod{p^\nu}$, for all primes p and integers $\nu \geqslant 1$, with $x_1, \ldots, x_n$ not all divisible by p, then $n \geqslant 20$, $h \geqslant 4$ are sufficient conditions to insure the solvability of

$$\phi(x_1, \ldots, x_n) = 0 \text{ in integers } x_1, \ldots, x_n.$$

3. Forms in many variables. If $Q = \sum_{j=1}^{5} \lambda_j x_j$ is an indefinite quadratic form with real coefficients, with at least one ratio λ_j/λ_k irrational, then, for every $\varepsilon > 0$ there exist arbitrarily large integers P, such that $|Q(x_1, \ldots, x_j)| < \varepsilon$ has more than γP^3 solutions in integers x_j, $1 \leqslant |x_j| \leqslant P$ with $\gamma > 0$.

If $Q(x_1, \ldots, x_n)$ is an indefinite quadratic form with real coefficients in at least 21 variables, for every $\varepsilon > 0$, the inequality $|Q(x_1, \ldots, x_n)| < \varepsilon$ is solvable in integers $x_1, \ldots, x_n$, not all zero. If the coefficients of Q are not all in rational ratios, then the set of values of Q (still for integral x_j's) is everywhere dense.

Let $G^*(k)$ be the least value of s such that $F(x_1, \ldots, x_n) \underset{\text{def}}{=} \sum_{j=1}^{s} c_j x_j^k = 0$ has infinitely many integral solutions for all sets of integers c_j, for which $F \equiv 0 \pmod{p^\nu}$ is solvable (i.e., the equation is solvable in every p-adic field). Also, let $\Gamma^*(k)$ be the least s that insures said congruence condition. Then, if $s = \max\{G^*(k), \Gamma^*(k)\}$ (and also, if k is even, if not all c_j's are of the same sign, so that the equation should be solvable over the reals) the Diophantine equation $F \equiv 0$ has infinitely many solutions. Davenport shows that $\Gamma^*(k) \leqslant k^2 + 1$ and, if $\delta > 0$, $G^*(k) \leqslant (4 + \delta)k \log k$. Furthermore, if $k \geqslant 18$, $G^*(k) \leqslant \Gamma^*(k)$. It may also be shown that, if $k \leqslant 6$, then $s = k^2 + 1$ (or lower values) suffice for the solvability of $F = 0$. One concludes that Artin's Conjecture (namely that s variables suffice if $s \geqslant k^2 + 1$) holds, except, perhaps, for $7 \leqslant k \leqslant 17$. Many other related nice results concerning, e.g., the simultaneous solvability of two cubics deserve to, but cannot be restated here.

4. Exponential and character sums. In the study of the distribution of quadratic residues and nonresidues one is led to the estimation of the sum $S_r(a_1, \ldots, a_r) =$

$$\sum_{n=0}^{p-1} \left(\frac{(n + a_1) \ldots (n + a_r)}{p} \right),$$

where $(\frac{a}{p})$ stands for the Legendre symbol. By elementary methods, Davenport shows that $S_r = O(p^{3/4})$, if $r = 3$ or 4. He also obtains similar results for higher power residues. The sum S_r occurs however also in a different context. If N_r stands for the number of solutions (x, y), distinct modulo p, of the congruence $y^2 \equiv (x + a_1) \ldots (x + a_r) \pmod p$, then $N_r = p + S_r(a_1, \ldots, a_r)$. Hence, Weil's proof of the Riemann hypothesis for curves, from which it follows that $S_r = O(p^{1/2})$, supersedes these results of Davenport and also his results on the sums $S_f = \sum_{\lambda=0}^{p-1} e(f(x)/p)(f(x) =$ polynomial of degree n with rational coefficients, or (with some restrictions) a rational function; $e(v) = e^{2\pi i v}$) and on the generalized Kloosterman sums $S' = \sum_{x=1}^{p-1} e(ax^n + bx^{-n})$. Davenport's results were (i) $S_f = O(p^{1-m^{-1}})$, where, for $n \geqslant 4$, m is the largest integer not in excess of n and of the form $2^g (g \geqslant 2)$, or $2^g \cdot 3 (g \geqslant 1)$; and (ii) $S' = O(p^{2/3})$ (identical to Salié's result for the classical Kloosterman sum [12]).

The study of exponential and character sums led Davenport to number theoretic results, such as the following:

A (technically elementary) proof of the fact that, if d is the least quadratic non-residue, then $d = O\{p^{1/2} \log p)^{1/\sqrt{e}}\}$ (the improvement over the previously known bound is small; the relevance of the paper resides in the "elementary" nature of its argument). More generally, if d_k is the least positive kth power nonresidue modulo p, $k > 2$, $p \equiv 1 \pmod k$, p large, then $d_k = O(p^{\alpha_k + \varepsilon})$, where $u = 1/2\alpha_k$ is the (unique) root of $\rho(u) = k^{-1}(\rho(u) = 1 - \log u$ for $1 \leqslant u \leqslant 2$, $u\rho'(u) = -\rho(u - 1)$ for $u \geqslant 2)$. Also, each class of cubic nonresidues modulo p contains a positive integer less than $p^{\gamma + \varepsilon}$, provided that $p \geqslant p_0(\varepsilon)$ and where $\gamma \cong .383 \ldots$ is defined by

$$-\log(2\gamma) + \int_1^{\gamma^{-1}} \frac{\log t}{1 + t} \, dt = 1/3.$$

Among Davenport's results on character sums over finite fields one finds, e.g., the following: Given a polynomial $P(x)$ of degree k, there exists $p_0(k)$ with the following property: If $p \geqslant p_0(k)$ and $P(x)$ is irreducible modulo p, then there exists an integer a, such that $x - a$ is a primitive root modulo $P(x)$. Equivalently, if θ is any generating element of the Galois field $GF(q)$ of

$q = p^k$ elements, then there exists an integer a, such that $\theta - a$ is a primitive root of $GF(q)$.

Finally, a perhaps isolated result, but which struck this reviewer as particularly nice, is the following: Let $n_1, \ldots, n_N$ be any N distinct integers and let $E = \sum_{j=1}^{N} e(n_j x)$. Then P. Cohen [4] proved that $\int_0^1 |E(x)| dx > C(\log \log N)$ ($C > 0$, some undetermined constant). Within hours after hearing a presentation of this result, Davenport proved that

$$\int_0^1 |E(x)| dx > \frac{1}{8}\left(\frac{\log N}{\log \log N}\right)^{1/4}.$$

This result has been superseded only very recently by S. K. Pichorides (see [8]; for related problems see also [9] and [11]) who increased the exponent to $1/2$ (the best possible exponent is 1).

5. The large sieve. Davenport did not write many papers on the large sieve and, as far as this reviewer knows, all were joint papers either with Bombieri, or with Halberstam (possible exception: a posthumously published paper, edited by Halberstam). But these papers are of greatest importance, especially for the clarification of the analytic basis of the method. While one just has to read the original papers, in order to fully appreciate their relevance, here are some of the (by now classical) results.

Let x_j be real numbers, a_n real, or complex ones, denote by $\|x\|$ the distance of x to the nearest integer, assume that, for $r \neq s$, $\|x_r - x_s\| \geqslant \delta > 0$, and set $S(x) = \sum_{n=M+1}^{M+N} a_n e(nx)$. Then $\sum_{r=1}^{R} |S(x_r)|^2 \leqslant K \sum_{n=M+1}^{M+N} |a_n|^2$. Here $K = K(N, \delta) \leqslant 2\max(N, \delta^{-1})$ and also $K \leqslant (N^{1/2} + \delta^{-1/2})^2$. If $N\delta$ is "large", $K \leqslant (1 + \varepsilon)N$; more precisely, if $N\delta > 1$, then $K \leqslant N(1 + 5(N\delta)^{-1})$, but if $c < 1$, then there exist sums for which $K = N(1 + c(N\delta)^{-1})$ is not sufficient. If $N\delta$ is "small", then $K \leqslant (1 + \varepsilon)\delta^{-1}$; more precisely, for $N\delta < 1/4$, $K \leqslant \delta^{-1}(1 + 270(N\delta)^3)$, but for some sums $K = \delta^{-1}(1 + (N\delta)^3/12)$ does not suffice.

These results were used to prove, among others, that

$$\sum_{q<X} \sum_{\substack{a=1 \\ (a,q)=1}}^{q} \{\psi(x; q, a) - x/\phi(q)\}^2 < Cx^2(\log x)^{5-A} \text{ for } X < x(\log x)^{-A},$$

where

$$\psi(x; q, a) = \sum_{\substack{n<x \\ n \equiv a (\bmod q)}} \Lambda(n);$$

and, for N sufficiently large and $k < N/3$,

$$\pi(M + N; k, l) - \pi(M; k, l) \leqslant \left\{2N/\phi(k)\log \frac{N}{k}\right\} \cdot \left\{1 + O\left(\log\log \frac{N}{k} \Big/ \log \frac{N}{k}\right)\right\},$$

where

$$\pi(x; k, l) = \sum_{\substack{p<x \\ p \equiv l (\bmod k)}} 1.$$

6. Series. Davenport's results on series are numerous and perhaps less focused upon a central theme than some of his other work. For that reason only a somewhat random sampling of the results is possible.

For fixed s and $\sigma(=\operatorname{Re} s)$ in $0<\sigma<1$, with χ a nonprincipal character modulo k, $L(s, \chi) = O(k^{(1-\sigma)/2})$. This improves the previously known bound by a logarithmic factor. This result has been improved since by Burgess (see [1], [2], [3]).

It had been known (see Ingham [7]) that

$$\lim_{T\to\infty} \frac{1}{T}\int_1^T |\zeta(\sigma+it)|^{2k}\,dt = \sum_{n=1}^{\infty} d_k^2(n)n^{-2\sigma}$$

($d_k(n)$ defined by $\zeta^k(s) = \sum_{n=1}^{\infty} d_k(n)n^{-s}$) holds for $0 \leqslant k \leqslant 2$, if $\sigma > 1/2$. Davenport proves that this holds for all integers $k > 1$, provided that $\sigma > 1 - (\nu+1)/(2k+2^\nu-2)$ (ν defined by $(\nu-1)2^{\nu-2}+1 \leqslant k \leqslant \nu\cdot 2^{\nu-1}+1$).

Davenport proved that certain Dirichlet series (among which are the Epstein and Hurwitz zeta functions, for certain values of their parameters) have an infinity of zeros with $\sigma > 1$; this contrasts sharply with the case of the Riemann, or Dedekind zeta functions.

Let $\{t\} = t - [t] - 1/2$ for nonintegral t, $\{t\} = 0$ for t an integer. A large number of series involving $\{nt\}$ were studied by Davenport. He showed, e.g., that $\sum_{n=1}^{\infty} \mu(n)n^{-1}\{n\theta\}$ converges uniformly to $-\pi^{-1}\sin 2\pi\theta$ and that $\sum_{n=1}^{\infty} \mu(n)n^{-2}\{n^2\theta\}$ converges for all real θ and is equal to $-\pi^{-1}\sum_1^{\infty}(\mu^2(n)/n)\sin 2\pi n\theta$. As a last example, assume that, for $n\to\infty$, one has $a_n \to 0$, $\sum_{-\infty}^{\infty}|a_n - a_{n+1}|$ and $\sum_{-\infty}^{\infty} a_n$ convergent, and that the partial sums of $\sum_{-N}^{N} a_n e(nt)$ are uniformly bounded for real t. Assume also that α is real and irrational and γ is real and set $g(t) = \sum_{-\infty}^{\infty} a_n e(nt)$. Then, for $|x| < 1$, the two series

$$\sum_{-\infty}^{\infty} \frac{a_n e(n\gamma)}{1 - xe(n\alpha)} \quad\text{and}\quad \sum_{\nu=0}^{\infty} g(\nu\alpha+\gamma)x^\nu$$

are both convergent and converge to the same value, which, for $x = re(k\alpha)$, $r\to 1$, is asymptotically equal to $a_{-k}e(-k\gamma)(1-r)^{-1}$.

7. Miscellanea. Much of the most beautiful and some of the most important work of Davenport has not yet been mentioned. But a review has to stop somewhere. To the slighted contributions belong those on the geometry of numbers, on the products of homogeneous linear forms with the beautiful "isolation theorems", on the Minkowski problem, on the study of binary and ternary cubic forms and on nonhomogeneous quadratics. Also the determination of all real quadratic fields with an Euclidean algorithm, much of Davenport's work on Diophantine inequalities and on Diophantine approximation, his work on polynomials, a beautiful paper on the product of power series, work on number theoretic problems (such as Euler's ϕ-function, small differences of primes, numeri abundantes, etc.) all this and much more has not even been mentioned.

Having gone through this long and incomplete list, one may well be tempted to reread one of the publications themselves, say a paper that one already knows well; it almost does not matter which. One will appreciate the clarity of the exposition and the precision, which leaves no room for uncertainty. The style has sometimes been characterized as austere or severe. It may, occasionally be also somewhat elliptic. The ideas are presented in a most economical fashion and the author does expect the reader to be able to fill in the more obvious details. This permits him to present the leading ideas in an uncluttered way.

Finally, while the ultimate verdict on the work, like everything human, belongs to history, those of us, who were fortunate enough to have known Harold Davenport, cannot help remembering also the man. While much of what he was–cultured, articulate, logical–is indeed reflected in his work, not everything is. He was generous with his time and enjoyed (or at least seemed to enjoy) showing Cambridge to his guests. While, to judge by his students, his standards must have been very high, he was quite patient with the more common brand of mankind and made genuine efforts to make himself understood by the less sophisticated reader (see, e.g., his book "The Higher Arithmetic"). In fact, this reviewer can recall only one outburst of impatience (or indignation?) of Davenport: it was with mathematicians who claim results, but never publish their proofs, either because they don't have any, or in order to keep their methods as private property of a small group of close collaborators. No names were named.

The reviewer wants to take this opportunity to thank Professor D. J. Lewis for a very helpful letter concerning Davenport which confirmed many and completed some of the reviewer's own recollections.

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EMIL GROSSWALD

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Automata-theoretic aspects of formal power series, by Arto Salomaa and Matti Soittola, Texts and Monographs in Computer Science, Springer-Verlag, New York, Heidelberg, Berlin, 1978, x + 178 pp., $16.50.

In the early sixties, stimulated by the discoveries of M. P. Schützenberger, a number of researchers at the University of Paris contributed to a new