

functions as the finite region expands to infinity. The resulting expressions satisfy the euclidean axioms and hence implicitly define a Wightman field theory but without uniqueness. The relativistic sharp time fields are however well defined. In Chapter 9, a close analogy is exploited between the lattice approximation to a  $a\Phi^4 + b\Phi^2 - \mu\Phi$  ( $\mu \neq \Phi, a > 0$ ) model and the Ising spin system. For this case, the half-Dirichlet theory is shown to satisfy spectral properties for the infinite region Hamiltonian required by Wightman's framework, and thus provides the desired nontrivial model of relativistic fields in two dimensional space-time.

Simon's book certainly provides a valuable and timely addition to the mathematical physics literature on quantum field theory. It makes more accessible many of the current ideas and techniques to which the author has made numerous contributions. Few details are bypassed, so with some supplement from the literature it is a good place to learn the subject, particularly for mathematicians who wish to short-cut learning unnecessary physics. Most enjoyable are Simon's incisive summaries at the beginning of each chapter on the thrust of current research and the careful documentation of individual results in the large bibliography. The full scope of constructive field theory is well represented by this work.

JOHN L. CHALLIFOUR

*La série génératrice exponentielle dans les problèmes d'énumération*, by Dominique Foata, University of Montréal Press, Montréal, 1974, 186 pp., \$5.00

These lecture notes contain seven chapters written by Dominique Foata and one chapter written by Bernard Kittel. The Foata chapters and the Kittel chapter have different notation and individual reference sections. For this reason we shall consider them separately.

Foata decided to present his lectures in the traditional format of formal power series. Thus Foata's presentation essentially follows that of John Riordan (*An introduction to combinatorial analysis*, Wiley, 1958 and *Combinatorial identities*, Wiley, 1968). Therefore the reader would be well advised to start with Riordan's books. On the other hand, Foata is able to give more recent results since his lecture notes were published six years after Riordan's latest book, *Combinatorial identities*. Foata chose not to use the operator theory setting as can be found in the presentation of Ronald Mullin and Gian-Carlo Rota (*On the foundations of combinatorial theory. III: Theory of binomial enumeration*, Graph Theory and Its Applications (B. Harris, editor), Academic Press, 1970).

Rota and several other authors have been publishing a series of papers dealing with the mathematical foundations of enumerative combinatorics. Foata's lecture notes also develop these foundations. Four years prior to the publication of the lecture notes considered in this review, Dominique Foata

and Marcel-P. Schützenberger published *Théorie géométrique des polynômes Eulériens*, Springer-Verlag Lecture Notes in Mathematics, no. 138, 1970. There is considerable overlap between the contents of the Foata-Schützenberger lecture notes and these Foata lecture notes.

What is Foata's contribution to the development of the foundations of enumerative combinatorics? After developing the elementary properties of exponential generating functions and the theory of formal power series, Foata discusses nonabelian and abelian partitional compositions. (Foata and Schützenberger introduced the partitional composition notation in their 1970 lecture notes, cited above.) A classical example of such a composition is a permutation of an  $n$ -set expressed as the abelian partitional composition of disjoint cycles. Foata obtains many classical enumerative results from what he calls the fundamental transformation. This transformation is a bijection of the symmetric group  $S_n$  onto itself such that a permutation with  $k$  excedances is mapped to a permutation with  $k$  descents.

What are some of the results that can be obtained from partitional compositions, the fundamental transformation, and exponential generating functions? Foata obtains generating functions for Eulerian polynomials and for Laguerre polynomials. He finds generating functions for permutations enumerated by the number of cycles and for permutations without fixed points. The fundamental transformation defined for permutations of an  $n$ -set is extended to all mappings of an  $n$ -set into itself. Finally, Foata obtains generating functions for acyclic functions enumerated by the number of fixed points, by the number of elements in each orbit, by height, and by the number of inversions.

Now, let us consider the chapter written by Bernard Kittel. He shows that six known probabilistic identities can be proven by an exponential generating function formula. It is exciting to see this interplay between these two areas of mathematics, probability and combinatorics. Kittel is applying combinatorial techniques to probability theory, in contrast to the usual application of probabilistic techniques to combinatorics.

Clearly, these lecture notes should be read by those studying the foundations of combinatorial enumeration. These notes seem too specialized to be of great interest to the mathematical community at large. This community needs a survey book which puts the various approaches to the foundations of enumeration in proper perspective. Such approaches include combinatorial mappings, combinatorial operators, generating functions, incidence algebras, and umbral (Blissard) calculus. At present such a book is not available.

EARL GLEN WHITEHEAD, JR.

*Theory of branching of solutions of non-linear equations*, by M. M. Vainberg and V. A. Trenogin, Monographs and Textbooks on Pure and Applied Mathematics, Noordhoff International Publishing, Leyden, 1974, xxvi+485 pp.