BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 80, Number 4, July 1974

A CLASS OF SQUARE-INTEGRABLE IRREDUCIBLE UNITARY REPRESENTATIONS OF SOME LINEAR GROUPS OVER COMMUTATIVE p-FIELDS[1]

BY R. G. LAHA

Communicated by François Treves, October 22, 1973

Let k be a commutative p-field which is not necessarily of characteristic 0 (cf. [2]). We denote by $\mathcal{O}$, $\mathcal{P}$ and $\mathcal{O}^{\times}$ the unique maximal compact subring of k, the unique maximal ideal contained in $\mathcal{O}$ and the group of invertible elements of the ring $\mathcal{O}$, respectively. Then the residue class field $\mathcal{O}/\mathcal{P}$ is a finite field of characteristic p ($p>1$ being a prime number). Let $n>1$ be a fixed positive integer. Let G be the subgroup of $\mathrm{GL}(n, k)$ consisting of those elements whose determinant belongs to $\mathcal{O}^{\times}$. Then $K=\mathrm{GL}(n, \mathcal{O})$ is a maximal compact subgroup of G.

Recently Shintani [1] constructed some square integrable irreducible unitary representations of G which are induced by suitable irreducible unitary representations of K, where these representations of K can be 'parametrized' by certain characters of suitable compact Cartan subgroups of G which are contained in K. However this construction is based on the assumption that n and p are relatively prime.

In the present note we show that an interesting subclass of these representations of G can be constructed without this assumption. Moreover we give a more explicit description of the structure of these representations of G in this case.

Formulation of the main result. Let k be a commutative p-field. Let $\mathcal{O}$, $\mathcal{P}$, $\mathcal{O}^{\times}$ be the maximal compact subring of k, the maximal ideal in $\mathcal{O}$ and the group of units in $\mathcal{O}$, respectively. Let π be a prime element of k and let q be the module of k. Then the residue class field $\tilde{k}=\mathcal{O}/\mathcal{P}$ is a finite field of characteristic p ($p>1$) containing q elements. For every $\nu \in Z$, we write $\mathcal{P}^{\nu}=\pi^{\nu}\mathcal{O}$ where $\mathcal{P}^{0}=\mathcal{O}$. Let $n>1$. Let G be the subgroup of the group $\mathrm{GL}(n, k)$ consisting of those elements whose determinant belongs to $\mathcal{O}^{\times}$. Let $K=\mathrm{GL}(n, \mathcal{O})$. Then G is a unimodular locally compact topological group and K is a maximal compact subgroup of G. We also note that the group G is totally disconnected. For every positive integer $m\geqq 1$, we set $K_m=\{x \in K : x\equiv I_n \pmod{\mathcal{P}^m}\}$. Then K_m is a compact open normal subgroup of finite index in K and moreover $K_1\supset K_2\supset\cdots$ form a fundamental system of neighborhoods of the identity I_n in K.

Let k' be an unramified extension of k of degree n over k. Let $\mathcal{O}'$,

---

AMS (MOS) subject classifications (1970). Primary 22E55; Secondary 33A75.

[1] This work was supported by National Science Foundation grant NSF-GP-35724 X.

$\mathscr{P}'$ and $\mathcal{O}'^{\times}$ be the maximal compact subring of k', the maximal ideal in $\mathcal{O}'$ and the group of units in $\mathcal{O}'$, respectively. Then we note that k' is generated over k by a primitive (q^n-1)th root of unity. Let $q'=q^n$. Let ζ be a primitive $(q'-1)$th root of unity. Then $\zeta \in \mathcal{O}'^{\times}$ and moreover we have $\mathcal{O}'=\mathcal{O}[\zeta]$ and $k'=k(\zeta)$.

For every positive integer $m\geqq 1$, we define the subgroup $\mathscr{Q}'_m$ of the group $\mathcal{O}'^{\times}$ by the formula $\mathscr{Q}'_m=\{a \in \mathcal{O}'^{\times}: a-1 \in \mathscr{P}'^m\}$. Then $\mathscr{Q}'_m$ is a compact open subgroup of finite index in $\mathcal{O}'^{\times}$ and moreover the subgroups $\mathscr{Q}'_1\supset\mathscr{Q}'_2\supset\cdots$ form a fundamental system of neighborhoods of the identity in $\mathcal{O}'^{\times}$.

Let ω be an arbitrary character of the multiplicative group $\mathcal{O}'^{\times}$ and let $\mathscr{P}'^r$ be the conductor of ω. We assume $r\geqq 2$ and set $s=[r/2]$. Let χ be a character of the additive group of k of order 0. Then there exists an element $x_\omega \in \mathcal{O}'$ such that the relation

$$\omega(a) = \chi(\mathrm{Tr}_{k'/k}(\pi^{-r}x_\omega(a-1)))$$

holds for all $a \in \mathscr{Q}'_{r-s}$. The character ω of $\mathcal{O}'^{\times}$ is said to be primitive if $x_\omega=\zeta$. Let ω be a primitive character of $\mathcal{O}'^{\times}$ and let $\mathscr{P}'^r$ be the conductor of ω. Let τ be the embedding of k' into $\mathscr{M}(n,k)$ as defined in [**1**]. Then $A=\tau(\mathcal{O}'^{\times})$ is a compact Cartan subgroup of G such that $A\subset K$. We now define the character ω of the group A by the formula $\omega(\tau(a))=\omega(a)$ for $a \in \mathcal{O}'^{\times}$.

We set $X=\tau(\zeta)$. Then $X \in \mathscr{M}(n,\mathcal{O})$. Then we define the one-dimensional representation χ^s_X on K_{r-s} by the formula

$$\chi^r_X(x) = \chi(\pi^{-r}(\mathrm{Tr}\, X(x-1))), \qquad x \in k_{r-s}.$$

Let K_X be the centralizer of χ^r_X in K. We now assume that r is even so $r=2s$ and hence $r-s=s$. Then $K_X=AK_s$. We now define the function λ_ω on K_X by the formula

$$\lambda_\omega(ax) = \omega(a)\chi^{2s}_X(x), \qquad a \in A,\, x \in K_s.$$

Then λ_ω is a one-dimensional representation of K_X which coincides with ω on A and with $\chi^{2s}_X \cdot 1$ on $K_s=K_{r-s}$.

Then the main result can be formulated as follows.

THEOREM. (i) *Let* $\sigma_\omega=\mathrm{Ind}_{K_X\uparrow K}\lambda_\omega$. *Then* σ_ω *is an irreducible unitary representation of* K.

(ii) *Let* $\pi_{\sigma_\omega}=\mathrm{Ind}_{K\uparrow G}\,\sigma_\omega$. *Then* π_{σ_ω} *is a square integrable irreducible unitary representation of* G.

REFERENCES

1. T. Shintani, *On certain square-integrable irreducible unitary representations of some P-adic linear groups*, J. Math. Soc. Japan **20** (1968), 522–565. MR **38** #2252.

2. A. Weil, *Basic number theory*, Die Grundlehren der math. Wissenschaften, Band 144, Springer-Verlag, New York, 1967. MR **38** #3244.

DEPARTMENT OF MATHEMATICS, BOWLING GREEN STATE UNIVERSITY, BOWLING GREEN, OHIO 43403