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ON THE DIOPHANTINE EQUATION $Y^2 + K = X^5$

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In this paper we shall discuss the integral solutions of the diophantine equation $Y^2+K=X^5$, where K is a square-free positive integer. We shall prove the following:

THEOREM. *If the class number h of the quadratic field $Q(\sqrt{-K})$ is not divisible by 5, and if $K\neq 8L-1$, then the equation $Y^2+K=X^5$ has no nonzero integral solutions with the exceptions of $K=19$, 341.*

In these cases the solutions will be as follows:

$$(22434)^2 + 19 = (55)^5,$$
$$(2759646)^2 + 341 = (377)^5. \quad \square$$

The ideal equation $[Y+\sqrt{-K}]\cdot[Y-\sqrt{-K}]=X^5$ leads to finitely many equations [see e.g. [3]] of the form $f(A, B)=m$, where f is a homogeneous polynomial of degree 5.

The case $Y+\sqrt{-K}=\omega^5$, where ω is an integer in $Q(\sqrt{-K})$ is reduced to solving $Y^2=20X^4+1$. This was discussed by W. Ljunggren [2] and J. H. E. Cohn [1].

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