the first derivative to essential singularities where all derivatives are bounded and continuous on either side of the critical point, but where a power series expansion about the critical point diverges. Up to now, progress in this field has been made by heavy applications of classical analysis, and it remains to be seen whether modern mathematical ideas have any relevance.

The last two chapters of the book are devoted to a modern development, namely, the identification of the states of a physical system by positive linear functionals on a suitable B^*-algebra. The translational, rotational and other invariances of the system manifest themselves as groups of automorphisms of the B^*-algebra. The great hope behind such an approach is to be able to avoid the problem of starting with a finite number of particles and tediously passing to the limit of an infinite system. The B^*-algebra approach to statistical mechanics does not at present meet with favor in all quarters, and it has a diffuse reputation of being arcane. It is another attempt to apply functional analysis to classical problems in physics. As Ruelle himself says, "the results are largely due to physicists with a background of axiomatic, relativistic, quantum field theory. This imparts a somewhat special flavor to the subject." To be candid, we must admit that so far no major breakthroughs in statistical mechanics have been achieved by such methods nor, on the other hand, has a dead end been reached. The next decade will perhaps decide the issue.

As I said in the beginning, I consider this work a milestone in statistical mechanics and in mathematics. It is also a work of considerable scholarship. The author has taken pains to be right and thorough in his bibliographical references. The book is not easy to swim through, however. Mathematicians will feel comfortable with the terse style, but they will probably be at a loss to understand the physical background and motivation of the material. Physicists, on the other hand, even specialists, may find the style terse almost to the point of opaqueness. The situation might have been alleviated to a considerable extent by providing a glossary of symbols and notation, but this has not been done. The hard work, both on the part of the author and on the part of the series reader, will be well repaid by the opening up of new and fruitful vistas in mathematical inquiry.

ELLIOTT LIEB

Markov Processes with Stationary Transition Probabilities, by Kai Lai Chung, Springer-Verlag, Berlin, Heidelberg, New York, x+301 pp. $14.00

Markov processes, whose definition goes back to Markov (1907) in a special case, have been intensively studied in recent years, in part because of their intrinsic interest, in part because of the close relation between Markov processes with stationary transition probabilities and other parts of mathematics, for example ergodic theory, semigroup theory, partial differential equations, potential theory. Chung's book is a treatment of Markov processes with countable state spaces and stationary transition probabilities. The hypothesis of a countable state space simplifies the theory, but forces attention on new problems.

The book is divided into two parts. Part I, comprising the first 118 of the 291 pages of the text, is devoted to the discrete parameter case. The basic facts about the asymptotic character of the sample sequences and related probabilities are proved. Asymptotic theorems involving $\sum_1^n f(x_j)$ for the chain $\{x_j, j \geqq 1\}$ are treated following Doeblin, using the fact that the partial sums between successive times the chain hits a specified state are mutually independent with a common distribution. Here as in Part II optional times (=stopping times = Markov times) are used explicitly and carefully.

Part II of the book is devoted to continuous parameter chains and includes a treatment of the measure theoretic foundations of continuous parameter processes thorough enough to support a painstaking analysis of the sample function smoothness properties of these chains. Many of the results here are due to the author himself. Several mathematicians, starting with Ray (1959) have immersed the countable state space of a continuous parameter Markov chain in a suitable topological space to get a strong Markov process with right continuous sample functions having left limits. This technique can be used to clarify and simplify some of the analysis in Chung's book, once the immersion has been carried through. (In this context the chain theory becomes a special case of Hunt's Markov process theory, so that the countable state space case is no longer a singular case, and the seemingly hopelessly irregular nature of the sample functions becomes reasonable.) The author did not use this technique, perhaps because it would have led him to the potential and boundary theory he was determined to avoid. The continuous parameter versions of results in the discrete parameter case, the continuity and derivability properties of the matrix transition function, conditions for the validity of the forward and backward differential equations are derived. If α is an optional time for the chain $\{x(t), t \geqq 0\}$, the 'post α process' $\{x(\alpha+t), t \geqq 0\}$ is analyzed in depth, and versions of the strong Markov property are proved.

Chung's book follows the modern trend in its balance between the study of sample functions and that of probabilities. (The latter study is still sometimes distinguished from the former by the adjective 'analytical', an identification which either rules probability out of analysis or the user of the adjective out of mathematics, depending on the sophistication of the audience.) In this balance and otherwise the book is a model of organization and elegance and is an indispensable basis for further research.

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