Stability in nonlinear control systems. By A. M. Letov. Translated from the Russian by J. G. Adashko. Princeton University Press, Princeton, 1961. 316 pp. $8.50.

This book is an accurate and clearly printed English translation of the Russian edition of 1955 by Academician A. M. Letov. The text treats the theory of asymptotic stability of real ordinary differential systems arising in the analysis of control processes with nonlinear feedback circuits. The main techniques are the canonical forms of Luré and the direct method of Liapounov.

The introduction contains a brief but adequate statement of the basic concepts of stability. The twelve following chapters are entitled:

I. Equations of Control Systems. Statement of Stability Problem.
II. First Canonical Form of Equations of Control Systems.
III. Second and Third Canonical Forms of the Equations of Control Systems.
IV. Stability of Control Systems.
V. Formulation of Simplified Stability Criteria.
VI. Inherently Unstable Control Systems.
VII. Programmed Control.
VIII. Problem of Control Quality.
IX. Stability of Control Systems with Two Actuators.
X. Two Special Problems in the Theory of Stability of Control Systems.
XI. Stability of Unsteady Motion.
XII. Control Systems Containing Tachometric Feedback.

Consider the real linear differential system

$$\dot{\eta}_k = \sum_{\alpha=1}^{m} b_{k\alpha}\eta_\alpha + n_k\mu \qquad k = 1, \cdots, m$$

where the constant matrix $b=(b_{k\alpha})$ and vector $n=(n_k)$ describe the effect of the regulating actuator $\mu(t)$ on the regulated vector quantity $\eta(t)$. The actuator $\mu(t)$ further satisfies

$$\dot{\mu} = f(\sigma)$$

where $\sigma=\sum_{\alpha=1}^{m} p_\alpha\eta_\alpha - r\mu$ and $f(\sigma)$ is any function selected from a specified class (A). Take (A) to consist of Lipschitz functions $f(\sigma)$ on $-\infty<\sigma<\infty$ with

$$f(0) = 0, \qquad \sigma f(\sigma) > 0 \quad \text{for} \quad \sigma \neq 0.$$

[The class (A) used by Letov is somewhat more general in that a dead zone interval about $\sigma=0$ where $f(\sigma)=0$ is allowed, and also some other modifications are permitted, but I simplify the problem slightly for exposition.] The constant real $m+1$ vector $P=(p_\alpha, r)$ is called the regulator constant or parameter. We consider the problem of local asymptotic stability of $\eta=0$, $\mu=0$ for the system

$$\text{(1)} \qquad \begin{aligned} \dot{\eta} &= b\eta + n\mu, \\ \dot{\mu} &= f(p\eta - r\mu). \end{aligned}$$

For given data b, n consider the subset $B=B(b, n)$ in the real $(m+1)$-space of regulator parameters P for which the corresponding differential systems are asymptotically stable at $\eta=0$, $\mu=0$ for all functions of class (A). The main goal of this theory is to describe and to estimate the Vyshnegradskii region $B(b, n)$.

In the special case where $f(\sigma)=\sigma$ this stability analysis reduces to the classical linear theory of feedback control. The nonlinearity of $f(\sigma)$, and the emphasis on the class (A), leads to new and important results in engineering control theory.

Luré has introduced a linear transformation $(\eta, \mu)\rightarrow(x, \sigma)$ so that the differential system (1) can be replaced by

$$\text{(2)} \qquad \begin{aligned} \dot{x}_k &= -\rho_k x_k + f(\sigma) \qquad\qquad k = 1, \cdots, m, \\ \dot{\sigma} &= \sum_{k=1}^{m} p_k' x_k - r' f(\sigma). \end{aligned}$$

Here we have assumed that the matrix b has distinct eigenvalues $-\rho_k$ with negative real parts, and $r>0$ and n are not equal to certain exceptional values.

To estimate $B(\rho)$ use a Liapounov function of the form

$$V = \Phi + F + \int_0^\sigma f(\sigma) d\sigma$$

where

$$F = \sum_{i=1}^{m} \sum_{k=1}^{m} \frac{a_k a_i}{\rho_k + \rho_i} x_k x_i$$

and Φ is also a quadratic form. A typical theorem is the following:

If the equations

$$p_k' + 2\sqrt{r'}a_k + 2a_k \sum_{i=1}^{m} \frac{a_i}{\rho_k + \rho_i} = 0, \qquad k = 1, \cdots, m,$$

are solvable for a_k (satisfying appropriate reality conditions), then the regulator parameters (p', r') lie in $B(\rho)$.

The text is primarily concerned with inherently stable mechanisms (where $-\operatorname{Re} \rho_k<0$), but there is some discussion of neutral or unstable mechanisms, especially in Chapter VI. Chapters VII–XII refer to generalizations of the above theory to cases concerning the stability of a moving equilibrium or cases where more complicated feedback processes are permitted.

The style of this book is in the spirit of the engineering sciences rather than mathematics. For example the main problem of determining the set $B(\rho)$ is only mentioned casually and the general properties of $B(\rho)$ are never considered (is $B(\rho)$ open, connected, convex, bounded; and how does $B(\rho)$ vary with ρ?). Instead the text begins (on the second page of Chapter I) with the analysis of examples of electromechanical actuators (although only the mathematical and not the engineering description of the mechanism is used). From an elaboration of examples and particular cases the author proceeds to a great number of calculations and formulas which apply in various special circumstances. For instance, in Chapter II there are 193 numbered formulas.

This wealth of special cases makes the text difficult for mathematicians but valuable for theoretical engineers. At every stage the author tests his latest formula against certain standard control problems of Bulgakov and this serves to compare the power of the various methods developed. In summary, this is a valuable text for engineering stability analysis and also a book which contains many ideas that should interest mathematicians.

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The theory of Lebesgue measure and integration. By S. Hartman, and J. Mikusiński. Translated from Polish by Leo F. Boron. Pergamon, New York, 1961. 176 pp. $5.00.

This brief text offers an introduction to that part of the theory of real functions which is concerned with measure and integration. Only linear Lebesgue measure is considered, except in the last chapters, which deal with plane measure. Because of this limitation it is possible to develop the theory by elementary techniques, with only a few set-theoretical preliminaries. Essentially nothing is presupposed beyond a good knowledge of the ϵ, δ fundamentals of calculus.

Following a line of development suggested by M. Riesz, the measure of an open set is defined as the sum of the lengths of its components; a set E is measurable if for every positive ϵ it can be represented in the form $O-A$, where O is open and A is a subset of an open set of measure less than ϵ; the measure of E is the lower bound of the measures of its open supersets. On this basis the properties of Lebesgue measure are obtained rather easily. The integral is defined for bounded measurable functions on bounded measurable sets in