finite difference equations, Ritz-Galerkin variational methods, approximate conformal mapping by polynomials, alternating procedure for conformal mapping. The methods are applied mainly to two-dimensional elliptic boundary value problems. In all cases there is a remarkably complete discussion of the error arising from approximation. Many examples are worked out in full numerical detail.

The following is a brief chapter-by-chapter outline: I. INFINITE SERIES: separation of variables for the Dirichlet problem, infinitely many equations in infinitely many unknowns, solution of boundary value problems by nonorthogonal systems of functions, improvement of rate of convergence of series solutions. II. INTEGRAL EQUATIONS: replacement by algebraic equations by means of mechanical quadrature formulas, Neumann series, analytic continuation, reduction to degenerate kernels, method of moments, method of Bateman. III. DIFFERENCE EQUATIONS: interpolation in one and two dimensions, replacement of elliptic boundary value problems in one and two dimensions by difference equations, uniqueness of solution of approximating equations, convergence to solution of original problem. IV. VARIATIONAL METHODS: variational formulation of boundary value and eigenvalue problems, Ritz and Galerkin methods, proof of convergence, applications. V. CONFORMAL MAPPING: approximation by polynomials, extremal properties, perturbation methods, Melentiev's graphical method, Green's function, relation to integral equations, mapping on canonical regions, Schwarz-Christoffel method (in great detail). VI. APPLICATION OF CONFORMAL MAPPING TO BOUNDARY VALUE PROBLEMS. VII. SCHWARZ AND NEUMANN ALTERNATING METHODS: relation to integral equations, proof of convergence.

The translation by C. D. Benster is somewhat literary, but with rare exceptions accurate and clear.

WILFRED KAPLAN

Asymptotic behavior and stability problems in ordinary differential equations. By Lamberto Cesari, Ergebnisse der Mathematik und ihrer Grenzgebiete, No. 16. Berlin-Göttingen-Heidelberg, Springer, 1959, 7+271 pp. DM 68.

To write a Monograph for the Ergebnisse series has common points with writing a seed catalogue—one must tell it all—and writing a treatise one must give a consistent and clear account of the topic. It is particularly arduous when the subject is the one considered here. For in the last two generations or so it has pulled from an almost dormant status to the position of one of the most active chapters of present day mathematics. The pioneers: Poincaré, Liapunov, Birkhoff have been followed especially in the Soviet Union by a host of high level mathematicians. Two directions at least have received particular emphasis in the USSR: the problems centering around stability—the Liapunov school, and the problems of asymptotic behavior. These are the precise topics to which Cesari's monograph is dedicated.

The Editors of the Ergebnisse Series have presented the author with a well nigh impossible task: to give a resume in a rather short space of so many lively contributions, a large part in Russian! Suffice to say that the author could not have acquitted himself better of the task: his little volume, distinguished by great carefulness and precision, is the guide par excellence in the labyrinth of recent work on ordinary differential equations. An important and additional merit is that it addresses itself not only to pure mathematicians but also to applied mathematicians and even to engineers. For instance, no little space is devoted to servo mechanisms and controls. Of great value also is a most extensive bibliography covering 60 pages! The Russian references are particularly abundant—a no mean advantage in view of their scattering and of the language barrier.

It is not profitable to give a detailed account of the very rich content of this Monograph, especially since, by and large, it is what one should expect. However, one may note more especially the following less expected features—no doubt a reflection of the reviewer's own taste and curiosity—a very careful treatment of Liapunov's stability theory and of his basic theorems, likewise of his type numbers; the inversion of his stability theorems by a number of authors mostly in the USSR but with Jose Massera in the lead; an extensive account of the author on his so-called L-diagonal process, and of Cesari and associates on periodic solutions of linear and nonlinear systems; the topological contributions of Waszewski and his group; the approximation methods of van der Pol and Krylov-Bogoliubov; the recent work on second order equations by Littlewood, Cartwright, by Levinson and by Levinson-Smith; the turning point theory of Langer; the work on asymptotic series by Wasow and by Turrittin.

In brief we have here a very careful survey of the work done on ordinary differential equations in the last few decades. The monograph is an invaluable source book on these activities and it will take a good deal of time before it is superseded.

S. LEFSCHETZ

Advanced calculus. By H. K. Nickerson, D. C. Spencer and N. E. Steenrod. Princeton, D. Van Nostrand Co., Inc., 1959. pp. ix+540, lithoprinted, $6.50.