and put $\nu(E)=\inf_\alpha H(E_\alpha)$. Then $\nu(E)<2\pi^{-1}H(E)$ for any measurable E, $\nu(E)\leqq 2^{-1}H(E)$ for connected E and $\nu(E)\leqq H(E)/(\sec\alpha+2\tan\alpha+\pi-4\beta-2\alpha)$, where $\tan\beta=2^{-1}\sec\alpha$ and $2^{-1}+\sin\alpha = 4\cos^2\alpha\cdot(1+4\cos^2\alpha)^{-1}$. The proof of these three inequalities takes 37 pages.

Problem 4 is Borsuk's problem in E^3, whether a set of diameter δ is always the union of four sets of diameter less than δ. The author prefers his involved solution as possibly generalizable to $n>3$ to Grünbaum's simple solution as a "lucky fluke."

The remaining problems concern bounded closed convex sets in E^2. Denote by $\delta(X, Y)$ the area of $X\cup Y-X\cap Y$ and put $T(X, n)=\inf\delta(X, P_n)$, where P_n traverses all convex polygons with at most n sides. Problem 5 considers several functions of this type, but the principal result is the convexity of $T(X, n)$ as function of n.

The other problems are easy to describe briefly: 6. Extremal problems for convex sets solved by triangles, 7. The asymmetry of sets with constant width, 8. Sets of constant width contained in a set of given minimal width, 9. Extremal properties of circumscribed triangles, 10. The closest packing of equilateral triangles.

Clearly, this book will not appeal to some, but will be most delightful to others. In any case, it proves the strength and vitality of plane geometry and should help to revise the attitude of those numerous educators in the U.S.A. who want to all but eliminate plane geometry from high school and college curricula.

HERBERT BUSEMANN

Die Lehre von den Kettenbruchen. Vol. II. *Analytisch-funktionentheoretische Kettenbruche.* By Oskar Perron. 3d ed. Stuttgart, Teubner, 1957. 6+316 pp. DM 49.

Earlier editions of this text on continued fractions contained in a single volume a part devoted to the arithmetic theory and a part devoted to function-theoretic aspects (analytic theory). In the third edition these two parts have been published in separate volumes. Volume I was published in 1954 and the publication of the present book, Volume II, marks the completion of the third edition. Because the second edition is well known and has served as a standard reference since its appearance in 1929, only the major changes occurring in Volume II are noted below.

Chapter I includes new sections on continued fractions with prescribed approximants (§3) and on a formula of Ramanujan (§8). Chapter II contains additional sections on new convergence critieria, including the parabola theorem, (§17), and on the convergence of the Ramanujan continued fraction (§16). Several theorems on convergence of continued fractions with positive and with real elements (§11 and §12) are deleted, the statement of two theorems is improved in §14, and §15 has been completely rewritten to unify the presentation of the Van Vleck-Jensen theorems with recent extensions of these theorems. Chapter III has new sections on C-fractions (§21) and some other expansions of power series into continued fractions (§31), and on the relation to J-fractions of polynomials whose zeros have negative real parts. The discussion of periodic continued fractions (§22) has been modified to include C-fractions, and parametric representation of two continued fraction transformations has been added (§26). Chapter IV has a new section on complete convergence (§38) which permits the statement of a necessary and sufficient condition for a determinate Hamburger moment problem in §39. Several sufficient conditions for determinate Stieltjes and Hamburger moment problems are deleted from §39. Chapters V and VI have been reproduced without essential change.

The stated objective of the book is to give in an easily intelligible way the present state of knowledge of the subject. The author has been confronted with the difficult task of selecting and coordinating the material of major importance and not all readers will agree with his selections. Any defects of the book are those of omission. The reviewer regrets the omission of the methods and viewpoint of positive definite continued fractions and, in particular, positive definite J-fractions. However, the numerous virtues of the book, among which are clarity of presentation, systematic citing of origins of theorems, and the many examples and formulas, will make it a valuable reference for many years to come.

W. T. Scott

Combinatorial topology. Vol. 2. *The Betti groups*. By P. S. Aleksandrov. Trans. by Horace Komm. Rochester, Graylock Press, 1957. 11 +244 pp. $6.50.

The original Russian edition of Aleksandrov's *Kombinatornaya topologiya* (Moscow-Leningrad, OGIZ, 1947) is a single volume consisting of five parts. The English translation of the first two parts has been published as Vol. 1 (See the Review in this Bulletin, Vol. 62, 1956, pp. 629–630). The present Vol. 2 is the translation of Part III (Chaps. VII–XII), which is devoted to homology and cohomology groups of locally finite abstract cell complexes and homology groups of compact metric spaces. It deals mainly with the construction of these groups and the proof of their topological invariance.

The entire book is intended as an introduction to the classical