groups, in particular the Dickson-Chevalley finite analogues of the exceptional simple Lie groups.

MARSHALL HALL, JR.

Games and decisions: Introduction and critical survey. By R. Duncan Luce and Howard Raiffa. John Wiley and Sons, Inc., 1957. 19+509 pp. $8.75.

This is a book written by mathematicians but "aimed primarily at those readers working in the behavioral sciences." It "attempts to communicate the central ideas and results of game theory and related decision making models unencumbered by their technical mathematical details: thus, for example almost no proofs are included." Despite these remarks from the authors' introduction, the book deserves the attention of professional mathematicians, behavioral scientists or anyone else interested in finding out about the subject matter of its title. Of these various groups I strongly suspect it is the mathematicians who will most readily understand its contents.

The book is unique in many respects. It is the first on the subject which attempts to cover the whole field, a feat it succeeds in doing with almost astonishing comprehensiveness. The authors have read, digested and present here in a lucid manner virtually every idea on games and decisions that has been put forth since these objects became the subject matter of a "theory." Even the game theory specialist will in all likelihood find branches treated here with which he is not familiar (this reviewer is grateful to the authors for the sections on statistical decision making).

The material of the book is divided up according to the following scheme: a general introductory chapter on games, a chapter on utility theory and then another more technical chapter on games, describing the extensive and normal forms. There follow three chapters on two-person games, the first on the, dare we call it, "classical" zero-sum theory, the next two on nonzero sum games, first the non-cooperative theory, then the cooperative. Already we note a sharp contrast with other books which have appeared since von Neumann and Morgenstern, and which have devoted all or almost all their attention to the zero-sum theory. The present authors hasten on to more unsettled and controversial parts of the theory which constitute their main object of study. Nevertheless, for the sake of completeness, the book includes no less than seven appendices to the two-person zero-sum chapter, covering such related topics as linear programming, infinite and sequential games among others. The next five chapters are devoted to n-person games and a presentation of the many concepts and theories which have been proposed for them. After this comes a chapter dealing with applications of the theory. The final two chapters are entitled *Individual decision making under uncertainty*, and *Group decision making.*

The book has novelty as well as comprehensiveness. At a conservative guess, half of the material it contains has not previously appeared in book form. Game theory is a rather diffuse subject, and the authors have done an admirable job of bringing under one roof many different ideas, mostly from mathematics and economics periodicals, and giving to this diversified material at least the semblance of unity. One of their unifying devices is consistent use of two approaches to the theories, the constructive approach: "Here is a theory. Let us investigate its properties," and the axiomatic approach: "Here are some desirable properties. Let us find a theory that has them."

We have described the book so far in its role as a survey of the subject, but this is only half of the story. Its subtitle reads: "Introduction and *Critical* Survey" (our italics) and accordingly, the book is about equally divided between presentation of material and critical analysis of it. Say the authors: "Our critical discussion and our examples are strongly colored—at least as far as a mathematician is concerned—by a social science point of view." Thus, along with straight information the reader will also be given the benefit of the authors' thinking on each subject. We shall comment briefly on this second aspect.

Game theory occupies a curious position among mathematical theories in that it is not intended to explain anything. In the language of the authors, ". . . we feel that it is crucial that the social scientist recognize that game theory is not *descriptive* but rather (conditionally) *normative.* It states neither how people do behave nor how they should behave in an absolute sense [I should hope not!] but how they should behave if they wish to achieve certain ends." Thus, by the authors' own statement, experimental verification is not to be expected. What then is to be one's basis in accepting or rejecting a proposed theory? The answer can only be that in the last analysis this is a subjective matter in which each person must decide for himself whether his Platonic ideal of a completely rational decision maker would or would not behave in accordance with a particular theory. The authors have given these matters considerable thought and their opinions are worth listening to. Nevertheless, as one theory after another is introduced and then taken to pieces, the reader may experience a certain sense of futility. As an illustration, Chapter 10 is entitled "ψ-stability" and concerns an interesting notion introduced by one of the authors to handle n-person games. After describing the theory in the first two pages, the authors proceed to demolish it with apparent relish for eight pages. There follows a section treating a special example which it turns out is introduced as a further illustration of the weakness of the theory. The positive results consisting of theorems which have been proved are included in fine print.

Actually it is not true that all of game theory comes down to "philosophical questions." There are instances, though admittedly few, where it is generally agreed that game theory gives the "right" answers. Perhaps the best illustration is the manner in which it settles the theory of bluffing as treated by von Neumann and Morgenstern and many others. This is a very special situation, true, and old stuff by now. Still, wouldn't it be a kindness to the reader to present at least one case in which the right theory applied to the right situation successfully explains a certain kind of behavior? Isn't this, after all, precisely the sort of thing the theory formulators are shooting at? Here, it would seem, is cause for celebration, yet this is one of the few points on which the book is silent. At times one almost feels that the main purpose of the theories is not to solve problems, but to provide grist for the critical mill, and that the construction and destruction of game theories has itself become a sort of super game. Game theory is at best a debatable tool for the solution of behavioral problems. Because of their zest for criticism the authors may have made it look even weaker than it really is.

We have a few lesser complaints. The book is at times unnecessarily verbose. As an illustration, in the chapter on two-person zero-sum games a whole section is devoted to *Compatibility of the pure and mixed strategy theories*. Surely any scientist, no matter how behavioral, would be satisfied with a few sentences to the effect that since a pure strategy may be considered as a special kind of mixed strategy, the latter theory includes the former. The trouble with belaboring the obvious is not only that it exhausts the reader, but makes it hard for him to determine which are the truly subtle points. Secondly, while the book contains a fair number of examples, I believe the authors could have been still more generous in this regard. For example, in presenting a half dozen or so different theories of cooperative two-person games, it would seem natural to take some single specific game and compare its solutions as prescribed by each of the theories. This, for some reason, is not done. Finally, in a book of this sort one expects some general concluding remarks. I was disappointed in not finding any discussion of such annoying but natural questions as, "have we gotten anywhere?", and "where is all this leading?". After presenting so much criticism on special topics, the authors really owe the reader some sort of general tying together of the pieces.

It is customary in these reviews to give a list of typographical errors as evidence of the conscientiousness of the reviewer. Let it be reported therefore that on page 315, line 2, the authors inadvertently toss a coil instead of a coin.

For our own concluding remark we return to our initial observation. The authors have made available and understandable a great mass of interesting material in a new field, an achievement that will surely earn them the admiration and gratitude of all who read their book.

DAVID GALE

Variational methods for eigenvalue problems. By S. H. Gould. Mathematical Expositions, no. 10, University of Toronto Press, Toronto, 1957. 14+179 pp. $8.75.

Until the publication of this book there had been no monograph at all on the study of variational eigenvalue problems, and it has always been hard for the student to get a suitable introduction to the subject. Hitherto, he has had to rely on expositions having the character of an engineering textbook and these have always been mathematically unsatisfactory.

In conformity with the other books of the Toronto series, the author does not presuppose much mathematical knowledge on the part of the reader and gives lengthy and detailed explanations whenevery they seem desirable. The nature of his subject, however, compels him to make use of the methods and fundamental concepts of Lebesgue integration and of the theory of linear transformations in Hilbert space, and it is therefore necessary for him to develop parts of these theories in very little space. These brief developments are satisfactory for the purpose of the book, but are of necessity incomplete.

In the introductory chapters, the author considers eigenvalue problems in finite dimensional space using such problems as that of the vibrating string as examples. The equivalence between the eigenvalue problem and the variational problem is carefully established and the Monotony principle on which the Rayleigh-Ritz and Weinstein methods depend is explained in detail. It is now necessary to develop the theory of Lebesgue integration and to construct the space L^2. The classical eigenvalue problems for membranes, plates, and rods are considered in a further chapter and another describes most of the Weinstein method in its original form as applied to the problem of the