RESEARCH PROBLEMS

The department of RESEARCH PROBLEMS will publish the statements of problems whose solution would make a significant contribution to mathematical research. Problems which are suitable for publication in the problem department of the American Mathematical Monthly will not be accepted for publication in RESEARCH PROBLEMS. Only problems whose solutions are unknown to the author should be submitted. Furthermore, the problems desired are those for which the solution will take the form of a research paper to be accepted on its merits and published in a research journal; since the BULLETIN does not accept contributed papers, it will not publish the solutions of its research problems. An attempt will be made, however, to publish references to papers which contain solutions.

The readers of the BULLETIN are invited to contribute problems to the department of RESEARCH PROBLEMS. Each problem should carry the name of the author and a brief title and should be written in a single paragraph in a form similar to an abstract, and in nontechnical language if possible. Relevant references should be included. All problems intended for publication should be sent to G. B. Price.

1. Einar Hille: *On the zeros of a certain class of Fourier transforms.*

In the theory of analytic continuation in the meromorphic star developed by H. von Koch in Arkiv för Matematik, Astronomi och Fysik vol. 12, no. 23 (1917) he encountered the integrals

$$f_n(x) = \int_0^\infty \frac{t^n}{\Gamma(t)} e^{itx} dt.$$

For his theory it was essential to show that $f_n(\pi) \neq 0$ for $n = 0, 1, 2, \cdots$, but this he was unable to do in spite of several efforts in which his pupils also shared. Prove or disprove this conjecture as well as the more general one that $f_n(x) \neq 0$ for real values of x. (Received October 8, 1953.)

2. R. E. Johnson: *Quotient rings.*

If R is a subring of the ring S having the property that $aR \cap R \neq 0$ for each nonzero $a \in S$, then S is called a (right) *quotient ring* of R. The set $Q(R)$ of all quotient rings of R has maximal elements by Zorn's lemma. *Question* 1. If $S \in Q(R)$, is $Q(S) \subseteq Q(R)$? *Question* 2. Are the maximal elements of $Q(R)$ isomorphic to each other? If the answer in either case is no, then it would be interesting to know for what types of rings the answer is yes. A general theory of quotient rings would make a significant contribution to the structure theory of rings. A few results on quotient rings are to be found in Proc. Amer. Math. Soc. vol. 2 (1951) p. 895 and in Duke Math. J. vol. 18 (1951) p. 808. (Received October 12, 1953.)

3. R. P. Boas, Jr.: *The absolute value of an absolutely convergent Fourier series.*

If $f(x)$ has an absolutely convergent Fourier series, does $|f(x)|$ have an absolutely convergent Fourier series? A negative answer would supply a simple example of a phenomenon discovered by Marcinkiewicz [Mathematica, Cluj vol. 16 (1940) pp. 66–73]: that there exist functions f and g with absolutely convergent Fourier series such that $g(f)$ does not have an absolutely convergent Fourier series. (Received October 16, 1953.)

4. R. P. Boas, Jr.: *The limit of the absolute value of an entire function.*

Let $f(z)$ be an entire function of exponential type. It is well known, and easily proved by Phragmén-Lindelöf arguments, that if $f(x)$ is bounded or approaches a limit as $x\to\infty$, then $f(x+iy)$ is bounded or approaches a limit, for each y, as $x\to\infty$. If $|f(x)|$ approaches a nonzero limit, does $|f(x+iy)|$ necessarily approach a limit? (Received October 16, 1953.)

5. R. P. Boas, Jr.: *Power series with positive coefficients.*

If $f(x)=\sum_{n=0}^{\infty}a_nz^n$ has $|z|=1$ as its circle of convergence, and $a_n\geqq 0$, then in a general way the behavior of $f(z)$ on an arc of the circle near $z=1$ governs the behavior of the function on the whole circle. If $f(z)$ has boundary values $f(e^{i\theta})$ belonging to L^2 on a small arc $|\theta|<\epsilon$, N. Wiener has shown (unpublished) that boundary values exist for all θ and $f(z)$ belongs to L^2 on the whole circle. Does this theorem extend to L^p for other values of p, and in particular for $p=1$? (Received October 16, 1953.)

6. R. M. Thrall: *Simultaneous congruence of skew symmetric matrices over a finite field.*

Let F be a finite field, and let A, B, C, D be skew symmetric F-matrices of degree n. Find necessary and sufficient conditions for the existence of a matrix P such that $PAP^{tr}=C$, $PBP^{tr}=D$. This problem arises in the study of metabelian groups. (Received October 19, 1953.)

7. R. M. Thrall: *Multiplication of Schur functions.*

In the theory of group characters, certain symmetric functions called *Schur functions* are important [Cf. D. E. Littlewood, *Theory of group characters*, Oxford, 1940, p. 82 ff.]. Each partition $\lambda:\lambda_1+\cdots+\lambda_n=n$, $\lambda_1\geqq\lambda_2\geqq\cdots\geqq\lambda_n\geqq 0$, of n determines a homogeneous symmetric function $\{\lambda\}$ of degree n. If μ is a partition of m, then

$$\{\lambda\}\{\mu\} = \sum k(\lambda, \mu, v)\{v\}$$

where the sum is over all partitions v of $m+n$. Find an analytic formula for the coefficients $k(\lambda, \mu, v)$. Combinatorial rules are known for the determination of these coefficients. (Received October 19, 1953.)